A novel semi-quantitative Fuzzy Cognitive Map model for complex systems for addressing challenging participatory real life problems

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\textbf{A B S T R A C T}

Fuzzy Cognitive Maps (FCM) are a promising approach for socio-ecological systems modelling. FCMs represent problem knowledge extracted from different stakeholders in the form of connected factors/variables with imprecise cause-effect relationships and many feedback loops. These typically large maps are condensed and aggregated to obtain a summary view of the system. However, representation, condensation and aggregation of previous FCM models are qualitative due to lack of appropriate quantitative methods. This study tackles these drawbacks by developing a semi-quantitative FCM model consisting of robust methods for adequately and accurately representing and manipulating imprecise data describing a complex problem involving stakeholders for pragmatic decision making. The model starts with collecting qualitative imprecise data from relevant stakeholders. These data are then transformed into stakeholder perceptions/FCMs with different causal relationship formats (linguistic or numeric) which the proposed model then represents in a unified format using a 2-tuple fuzzy linguistic representation model which allows combining imprecise linguistic and numeric values with different granularity and/or semantic without loss of information. The proposed model then condenses large FCMs using a semi-quantitative method that allows multi-level condensation. In each level of condensation, groups of similar variables are subjectively condensed and the corresponding imprecise connections are computationally condensed using robust calculations involving credibility weights assigned to variables (variables' importance). The model then uses a quantitative fuzzy method to aggregate perceptions/FCMs into a stakeholder group or social perception/FCM based on the 2-tuple model and credibility weights assigned to FCMs (stakeholders' importance). Thereafter, the structure of produced FCMs is analysed using graph theory indices to examine differences in perceptions between stakeholders or groups. Finally, the model applies various what-if policy scenario simulations on group FCMs using a dynamical systems approach with neural networks and analyses scenario outcomes to provide appropriate recommendations to decision makers. An example application illustrates method's effectiveness and usefulness.

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1. Introduction

Soft computing approach, coined by Zadeh in (1994) [62], has been proposed to deal with reasoning and approximation and exploit the tolerance for imprecision and uncertainty of complex problems. Zadeh (1994) also pointed to the key techniques of soft computing: fuzzy logic, neural network and probabilistic reasoning, with emphasis on fuzzy logic technique for approximate decision making. The last three decades have witnessed the growth of several soft computing approaches such as intelligent systems and expert systems proposed to address real life ecological problems. For example, Yamada et al. [58] used expert systems for eliciting expert knowledge for wildlife habitat modelling. This study provided promising results for wildlife habitat management. However, the authors found out that the elicitation of knowledge base was complicated and very time consuming. Another example of addressing ecological problems using intelligent systems was in Chen and Liu [9]. In this study, artificial neural network and multilinear regression models were developed for modelling water quality in reservoirs. It provided valuable information that could help decision-makers in managing reservoir water quality. However, this study requires many complex calculations and large amount of data for the water quality variables needed for model...
simulation. Moreover, it lacks an effective method for the determination of sufficient variables needed to deal with the complexity and dynamicity of the problem.

Many real-life problems such as ecosystems management are qualitative, complex and dynamic such that their knowledge can only be represented in the form of relevant factors linked to each other through imprecise and uncertain cause-effect relationships and many feedback loops. Fuzzy Cognitive Map (FCM) presented by Kosko [31] is one such directed graph that represents domain knowledge on a map by way of nodes and imprecise directed connections between them to represent the factors and relationships, respectively. FCM incorporates fuzzy logic to deal with uncertain connections and allows feedback loops to appropriately represent system dynamics. Compared to various soft computing approaches such as intelligent systems and expert systems, FCM model is simpler and quicker to extract knowledge from diverse sources and different levels of stakeholder experience at the lowest cost. This is because FCM model can represent and deal with the knowledge of various complex problems in an intuitive, appropriate and flexible way that other intelligent systems cannot. It represents knowledge as factors and relationships, rather than using complicated IF/THEN rules as expert systems. It has the ability to deal with ill-defined variables and relationships. It is also useful if detailed scientific data or knowledge are lacking or insufficient but local knowledge of people is available [43] thus making them highly valuable for such domains in a way that neural networks are not. Further, FCM model can combine diverse knowledge sources (individual FCMs) into a Group map that represents overall knowledge. Additionally, it allows simplification of a large complex map into a simpler map so that the important processes of the domain become easily understandable, traceable and predictable. Finally, FCM model can be treated as a dynamical system that helps making inferences from system outcomes using simple operations and scenario simulations instead of using complex and costly calculations as in most other applicable soft computing and quantitative methods.

Fig. 1 shows an example FCM from a water scarcity problem showing a number of factors (water resources, demand, wastage and economic, legal, technological factors etc.) affecting the water scarcity situation and their relationships (positive, negative and feedback). Feedback loops in an FCM mean that the effect of change in one node on other nodes can, in turn, affect the node initiating the change. FCM with feedback therefore is a dynamical system and it can be used not only to understand systems properties and behaviour but also to conduct what-if policy scenarios simulations that can be the basis for recommending effective policies for improved management of the system concerned.

FCM model is a useful approach to model and reflect the reality of complex ecological or environmental problems involving humans. In Papageorgiou et al. [44,45], the researchers developed FCM models using ‘expert’ knowledge to capture the effective factors and causal relationships in agriculture applications. These models could assist decision makers in agricultural yield management. However, the drawback is that these FCM models only capture experts’ knowledge which could narrowly represent the knowledge of complex domains. For example, addressing complex ecological problems could involve many relevant stakeholders with varying levels of knowledge and perceptions that often exist in an environment of conflict. Perceptions (point of view) of each ‘expert’ stakeholder could be represented by an individual FCM as in Fig. 1 but the development of FCM models based solely on experts cannot be completely representative of the entire domain knowledge. In such cases, FCM model can benefit from local people’s knowledge to extend or complement experts’ knowledge in knowledge extraction [43]. Several studies have addressed the limitation of the dependency on experts’ knowledge alone in developing FCM models in ecological problems. Kafetzis et al. [19] combined the perceptions of farmers, local residents and government officials to develop a social FCM model to analyse conflicts of stakeholder views on water problems [19]. Further, owing to the diversity of opinions of the stakeholders, the individual and aggregated perceptions/FCMs are often large and complex. Such large FCM needs to be simplified (condensed) before or after aggregation in order to be understood, analysed and later simulated in search of effective policy options that could alleviate the ecological problem concerned. Ozesmi and Ozesmi [43] suggested a novel multi-step FCM model for ecological problems. This model can extract different types of knowledge from experts as well as from lay experts, combines the perceptions of different stakeholder groups and allows simplification/condensation of large FCMs. The approach has proven to be useful and gained good results in addressing a number of environmental problems ([61,54]).

Despite its capabilities in modelling ecosystems management, currently FCM model has a number of limitations and building a robust and comprehensive model is still a challenge. Most important challenges can be summarised as follows. The participants, especially humans, would like to express their knowledge in a way that is natural to them rather than in ways that are artificial or enforced by the researcher (system developer). FCM model lacks satisfactory mathematical methods to represent and deal with different imprecise numeric values and/or linguistic expressions of various perceptions that are more likely to arise due to different levels of stakeholder experiences. When an FCM model deals with different perceptions of various participants and stakeholders, there is a need to combine these perceptions in order to obtain a group or social perception [33,42,43]. FCM model lacks a fuzzy persuasive method to aggregate conflicting and shared perceptions and also suffers from the inability to assign acceptable credibility weights to these perceptions during the aggregation process in order to successfully attain a more realistic group consensus. After aggregating FCMs into a group or social FCM, the result of the aggregation process is often a complex map and it is very difficult to gain insights and understanding from it. Even individual stakeholder FCMs may be large and complex in a large problem domain. In such cases, there is a need to simplify a complex map into a simpler one. A qualitative condensation method has been previously used to combine, subjectively, the variables into a higher level of abstraction [43]. This qualitative condensation lacks a suitable approach to multi-level condensation as well as adequate representation and calculation of the connections resulting from the condensation process. Finally, FCM model lacks sound objective criteria for evaluating the simulated maps for choosing the most effective policies.

The above discussion makes the case that FCM representation and calculations involved in the condensation and aggregation processes as well as the evaluation of its outcomes should be carried out using appropriate methods that can deal with imprecise or fuzzy values, avoid loss of information, reflect process validity, and lead to accurate outcomes. Therefore, improvements are needed in approaches to all aspects of FCM ranging from representation, development, synthesis, condensation, simulation and evaluation. The main goal of this article is to develop an advanced FCM model that improves FCM representation, development, synthesis, condensation and evaluation in a systemic way to provide a robust semi-quantitative FCM model for addressing and modelling complex real-life problems. The main contributions of the article can be summarised as follows. It uses an appropriate fuzzy representation method to deal with imprecise numeric values as well as linguistic expressions, individually or together. It also benefits from various intelligent systems, especially graph theory and fuzzy logic approaches, in handling imprecise data during calculations. In terms of FCM aggregation process, this article enhances this process by, firstly, using an appropriate fuzzy aggregation method, and
secondly, proposing a new method for calculating the credibility weights for variables and FCMs and then using credibility weights of FCMs in the aggregation process. In terms of FCM condensation, this article replaces the qualitative or subjective condensation method with a semi-quantitative or subjective-objective condensation method and uses more than one level of condensation. The subjective part of the proposed condensation method is identification of the condensed variables in the higher levels and the objective part is in using suitable calculation methods for calculating new connection weights and credibility weights for variables at each level of condensation. Furthermore, the study proposes an insightful, objective and effective multiple criteria evaluation approach for assessing FCM simulation outcomes to select the most appropriate policies. These improvements are expected to make the whole FCM development process robust leading to effective strategy/policy development from map simulations.

This article is structured as follows: Section 1 provides an introduction to socio-ecological systems, challenges to their modelling and the important role models play in our understanding and managing these systems. It summarises current modelling approaches and their limitations, introduces FCM as an alternative and highlights the attractive properties of FCM for modelling socio-ecological systems relative to other soft computing and other approaches. Section 2 gives a brief introduction and background to FCM highlighting specific limitations and issues in current FCM models and formulates the research problem. Section 3 provides an overview of the methods and concepts used to develop the proposed advanced FCM model based on a fuzzy 2-tuple method, graph theory and social network theory to address the current limitations and deliver the aforementioned contributions. Section 4 presents the whole model. Finally, Section 5 presents an application of the developed model to a real life problem involving ‘Mitigating Groundwater Degradation and Depletion in Jordan’ before we conclude the article and suggest some future work.

2. Fuzzy Cognitive Maps (FCM)

FCM is a hybrid soft computing approach incorporating properties of fuzzy logic, nonlinear systems dynamics models and neural networks techniques [31]. As stated before, the structure of FCM is a directed graph with feedback loops. This structure gives FCM approach significant capability to represent domain knowledge of qualitative complex problems, capture their nonlinear systems dynamics, and help generate potential solutions to domain problems. Human perceptions are rife with uncertainty, imprecision and incompleteness that necessitate a cognitive model to deal with these data. FCM with fuzzy weights has the ability to deal with uncertain and imprecise data [33,35], depict the knowledge from human perceptions and represent this knowledge in a symbolic manner. The significance of the FCM model lies in representing the factors and relationships of the problem in a fuzzy (imprecise) way. In other words, FCM consists of nodes, also known as variables or concepts, and imprecise causal connections, also known as cause-effect relationships among nodes (see Fig. 1). The nodes represent the related causal factors that describe the states, behaviours or characteristics of the domain knowledge. These factors correspond to variables, concepts, events, input, output, trends and other characteristics of the domain knowledge being modelled [12]. The imprecise connections among nodes represent their cause and effect relationships. A connection between two nodes has a sign, magnitude and arrow to express type, strength and direction of the connection, respectively. The experts/designers draw their FCMs to represent a variety of domain problems. This process is called FCM development or domain knowledge representation.

Due to the flexibility and ability of FCM to represent human knowledge, FCM has been used for modelling numerous different ill-structured domains such as: political decisions and developments [53,56], socio-ecological systems [14,17,21,26,42,43,54], socio-economical systems [10], agriculture [27,44], medical decision-making [1,23,52], catastrophic natural events, e.g., earthquakes [50,51], and water and drought management [17–19]. FCM model can represent the knowledge of various complex problems and provide pragmatic solutions in a way that other intelligent systems cannot. It can also play a significant role in helping decision-makers devise appropriate strategies through simulating complex dynamics of the system, particularly if the system consists of different participants or stakeholders. Importantly, FCM model can combine perceptions of different stakeholders into a Group stakeholder map that represents overall Group perception. This process is called FCM combination or aggregation. The aggregation process, usually, produces a large complex FCM with a large number of nodes and connections. FCM model allows simplification of the large complex FCM into a simpler one with a small number of nodes and connections [43]. This process is called FCM

![Fig. 1. An example FCM depicting the perception of a stakeholder on a Water Scarcity Problem. For clarity, the weights of connections are not shown on the graph; solid and dashed arrows indicate negative and positive influences, respectively.](image-url)
condensation. Furthermore, FCM model can utilize graph theory indices and/or statistical methods to analyse the structure of the FCM [42,43]. This process is called statistical analysis of the structure of the FCM. FCM model can reveal the effects of nodes on each other and the whole system by simulating the FCM as a dynamical system based on known initial state of nodes in the system to find their final state. This process is called FCM inference. Finally, it can perform different scenario (policy) simulations using "what if" questions to see what state the model will reach if the initial state of some specific nodes are altered [33] in order to find more desirable outcomes for the system concerned to help in decision making. Considering the versatility and flexibility of FCM, there is a great scope for and advantages in advancing FCMS in all its aspects described above. The specific aspects advanced in this research are addressed in the next sub-sections.

2.1. Complete representation of the forms and range of perceptions: enhancing the versatility of FCMS

FCM is a network of concepts and their relations (weights). In FCMS for complex real-life participatory problems, elicited relations between concepts can be a mix of positive or negative linguistic statements and numeric values depending on the participants’ level of understanding and skills. Therefore, representing all such relations in a uniform format greatly aids and simplifies any subsequent FCM processing, be it condensation, aggregation or simulation. Currently, methods to unify weights presented in a mix of linguistic statements and numeric values in FCMS are lacking. Fuzzy 2-tuple model has been proposed to represent symbolic or/and numeric values by a pair of values called 2-tuple—a fuzzy set and a symbolic translation [25], which can be useful here (the details of this model are presented in Section 3). However, the current 2-tuple format deals only with positive linguistic information in a linguistic term set. Therefore, one of the main objectives of this article is to propose enhancements to this model to accept negative symbolic or numeric information while keeping model functions operating as they are. These enhancements do not change model’s purpose or conflict with its definitions, propositions and rules. Importantly, we make use of the proposed enhanced 2-tuple representation to conduct all FCM processing – from map representation to aggregation – in a unified and robust manner.

2.2. Finding key players in the system: important nodes in an FCM that control information flow

Not all nodes in an FCM have equal value; there can be highly influential nodes that control information flow and therefore, define the final state reached by the system. Understanding these nodes cannot only be useful in understanding the structure of an FCM but also in affecting changes to move the system to a more desirable state, such as water abundance in the case of water scarcity. In the past, only a local centrality measure, degree centrality, has been used to identify important/central nodes [31,43]. This measure is based on the connectivity of a node to other nodes in the system and highly connected nodes are assumed to be central. However, high connectivity (local measure) alone may not accurately identify important/central nodes. Global measures such as how quickly a node reaches or is reached by other nodes (Closeness measure) and how often information flows through a node (Betweenness measure) are likely to play an influential role here. This article provides a novel method to derive a new centrality measure, called “Consensus Centrality Measure” (CCM) [40] to define a node’s centrality in its FCM based on the local and two global centrality measures mentioned above. Additionally, this article defines a CCM for each FCM (in a group of FCMS) that helps identify the influence of FCMS (stakeholders) on system outcomes.

2.3. Representing whole community perception: FCM aggregation

Large applied problems, such as socio-ecological problems, constitute different perceptions from various relevant stakeholders/participants. Thus, it is necessary to integrate varied types of knowledge (i.e., from lay experts to experts) and different levels of perception (i.e., from local people to high level managers) from different stakeholders [43]. Such participatory knowledge would organize the complexity of a problem [21], and then lead to the encapsulation of a successful, flexible and adaptable system [42]. FCM is an effective and useful approach for handling such problems. FCM allows any number of participants to take part and each participant can build a different FCM with different variables and connections. Each participant can easily identify the relevant variables and connections among them, but identifying the relevant strengths of the connections, e.g., accurate magnitude of strength, is a challenge. This is compounded by the fact that the perceptions of individual stakeholders are generally dominated by their knowledge and preferences.

That notwithstanding, a large sample size of FCMS or perceptions leads to finding out the relevant variables and connections among variables with more certainty [37], reducing additional new variables, stable connection strength values as well as high reliability in the combined knowledge [33], and a better representation of the problem [13]. A collection of different perceptions is particularly useful for decision making as it encapsulates a wide range of skills, interests and knowledge representation [55], and it leads to useful information [39]. Therefore, to improve the accuracy of outcomes and obtain a reliable and acceptable consensus representation of knowledge from several individual stakeholders, the individual perceptions (FCMs) are aggregated into a group or combined FCMS [32,34].

Fig. 2 shows a simple example of graphical representation of FCM aggregation. It shows two FCMS in relation to a water scarcity problem, each described by a member of the stakeholder groups of experts and local people, respectively, with imprecise numeric and linguistic values. They reveal how the two stakeholders view water scarcity situation as being affected by factors such as water resources, technology, water demand and wastage of water etc. with only some variables overlapping between the two. Fig. 2 also shows that the group FCM resulting from the aggregation of the two maps includes all distinct variables in the individual FCMS and all possible connections among them. Different methods have been used to combine individual FCMS into a group FCM and define the values of the new connections.

The most common method that has been used for aggregating individual FCMS into a combined one is additive superimposition method [12,20,33,43,50]. In this method, each adjacency matrix $W_k$, of individual FCM connection strengths or weights is augmented by including all distinct variables from all individual FCMS. Every new variable in the augmented matrix of an FCM is encoded by zeros in its column and row. Then all augmented matrices are added together to form a combined or group FCM connection matrix as:

$$W = \sum_{k=1}^{M} W_k$$

where, $W$ is the group FCM adjacency matrix (or FCM matrix), $M$ is the number of augmented FCMS, and $W_k$ is the augmented matrix of $k$th FCM.
The group FCM could be normalized to present its connection weights \( w_{ij} \) in the interval \([-1, 1]\). This could be done by averaging the group FCM to produce a group mean FCM matrix \([35]\) as

\[
W = \frac{1}{M} \sum_{k=1}^{M} W_k
\]

(2)

where, \( W \) is the group mean FCM matrix.

This method is easy and requires simple mathematical calculations. It also allows embedding any new FCM easily by adding its augmented matrix to the collection of FCMs. However, the main drawback of this method is that such combination of the causal connections from multiple individuals or experts may incorrectly reflect either agreement or disagreement of the individual FCMs. This is called conflicting causal connection. For example, when connection between two variables has opposite signs (disagreement) in two different maps, this will lead to weakening the combined connection or cancelling each other which is not a true representation of the situation. On the other hand, the agreement of connection between two variables will lead to reinforcing the combined connections.

An appropriate way to combine conflicting knowledge of group participants is to involve their credibility weights in the combination \([32]\). Each participant is assigned a non-negative credibility weight \( b_i \) in the range \([0, 1]\) \([33,53]\) and a weighted Group FCM matrix \( W \) is defined by

\[
W = \sum_{k=1}^{M} b_k \times W_k
\]

(3)

where \( b_i \) is the credibility weight of \( k \)th participant (the developer of \( k \)th FCM), and \( b_1 + b_2 + \ldots + b_M = 1 \).

Engaging credibility weights in the combination process gives a better representation of different levels of experience of participants in extracting knowledge. It also produces a weighted FCM that can better reflect the conflicts in the knowledge of the participants \([33]\). However, a suitable approach to obtain realistic credibility weights is lacking. In this article, we present an objective method to determine FCM credibility weights and then use these weights in FCM aggregation to obtain weighted Group FCM. Proposed credibility weights are derived from the Consensus Centrality Measure (CCM) described previously.

2.4. Simplifying knowledge network: FCM condensation

When an FCM includes a large number of nodes, it becomes complex making it harder to understand its structure and gain insight from it. The ideal number of nodes in an FCM is around 12 \([6]\), and then on, the more the number of nodes, the more complex FCM is. A large number of nodes might be identified in an individual FCM. After combining the individual FCMs into a group FCM, even larger number of nodes and interconnections result in the group FCM. Therefore, there is a need to simplify the complex FCM into a smaller FCM with fewer nodes and connections. This process is called condensation. The condensed FCM is more understandable, traceable and predictable than the larger one, and thereby leads to improved outcomes. There are two types of condensation: qualitative and quantitative condensation.

In the qualitative condensation approach, as has been used in Refs. \([39,43,50,51,59]\), the experts or developers of the system con-
cerned use their knowledge and judgement to group, in a subjective way, similar or related variables into high level categories (groups) [23, 49]. Thereafter, the connections among the variables are also condensed and then transferred to new connections among the groups. Ozesmi and Ozesmi [43] suggested the cognitive interpretation diagram (CID) to identify connections among the groups and their signs and magnitudes. Another suggestion for identifying the weights of connections among groups was presented in Young et al. [59]. In the latter article, the researchers converted the connection weights between variables in two groups into linguistic (fuzzy) variables and summed them into an overall linguistic value, and then defuzzified this linguistic value into a numerical value in the interval [−1, 1] that became the new connection weight between the two groups of variables.

In the quantitative aggregation, similar nodes are objectively aggregated into a single unit [23]. Here, the map which is a digraph is divided into sub graphs. Each sub graph encompasses a group of strongly connected variables. Then each sub graph is replaced by a single unit and the connections between sub graphs are maintained. Another quantitative approach used by Samarasinge and Strickert [47] to aggregate FCM concepts is Self-Organising Map (SOM) that assembles nodes with similar connection weight vectors based on a distance metric. Both of the above approaches require a further step to condense original weights between variables into new weights between the discovered groups. In all previous condensation methods, there are limitations; especially, in the way new weights between the groups are determined. There is information loss in the transfer of weights from the node to group level. In this article, we propose a robust semi-quantitative FCM condensation method. This method can use more than one level of condensation and at each level group of similar variables are subjectively condensed. In particular, their corresponding connections are condensed using robust calculations in such a way that information at the lower (node) level is preserved in the higher (group) level. In the next section, methods used to accomplish the proposed advancements are presented.

3. Materials and methods

The main aim of this article is to propose a robust semi-quantitative FCM model. This section reviews the effective and robust methods and models that contribute to the development of this proposed model. It, first, discusses the concept of the 2-tuple fuzzy linguistic representation model and its efficiency in representation and computation of imprecise numeric and fuzzy values in a unified format. Then, it gives an overview of some social network measures and discusses how these measures can be exploited to develop a new significant measure, Consensus Centrality Measure (CCM), to be used in various aspects of FCM processing including the determination of credibility weight for concepts as well as FCMs to allow realistic incorporation of diverse perceptions.

3.1. 2-Tuple fuzzy linguistic representation method

One of the important objectives of this article is to extend a 2-tuple fuzzy linguistic approach [25] for representing linguistic and numeric imprecise connections uniformly in a novel fuzzy way that can be effectively used in various subsequent computational processes of FCMs. The proposed model can represent and deal with imprecise values without any loss of information, and it keeps the consistency of these values throughout any subsequent computational processes. Specifically, it represents each fuzzy value (linguistic or numeric) by a pair of symbolic values called a 2-tuple. It can combine linguistic and numeric imprecise or fuzzy values with different granularity and/or semantic and retain their fuzzy representation during the combination process.

The 2-tuple fuzzy linguistic representation model is based on the concept of a symbolic translation [25]. It takes a symbolic linguistic model proposed in Delgado et al. [11] as the basis for representing the symbolic translation. The symbolic model represents linguistic information by an ordered linguistic term set, \( S = \{s_0, \ldots, s_g\} \), where \( g + 1 \) is the number of linguistic terms (or fuzzy sets) in \( S \) (cardinality). It uses the ordered structure of the set (labelled index \( i = 0, \ldots, g \)) to perform the calculations.

For example, a linguistic term set \( S \) with seven terms could be given as: \( S = \{s_0 = \text{‘extremely low’}, s_1 = \text{‘very low’}, s_2 = \text{‘low’}, s_3 = \text{‘medium’}, s_4 = \text{‘high’}, s_5 = \text{‘very high’}, s_6 = \text{‘extremely high’}\} \). The following definitions apply to linguistic term set \( S \).

**Definition 1.** Let \( S = \{s_0, \ldots, s_g\} \) be a linguistic term set and \( \beta \in [0, g] \) the numerical result of a linguistic symbolic aggregation of labelled indices of linguistic terms \( i \) in the linguistic term set \( S \), i.e., the result of a symbolic aggregation operator (see Eqs. (8) and (9) for more clarity). Let \( i = \text{round}(\beta) \), where \( \text{round}(\cdot) \) is the mathematical rounding operation, and \( \alpha = \beta - i \) be two values, such that \( i \in [0, g] \) and \( \alpha \in [−0.5, 0.5] \); then \( \alpha \) is called a symbolic translation [25].

From Definition 1, \( \alpha \) value represents the difference between \( \beta \) value resulting from a symbolic aggregation operation and the index value \( i \) of the linguistic term \( s_i \) in \( S \) closest to \( \beta \). Then, a pair of symbolic values \( (s_i, \alpha) \), where \( s_i \in S \) and \( \alpha \in [−0.5, 0.5] \), which represents the means of the 2-tuple linguistic representation model is defined.

**Definition 2.** Let \( S = \{s_0, \ldots, s_g\} \) be a linguistic term set and \( \beta \in [0, g] \) represents the result of linguistic symbolic aggregation, then the 2-tuple \( (s_i, \alpha) \) equivalent of \( \beta \) denoted as \( \Delta(\beta) \) is obtained with the following function [25]:

\[
\Delta(\beta) = \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [−0.5, 0.5] \end{cases}
\]

(4)

where \( s_i \) has the closest index label to \( \beta \) and \( \alpha \) is the value of the symbolic translation.

**Definition 3.** Let \( S = \{s_0, \ldots, s_g\} \) be a linguistic term set and \( (s_i, \alpha) \) be a 2-tuple, then the equivalent numerical value \( (\beta) \) to 2-tuple \((s_i, \alpha)\) denoted as \( \Delta^{-1}(s_i, \alpha) \) is obtained with the following function:

\[
\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta
\]

(5)

**Definition 4.** Let \( s_i \in S \) be a linguistic term, then its equivalent 2-tuple representation is obtained by adding a value \( 0 \) as a symbolic translation [25]:

\[
\Delta(s_i) = (s_i, 0)
\]

(6)

**Definition 5. Negation Operator of a 2-tuple:** Let \( S = \{s_0, \ldots, s_g\} \) be a linguistic term set and \( g + 1 \) is the cardinality of \( S \), the negation operator over 2-tuples is defined as follows:

\[
\text{Neg}(s_i, \alpha) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))
\]

(7)

As stated before, the 2-tuple deals with only positive linguistic information in a linguistic term set and there is a need to extend its range to incorporate negative linguistic and numeric imprecise values for FCM connection weights. This article therefore proposes enhancements to fuzzy 2-tuple model that do not change its purpose or conflict with its definitions, propositions and rules; and they are:

1. The linguistic information is represented by \( S = \{s_g, \ldots, s_0, \ldots, s_2\} \) linguistic term set, where \( 2 \times g + 1 \) is the number of linguistic terms in \( S \) (cardinality) and \( \beta \in [−g, g] \)
2. \( \text{Neg}(s_i, \alpha) = \Delta(-\Delta^{-1}(s_i, \alpha)) \)
In this research, 2-tuples based on a linguistic term set $S = \{s_g, \ldots, s_0, \ldots, s_s\}$ are used to represent linguistic and numeric imprecise connection weights in the FCM and their equivalent $\beta$ numeric values are used to perform calculations (i.e., aggregation and condensation calculations). Each linguistic term $s_i$ in $S$ is represented by real valued triangular parameters $(a_i, b_i, c_i)$ (i.e., triangular membership function). Now, we discuss how this model can be used to represent linguistic and numeric weights in the unified format of 2-tuple and $\beta$. The 2-tuple value of a linguistic weight represented by a linguistic term $s_i$ in $S$, is directly obtained using Eq. (6), and its equivalent $\beta$ value is calculated using Eq. (5). It is important to mention here that different linguistic sets (e.g., if different linguistic sets are used for different FCMs) are normalized into one standard set before any calculations. In case of a linguistic weight being represented by a numeric imprecise value in the range $[-1, 1]$, its $\beta$ value is calculated first and then its equivalent 2-tuple value is obtained using Eq. (4). To find $\beta$ value of a numeric imprecise value $(n)$, first the membership function of value $n$ associated with $s_i$ (i.e., $\mu_s(n)$) is calculated using the following equation:

$$
\mu_s(n) = \begin{cases} 
\frac{n-a_i}{b_i-a_i}, & a \leq n \leq b \\
\frac{c_i-n}{c_i-b_i}, & b < n \leq c \\
0, & \text{otherwise}
\end{cases}
$$  \hspace{1cm} (8)

Then, $\beta$ value is the result of a symbolic aggregation of membership functions over labels $i$ assessed in $S$ and obtained using the following equation:

$$
\beta = \sum_{i=0}^{g} \frac{\mu_s_i}{g} \hspace{1cm} \text{(9)}
$$

Now we demonstrate the application of above definitions in the 2-tuple representation to the two FCMs from the ‘expert’ and ‘local people’ stakeholder groups described in Fig. 2. Suppose numeric and linguistic imprecise connection weights in these FCMs are expressed using $[-1, 1]$ numeric interval and ‘very highly negative’, ‘very highly negative’, ‘highly negative’, ‘medium negative’, ‘low negative’, ‘very low negative’, ‘none’, ‘very low positive’, ‘low positive’, ‘medium positive’, ‘highly positive’, ‘very highly positive’ linguistic expressions, respectively.

Let the linguistic set $S = \{s_{-6}, s_{-5}, s_{-4}, s_{-3}, s_{-2}, s_{-1}, s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ is used to represent these weights in 2-tuple and $\beta$ values using 2-tuple model. Let linguistic terms in the linguistic set $S$ are represented by triangular membership functions (fuzzy subsets) as shown in Fig. 3.

The connection weights in the ‘local people’ FCM are linguistic values, and hence, their 2-tuple and $\beta$ values are calculated using Eqs. (6) and (5), respectively. For example, on this FCM, the influence of Water Resources on Water (scarcity) Situation is very high (VH). Therefore, from Fig. 3, $s_5 = s_9$ and from Eq. (6), $i = 5$ and $\alpha = 0$; therefore, 2-tuple is $(s_5, 0)$ and from Eq. (5), $\beta = 1 + \alpha = 5$ (refer to vertical line on the right hand side of Fig. 3). On the other hand, the connection weights on ‘expert’ FCM are numeric values, and hence, their $\beta$ values are calculated first using Eqs. (8) and (9), and then equivalent 2-tuple values of these $\beta$ values are calculated using Eq. (4).

For example, on this FCM, Technology has a negative influence $(-0.3)$ on Water Demand. This value belongs to two membership functions ($-L$ and $-VL$) in Fig. 3 (refer to the vertical line placed at $-0.3$ in the Universe of Discourse). The corresponding indices of the fuzzy sets are $i = -2$ and $-1$ and membership function values calculated from Eq. (8) are approximately 0.8135 and 0.1865, respectively. The resulting $\beta$ value from Eq. (9) is $-1.813$ and the corresponding 2-tuple from Eq. (4) is $(s_{-2}, 0.187)$. Table 1 and Fig. 4 show the resulting representation of the connection weights of both FCMs in $\beta$ and 2-tuple values, respectively, using the 2-tuple representation model. As stated before, 2-tuple is used for representing both linguistic and numeric connections in a uniform format so that they can be easily handled together and all subsequent computations are carried out using their equivalent numeric values of $\beta$ as will be explained in detail in the next segment of the article.

3.2. Centrality measures—inspirations from social networks

A common and suitable way to represent and analyse complex interconnected systems with many connections and feedbacks in general is by using network models. Social networks are network models which have grown rapidly in the last decades in the Social Sciences [29] and as such advances in social network studies can be brought to bear on FCMs. Typically, socio-ecological systems are represented by coherent social networks that can represent interactions in both social and ecological systems as well as interactions between them. The key aspects of social networks are network analysis and connections that determine the interactions among nodes [48,57]. Social networks are modelled by graphs of nodes and connections between these nodes [4,7], which can then be easily analysed using graph theory methods [60]. A graph could be directed or undirected or weighted or unweighted. The attention of this article is a directed weighted graph as FCM belongs to this category.
FCM is a digraph consisting of a network of nodes and weighted connections that can represent any social structure [40]. Kosko [31] introduced the concept of centrality to depict and understand the contribution of a node in an FCM. He used the degree centrality measure to reflect the importance of a node; i.e., a node with higher degree value is of higher importance to the causal flow of information in the FCM. Beyond Kosko, few researchers have used the degree centrality to identify important nodes in FCMs and used this information for various purposes. Ozesmi and Ozesmi [42,43] utilized the degree centrality to characterize the most central/important nodes of their social FCMs in order to understand and analyze the structure of these FCMs. Ozesmi and Ozesmi [43] pointed out that the important nodes play a large role in an FCM by influencing and/or being influenced by other nodes.

Another attempt to use the most important nodes based on their degree centrality in social group FCMs was presented in Strickert et al. [50]. In this article, the authors utilized the importance of nodes to define decision nodes that could be useful in scenario simulations, i.e., the altered state of these decision nodes represents a particular policy scenario and these nodes are fixed (or clamped) at their respective altered state throughout the simulation process to ascertain the final state of the other nodes in the system under this policy scenario. In the feedback cycle of FCM, the effect of a node in one cycle may influence other nodes through a chain of connections among them. This means that the influence of a node is not only on its adjacent nodes, but it may extend far beyond in complex ways affecting and/or transferring the accumulated effects of upstream nodes to all other nodes existing in the path of the feedback chain. This issue poses a challenge that lies in two questions: what are the most important nodes in an FCM that can make influential changes? And how can such nodes be defined? This issue inherent in the dynamics of an FCM should be considered when identifying the most important nodes in the FCM. The previous studies show that the only measure that has been used to define the importance of a node in an FCM is the degree centrality measure.

The degree centrality measure is considered a straightforward and efficient local measure to identify the centrality of a node in a social network (and an FCM). It articulates the direct connections of a node in an FCM to other nodes [43]. It measures the contribution or activity of a node in an FCM and does give a strong indication of the effective nodes that can be used for understanding and analysing the structure and other characteristics such as system dynamics of the FCM. However, a node connected with a small number of highly effective nodes may be more powerful and influential than a node connected with a large number of lowly effective nodes. In this case, local measures, such as the degree centrality measure, are less relevant and therefore, global measures, such as closeness and betweenness, can be more suitable for identifying a node’s centrality and can give better results as has been reported in Social Science literature [8]. Specifically, a node may have a strategic location, i.e., if a node is located in between and/or close to other nodes, it may have a considerable influence on the flow of information (communication control) through that node. This feature can be taken into account through the consideration of the concept of the shortest paths rather than direct connections between nodes. Closeness and betweenness centrality measures take into account the indirect connections of a node by emphasizing the shortest paths between this node and the other nodes to show how a node could be globally quite central in the whole network rather than just locally [22]. As such, these latter measures reflecting global centrality of a node can be more effective than the degree centrality measure which only emphasizes the direct connections of a node and shows how much the node is central in a local neighbourhood.

Table 1

<table>
<thead>
<tr>
<th>Connection weights on Expert FCM represented in β values</th>
<th>Water situation</th>
<th>Water resources</th>
<th>Water demand</th>
<th>Technology</th>
<th>Wastage of water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water situation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Water resources</td>
<td>5.412</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Water demand</td>
<td>−6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Technology</td>
<td>0</td>
<td>3.588</td>
<td>−1.813</td>
<td>0</td>
<td>−4.188</td>
</tr>
<tr>
<td>Wastage of water</td>
<td>−2.412</td>
<td>0</td>
<td>1.188</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connection weights on Local People FCM represented in β values</th>
<th>Water situation</th>
<th>Water resources</th>
<th>Water demand</th>
<th>Water projects</th>
<th>Economic situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water situation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
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<tr>
<td>Water demand</td>
<td>−6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Water projects</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>−5</td>
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<tr>
<td>Economic situation</td>
<td>0</td>
<td>0</td>
<td>−3</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
In fact, the selection of a suitable centrality measure or a combination of the above three measures depends on the context of the application concerned [16]. Therefore, this research employs a novel method to derive a new centrality measure, called “Consensus Centrality Measure” (CCM) that we introduced in Obiedat et al. [40], to define a node’s centrality in its FCM. It combines the degree centrality measure with the two significant centrality measures just described—closeness and betweenness. CCM combines the three measures using the weighted average operator to take into account the relative importance of a certain measure over other measures. These ideas are presented in the next sections.

3.2.1. Degree centrality measure

The degree centrality measure is the simplest measure used in social network analysis. In a directed graph, two components, in-degree and out-degree centrality measures, are used to find the degree centrality of a node [5,23,30]. In a signed weighted digraph like FCM, the in-degree centrality of a node equals the sum of its absolute incoming connection weights from its neighbours:

\[
id(c_i) = \sum_{j=1}^{N} |w_{ji}| \quad (10)
\]

where \(id(c_i)\) is the in-degree centrality of node \(c_i\), \(N\) is the number of nodes connected to node \(c_i\) in FCM, and \(w_{ji}\) is the weight of the connection entering node \(c_i\) from node \(c_j\). In contrast, the out-degree centrality of a node in an FCM equals the sum of its absolute outgoing connection weights to its neighbours:

\[
\od(c_i) = \sum_{j=1}^{N} |w_{ij}| \quad (11)
\]

where \(od(c_i)\) is the out-degree centrality of node \(c_i\). The overall degree centrality of a node in FCM is the summation of its in-degree and out-degree [5,31]:

\[
\text{Cen}_D(c_i) = \id(c_i) + \od(c_i) \quad (12)
\]

where \(\text{Cen}_D(c_i)\) is the degree centrality of the node \(c_i\).

3.2.2. Closeness centrality measure

The closeness centrality is considered a global centrality measure because it is based on the shortest paths (geodesic distances) concept [3,46]. It finds the sum of geodesic distances between a given node and the remaining ones. In this sense, the lower the sum is, the higher the centrality. To reflect this idea, the inverse of the sum is taken to express the closeness centrality. As with degree centrality, this research considers two aspects of closeness centrality measure, in-closeness and out-closeness, to obtain the closeness centrality of a node in an FCM. In addition, the distances between nodes are to be represented by the absolute value of the connection weights. Thus, the in-closeness centrality of a node in an FCM measures how much other nodes are close to this node, and it is expressed as follows:

\[
ic(c_i) = \frac{1}{\sum_{t=1}^{N} d_G(t, c_i)} \quad (13)
\]

where \(t \neq c_i\), \(\nic(c_i)\) is the in-closeness centrality of the node \(c_i\), \(N\) is the number of nodes in FCM, and \(d_G(t, c_i)\) is the shortest path from node \(t\) to node \(c_i\). The out-closeness centrality of a node in an FCM measures how much this node is close to other nodes, and it is expressed as follows:

\[
\noc(c_i) = \frac{1}{\sum_{t=1}^{N} d_G(c_i, t)} \quad (14)
\]

where \(t \neq c_i\) and \(\noc(c_i)\) is the out-closeness centrality of the node. The closeness centrality of a node in FCM is the summation of its in-closeness and out-closeness, and it is defined as follows:

\[
\text{Cen}_C(c_i) = \nic(c_i) + \noc(c_i) \quad (15)
\]

where \(\text{Cen}_C(c_i)\) is the closeness centrality of the node \(c_i\).

3.2.3. Betweenness centrality measure

Like closeness, the betweenness is a global centrality measure based on the concept of shortest paths. The betweenness centrality of a node is determined by summing the proportion of shortest paths between all node pairs that go through a particular node [2,15]. To find the betweenness centrality of a node in an FCM, the absolute value of the connection weights between nodes represents the distances between these nodes, and hence, the betweenness centrality of a node is defined as:

\[
\text{Cen}_B(c_i) = \sum_{s,t=1}^{N} \frac{\sigma_{st}(c_i)}{\sigma_{st}} \quad (16)
\]

where \(s \neq t \neq c_i\), \(\text{Cen}_B(c_i)\) is the betweenness centrality of the node \(c_i\), \(\sigma_{st}\) is the number of shortest paths from node \(s\) to node \(t\), and \(\sigma_{st}(c_i)\) is the number of shortest paths from node \(s\) to node \(t\) that pass through node \(c_i\).

3.2.4. Consensus centrality measure (CCM) of a node and FCM

We proposed a novel CCM measure for a node and FCM in Obiedat et al. [40] and Obiedat and Samarasinghe [41], respectively. They were obtained from the above degree, closeness and betweenness measures. CCM of a node is obtained as [40]:

\[
\text{Cen}_{Cons}(c_i) = \text{Cen}_D(c_i) + \text{Cen}_C(c_i) + \text{Cen}_B(c_i) \quad (17)
\]

where \(\text{Cen}_{Cons}(c_i)\) is the CCM of node \(c_i\), \(i = 1\) to the number of nodes in FCM, \(\text{Cen}_D(c_i)\), \(\text{Cen}_C(c_i)\) and \(\text{Cen}_B(c_i)\) are the degree, closeness and betweenness centrality values of node \(c_i\), respectively, and \(\text{Cen}_D\), \(\text{Cen}_C\) and \(\text{Cen}_B\) are the prioritization weights for the degree, closeness and betweenness measures, respectively, where \(\text{Cen}_D + \text{Cen}_C + \text{Cen}_B = 1\).

Similar to the centrality of a node in an FCM, centrality of an FCM in a group of FCMS can indicate the influence of that FCM on the overall group system behaviour. To obtain CCM for an FCM (\(\text{Cen}_{Cons}(\text{FCM})\)), the degree (\(\text{Cen}_D(\text{FCM})\)), closeness (\(\text{Cen}_C(\text{FCM})\)) and betweenness (\(\text{Cen}_B(\text{FCM})\)) centrality measures of FCM need to be obtained. Each of these measures is calculated based on the corresponding node centrality measures in respective FCMS. Let \(\text{Cen}_D^\ast\), \(\text{Cen}_C^\ast\) and \(\text{Cen}_B^\ast\) are the maximum degree, closeness and betweenness centrality values of nodes in an FCM respectively. Then, the degree, closeness and betweenness for the FCM are calculated from the following Eqs. \((18)-(20)\), respectively [40]:

\[
\text{Cen}_D(\text{FCM}) = \frac{\sum_{i=1}^{N} (\text{Cen}_D^\ast - \text{Cen}_D(c_i))}{N - 1} \quad (18)
\]

\[
\text{Cen}_C(\text{FCM}) = \frac{\sum_{i=1}^{N} (\text{Cen}_C^\ast - \text{Cen}_C(c_i))}{(N - 1)(N - 2)(N - 3)} \quad (19)
\]
Table 2
Adjacency matrix of the FCM in Fig. 1.

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>C₆</th>
<th>C₇</th>
<th>C₈</th>
<th>C₉</th>
<th>C₁₀</th>
<th>C₁₁</th>
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<td>0</td>
<td>0</td>
<td>−0.31</td>
<td>−0.26</td>
</tr>
<tr>
<td>c₃</td>
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<td>−0.13</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>−0.36</td>
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<td>0</td>
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</tr>
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<td>c₄</td>
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<tr>
<td>c₅</td>
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<td>0</td>
<td>−0.34</td>
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<td>0</td>
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</tr>
<tr>
<td>c₆</td>
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<td>−0.48</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
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<td>0.29</td>
<td>−0.17</td>
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<td>0</td>
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<td>0.58</td>
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<td>0.63</td>
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<td>0</td>
<td>0</td>
<td>−0.17</td>
<td>0.42</td>
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</tr>
</tbody>
</table>

Table 3
Values for Degree, Closeness, Betweenness, and CCM centrality measures of FCM nodes considered in Example 1.

<table>
<thead>
<tr>
<th>Node</th>
<th>Degree centrality</th>
<th>Closeness centrality</th>
<th>Betweenness centrality</th>
<th>CCM centrality</th>
</tr>
</thead>
<tbody>
<tr>
<td>c₁</td>
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<td>4.12</td>
<td>1.27</td>
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</tr>
<tr>
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<td>5.45</td>
<td>2.81</td>
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</tr>
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</tr>
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</tr>
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</tr>
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<td>6</td>
<td>5.24</td>
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</tbody>
</table>

\[
\text{Cen}_b(\text{FCM}) = \frac{\sum_{i=1}^{n} (\text{Cen}_{b_i} - \text{Cen}_{b(c_i)})}{n-1} \tag{20}
\]

Then, the CCM of the FCM is obtained as follows [40]:

\[
\text{Cen}_{\text{CCM}}(\text{FCM}) = b_D \times \text{Cen}_D(\text{FCM}) + b_C \times \text{Cen}_C(\text{FCM}) + b_B \times \text{Cen}_B(\text{FCM}) \tag{21}
\]

CCM is a weighted average of the three centrality measures with weights indicating their priority. Prioritisation weights are subjectively assigned by the developer of the system. The developer assigns a non-negative weight for each measure according to the measure’s effectiveness in determining node or FCM centrality. This gives the FCM developer flexibility, depending on his/her in-depth understanding of the context of FCM, to prioritize between these measures by assigning a non-negative weight to each in the interval [0, 1]. This means that the CCM can endow a measure with a preference over others if the weight of this measure takes on a high value, for example, close to 1, or even ignore one or two measures from calculations if they are given zero or very low values.

To illustrate the generation of centrality measures, consider as an example FCM nodes in Fig. 1 and its adjacency (weight) matrix presented in Table 2 below. As FCM weight values are represented in numeric values in the interval [0, 1], this requires first converting them to \( \beta \) values based on the degree of membership in relevant fuzzy sets (Eqs. (8) and (9)) as described previously. Resulting \( \beta \) values of the weights are used in calculations of the centrality measures using Eqs. (10)–(16). Tables 3 and 4 show the degree, closeness, betweenness, and CCM centrality measures of FCM nodes (Eqs. (10)–(17) and FCM itself (Eqs. (18)–(21)), respectively, represented in \( \beta \) values. In this example, we consider that degree, closeness and betweenness measures are equally important and so have the same weight (i.e., 0.33).

3.2.5 Credibility weight of a node and FCM

The credibility weights (CW) are assigned to nodes and FCMs based on the calculated CCM of nodes and FCMs, respectively, represented in \( \beta \) values. This proposition has been developed from the idea that if a node/FCM is more central (important) than other nodes/FCMs, this means that it occupies a higher or more powerful position with greater ability to influence and change the outcomes of the domain of concern compared to other nodes/FCMs. Consequently, this node/FCM should take a higher credibility value than other nodes/FCMs, and so on. The CCM is more expressive about the centrality than other measures because it combines the three centrality measures incorporating local and global influence of nodes. Therefore, this measure is utilized to assign credibility weights in the interval [0, 1] to both nodes and FCMs. First, the CCM of nodes in FCM and FCMs in the system are normalized in the interval [0, 1]. For this calculation, we use the already described 2-tuple model as it has the advantage of being able to convert \( \beta \) values into [0, 1] or \([-1, 1]\). The transformation process is presented in Herrera and Martínez [25]. It uses the function \( \delta \) to compute two 2-tuples from CCM \( \beta \) value based on the degree of membership in linguistic terms that supports the same counting of information as follows:

\[
\delta(\beta) = \{(S_n, 1 - \gamma), (S_{n+1}, \gamma)\} \tag{22}
\]
where \( h = \text{trunc}(\beta) \), \( \text{trunc} \) is the usual trunc operation, \( \gamma = \beta - h \), and \( S_h \) and \( S_{h+1} \) are linguistic terms in \( S \), in which \( \beta \) has membership.

Then it uses the function \( \kappa \) to convert these two 2-tuples into a numerical value assessed in the interval [0, 1] or [-1, 1] as follows:

\[
\kappa((S_h, 1-\gamma), (S_{h+1}, \gamma)) = CV(S_h) \times (1-\gamma) + CV(S_{h+1}) \times \gamma \tag{23}
\]

where \( CV(.) \) is a function providing a characteristic value like one of the defuzzification techniques.

For example, considering CCM (\( \beta \) values) of nodes in FCM in Table 3 (last column and repeated in Table 5 column 2), Table 5 (column 3) shows these \( \beta \) values represented in [0, 1] after applying the above transformation using Mean of Maximum (MoM) as defuzzification method.

Once the CCM values in the interval [0, 1] are obtained from the original CCM \( \beta \) values, the credibility weight of each node (cwFCM) in the FCM is calculated by dividing its CCM by the sum of CCMs of all nodes in the FCM. The credibility weight values of nodes in the example FCM, resulted from this calculation, are shown in column 4 of Table 5. Similarly, the credibility weight of each FCMcwFCM in the system is calculated by dividing its normalised CCM by the sum of CCMs of all FCMS in the system.

### 4. Proposed robust semi-quantitative FCM model

This article enhances the existing qualitative FCM approaches to produce a robust semi-quantitative approach. The proposed model includes several coherent and consistent steps of reasoning and calculations based on robust methods and models making it more contextually realistic and reasonable and computationally practical and manageable. Accordingly, the proposed model is suitable for, and effective in, addressing a range of currently challenging complex dynamical problems dominated by uncertainty and imprecise data such as participatory real-life or environmental problems. Specifically, the model includes eight advanced processes that make it coherent and robust. Fig. 5 shows in sequence the configuration of the 8 steps involved in the proposed FCM model. It starts from the phase of the data collection and FCM development in the form of ambiguous perceptions based on in-depth interviews with relevant stakeholders (steps 1–3), and then FCM data manipulation without loss of information throughout different stages of FCM processing (FCM representation, condensation and aggregation) using the novel computational approaches proposed in this research (steps 4–6), to the phase of knowledge extraction in the form of map structure and properties (step 7) as well as generating solutions or recommendations through simulations (step 8). These steps are described in the next sections.

#### 4.1. Interview process (step 1)

In problem solving involving participation of people (participatory systems), one of the most common methods to collect useful data about the problem is to conduct qualitative interviews with relevant respondents [43]. Qualitative interviews are attempts to understand the problem from people’s point of view and to uncover the meaning of their experiences [36]. The first step of the proposed model is to collect significant qualitative data about the problem from relevant stakeholders using in-depth semi-structured qualitative interviews based on open-ended questions. Depending on the complexity and domain of the problem, the relevant stakeholders can be divided into groups based on their attributes (e.g., experts, non-experts etc.). An interview may take a long time and generate a lot of data, and hence it is highly recommended that the interview is recorded and useful notes are taken. This helps in the subsequent processes of Development (step 2) and Review (step 3) of FCM.

#### 4.2. FCM development process (step 2)

After each interview with a stakeholder, the raw data collected from it are reported. The second step of the proposed model is to depict the collected data in the form of FCM of the stakeholder. The data are converted into causal factors/variables and relationships/causality. Some of these variables may represent actions, events, inputs, outputs, and other states that could affect each other either negatively or positively, and this in turn describes the behaviour of the domain problem. The variables are transferred into nodes and the causality into connections to draw the FCM in a suitable media such as a whiteboard. The structure of the FCM shows the influence of domain variables on each other.

#### 4.3. FCM review process (step 3)

Typically, an FCM developed immediately from an interview may not be comprehensive or reflective of all the important data that have been made available in the interview. One of the reasons for this situation is the generally limited time available for the interview and FCM development processes considering the enormity of the domain problem tackled. Eliciting knowledge from participants usually takes a long time, particularly if the problem concerned is substantial. An interview contains many open-ended questions to investigate comprehensively most of the influential factors and relationships between them in relation to the problem, which requires much reflection and probing. Accordingly, the development of FCM may also take a long time if many factors and connections are defined. Another reason is the difficulty in depicting all the identified factors as nodes in FCM and then tracing the connections between them. To handle this issue, the third process of the proposed model is to hold an FCM review by the system developer (person who conducted the interviews). The review process includes reviewing each developed FCM, recorded interview and written notes to determine if there are any missing important factors and connections mentioned in the interview but not included in the developed FCM. Accordingly, the system developer makes any necessary updates to the FCM so that it includes all important factors and connections, and thus the FCM becomes reflective/representative of all that has been revealed in the interview.

#### 4.4. FCM fuzzy representation process (step 4)

Fourth step initiates data manipulation and calculations that continue through to step 8. The purpose of step 4 of the proposed model is to represent a variant (numeric and linguistic) imprecise data in a uniform way to enhance the quality of data representation.
in FCM. For this purpose, this article uses the extended 2-tuple fuzzy representation model stated earlier. The 2-tuple model can represent and deal with imprecise numerical and vague linguistic data either separately or together. It represents these data in the form of pairs of symbolic values. It can also transform these pairs of symbolic data into numeric \( \beta \) values and vice versa for easier handling in further computations. The option of selecting between numerical values and linguistic words for connection strengths gives the FCM designer/participants the freedom in the expression of FCM connection weights, especially when it is difficult to give a straightforward or explicit value.

The first step of the FCM fuzzy representation method is to determine a linguistic term set, \( S \), consisting of an expressive number of linguistic terms (membership functions) for all FCM connection weights. In the case of FCM weights being expressed in linguistic values, Eq. (6) is used to represent FCM weights in 2-tuple values (pairs of \( (s, \alpha) \)) and then find equivalent \( \beta \) values to these 2-tuple values using Eq. (5). In the case of FCM weights being expressed in numeric values in the interval \([-1, 1]\), first, they are
Table 6  
The β values representing the adjacency matrix of connection weights in the example FCM in Fig. 1.

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Table 7  
The 2-tuple values that are equivalent to β values of weights in Table 6.

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</table>

Based on the membership values μ(wᵢⱼ), we obtain a (numerical) β value for (wᵢⱼ) using Eq. (9) [25] which for this case is:

\[ β(wᵢⱼ) = \frac{1}{6} \sum_{i=6}^{i=6} i \times b_i \]

where \( β(wᵢⱼ) \) is the numerical β value of the FCM connection weight (wᵢⱼ). Then, we apply the previous step to all FCM connection weights in the example to obtain the β values for all FCM connection weights and these β values for weights are shown in Table 6. Finally, from the β values, we obtain their equivalent 2-tuple (sᵢ, α) values using Eq. (4). The 2-tuple (sᵢ, α) values of the weights are shown in Table 7.

4.5. Fuzzy condensation of FCM (step 5)

In large complex participatory problems, especially if the problem includes a range of stakeholder groups, many factors and relationships among these factors are likely to be defined. As a result, the FCMs designed by the relevant stakeholders could be very complex and include a large number of nodes and connections. In addition, these FCMs may need to be aggregated to obtain stakeholder group FCMs or social FCM (sum total of all FCMs). In group or social FCM, the number of nodes could reach hundreds and the number of connections could be up to thousands. With such FCMs, it is a big challenge to understand, analyze or gain insights from them. To handle this challenge, the fifth step of the proposed model is the FCM condensation process. It is a semi-quantitative method and simplifies a large FCM with a large number of nodes (variables) and connections into a smaller FCM with a small number of higher level categories (groups) of variables and connections.
The proposed condensation method is based on multi-level condensation determined by the system developer. The system developer should take into account several issues when choosing the number of condensation levels, such as the size of FCM, the strength of interdependence between nodes, the smoothness and easiness of the transition process from the lower level of condensation to a higher level, and how to categorize the nodes and connections at the lower level into new nodes (groups) at the higher level without loss or distortion of information in the FCM. The condensation method proposed in this article uses a number of robust calculations and utilizes the credibility weights of nodes at the lower level in the process of transferring the nodes and their connections to a higher level. Moreover, the values used in this method are represented by fuzzy numeric $\beta$ values in all computational processes to avoid loss of information. Consequently, this novel method can be considered as a semi-quantitative fuzzy method proposed to overcome the shortage of previous qualitative condensation methods.

The proposed method consists of two major phases in each level of condensation. The first phase is subjective (qualitative); it involves identification of the groups of nodes at the higher level. The system developer uses their knowledge and experience in the problem domain to identify similar nodes in the lower level to be condensed into groups at the higher level of FCM condensation and identifies the names of these groups. The second phase is objective (quantitative); it includes steps and calculations to ensure valid and accurate transition of weights from the lower level to the identified higher level. It uses the credibility weights of nodes at the lower level, which are assigned based on the CCM of these nodes, to calculate the connection values at the higher level. Typically, the FCM may have nodes and connections at the lower level, which are condensed to the same group at the higher level. However, some nodes in the same group may, through each other, have indirect connections to nodes in other groups; and some of these nodes in turn through other nodes in the same group may already have indirect links to nodes in other groups. Rather than eliminating or disregarding such indirect connections, the proposed method uses a proper way to refine these connection weights before proceeding with the condensation process. The refinement process redistributes these indirect links to direct links between appropriate nodes in different groups. This helps avoiding some nodes to have connections to themselves (self-reference connections due to connections within a group) in the resulting condensed FCM and avoiding the loss of indirect interactions between nodes prevailing at the lower level before condensation.

The refinement process is performed for each node $c_i$ in a group of nodes that require redistribution of indirect connections at the lower level to nodes in other groups. It starts after the group of nodes at the higher level and their nodes at the lower level are defined and its steps are described in Algorithm 1.

**Algorithm 1.** Steps in the refinement process of the indirect connections of node $c_i$.

1. For each node $c_j$ in the same group that node $c_i$ belongs to do the following:

   A. If there is a connection between $c_i$ and $c_j$ ($w_{ij}$), then

      i. If there is a connection between $c_j$ and any node $c_k$ ($w_{jk}$) outside this group in the FCM, then

         a. assign a new connection between $c_i$ and $c_k$ as follows:

            $$\text{New}_{w_{ik}} = \text{New}_{cw_i} \times w_{ij} + \text{New}_{cw_j} \times w_{jk}$$

   b. If the absolute value of $\text{New}_{w_{ik}}$ is greater than the absolute value $w_{ik}$, then $w_{ik} = \text{New}_{w_{ik}}$

   ii. Remove the connection $w_{ij}$

2. Repeat step 1 for other nodes in the group

The advantage of the proposed refinement of every connection between any two nodes located in the same group over its cancellation is that the cancellation may adversely affect the behaviour of the FCM, for example, when a node influences another node in the same group and the latter in turn influences other nodes in other groups. In this novel method described in Algorithm 1, the indirect influences of connections between nodes in different groups are converted to direct influences between them. The method calculates the weights for the newly created direct influences by considering the connection weight and credibility weights of nodes that established the indirect influences. Eq. (26) expresses this calculation. Finally, if the nodes which formed the newly created direct influences already have connection weights between them, then the highest of the absolute value of the newly created direct influence and the existing weight is assigned as the connection weight between these nodes.

After the refinement process, FCM condensation process for each level of condensation resumes with the Algorithm 2 shown below. The process involves computing new weights between groups at the higher level. The first step in this process is to initialize to zero the connection weights between groups at the higher level. Then, for the connection between two groups, we identify at the lower level only the non-zero connections between nodes in these groups. For the identified nodes in each group, we utilize their calculated credibility weights at the lower level to calculate a new credibility weight for them using Eq. (27). Then, Eq. (28) uses these new credibility weights of nodes and their connections at the lower level to calculate a condensed connection between the two groups at the higher level. These steps are repeated until all connections between all groups at the higher level are calculated. In this way, the adjacency matrix of the condensed FCM is created. It is easy from this matrix to draw the condensed FCM and show the connections between its nodes (groups).

**Algorithm 2.** Steps in the fuzzy condensation process for FCM at each level of condensation.

1. Initialise an adjacency matrix ($G$) for the groups of condensed nodes at the higher level and fill its elements (connection weight values between groups) with zero values

2. For each connection weight $g_{ij}$ in the matrix $G$ between group $g_i$ and group $g_j$ do the following:

   A. Obtain the nodes at the lower level that belong to $g_i$ and $g_j$ groups at the higher level

   B. Initialise two dimensional matrix (Grp$_{ij}$), $i = 1$ to the number of nodes in $g_i$ and $j = 1$ to the number of nodes in $g_j$

   C. Store in Grp$_{ij}$ the connection values at the lower level from nodes in $g_i$ to nodes in $g_j$

   D. Select from Grp$_{ij}$ the nodes in both $g_i$ and $g_j$ groups that have at least one non-zero connection value and reassigned new credibility weights to each node using the following equation:
\[
\text{New}_{cw_{ni}} = \frac{cw_{ni}}{\text{sum}(cw_{nj})}
\]  
(27)

where \(\text{New}_{cw_{ni}}\) is the new credibility weight of a selected node \(n\) in \(g_i\), \(n = 1 \ldots N\), \(N\) is the total number of selected nodes in \(g_i\), \(cw_{ni}\) is the lower level credibility weight of the selected node \(n\) in \(g_i\), and \(\text{sum}(cw_{nj})\) is the sum of credibility weights at the lower level of all selected nodes in group \(g_i\).

**Hint:** the above equation calculates the new credibility weight of a selected node in group \(g_i\), taking into account in this case that \(N\) is the total number of selected nodes in \(g_i\).

3. Assign new weight value to the connection \(g_{ij}\), between \(g_i\) and \(g_j\) groups at the higher level as follows:

\[
g_{ij} = \text{sum}(\text{New}_{cw_{mi}} \times \text{New}_{cw_{mj}} \times w_{ij})
\]  
(28)

where \(\text{sum}(\text{New}_{cw_{mi}} \times \text{New}_{cw_{mj}} \times w_{ij})\) is the sum of all connection weights \(w_{ij}\) at the lower level between nodes in \(g_i\) and nodes in \(g_j\) after multiplying each connection weight between two nodes by their new credibility weights assigned using Eq. (27).

4. Repeat step 2 for all connection weights in matrix \(G\) at the higher level

4. Construct from the matrix \(G\) the condensed FCM at the higher level

Once the condensed FCM is constructed at this level, the CCM and credibility weights values for its nodes can be easily obtained using calculations stated before. These values are required for the next level of FCM condensation process. After the completion of all levels of the FCM condensation process, a simpler FCM with fewer nodes and connections is constructed. In this case, it is easy to gain insights from FCM and analyse its structure as well as apply it for simulating what-if scenarios. It also helps produce a simpler group FCM that would result from combining condensed individual FCMs. Next section presents the FCM aggregation process that combines FCMs in a group (such as a stakeholder group) into the corresponding group FCM as well as combine multiple group FCMs (such as multiple stakeholder groups) into a social FCM.

### 4.6. Fuzzy aggregation process of FCM (step 6)

According to literature, obtaining a consensus FCM from a group of different FCMs pertaining to stakeholders with diverse perceptions is currently a challenge. The FCMs are typically developed for complex domains characterised by uncertain and imprecise knowledge. As such, the issue of conflicting perceptions between the stakeholders (designers of FCMs) is natural to arise. In real-life problems, incorporation of the human dimension through human perceptions/opinions has become a necessity to fully characterise and study such problems. However, people vary in the level of knowledge and experience and their opinions should be taken into account accordingly when their FCMs are combined into an overall expression for a group. Hence, the FCMs that depict these perceptions should be weighted according to an objective credibility measure. This issue constitutes a barrier for developing a valid and accurate group FCM that describes a consensus perception. Another issue that has been stressed throughout this article is that different people may need different measures to express their knowledge in the form of nodes and connection between nodes. Some may prefer to use linguistic terms while others prefer numeric values; or they may use different scales of linguistic and numeric measures. Finally, as complex problems addressed by FCM are typically characterised by ambiguity and uncertainty, the imprecise values describing the connections between nodes in FCM should be represented by a suitable model that retains the accuracy when combining weights expressed in different formats in different FCMs. To overcome the stated challenges, this article utilizes the FCM aggregation method we proposed in Obiedat et al. [40] to introduce the sixth process of the proposed model — robust fuzzy aggregation of FCM.

The introduced FCM aggregation method can be applied on the FCMs before or after condensation process, or even at any level of it. This method uses the 2-tuple model, described previously, to represent the imprecise connection values between nodes in FCMs in fuzzy \(\beta\) values. It also takes into consideration the importance of these connection values by weighting them according to the credibility weight of their FCMs. Credibility weights of FCMs address the importance of different levels of knowledge of the stakeholders and are used to prioritize the importance of the connections before they are aggregated with the connection values in other FCMs. Finally, the proposed fuzzy aggregation method can deal with multiple linguistic and numeric scales used to describe imprecise connection values in different FCMs. In other words, the method allows describing imprecise connections by different fuzzy sets as appropriate for the stakeholders. Fuzzy sets could include different number of linguistic terms represented by different membership functions (fuzzy subsets) to deal with multiple linguistic and/or numeric scales. The different linguistic sets are to be converted into one uniform set called Base Linguistic Term Set (BLTS) [24,25].

The BLTS is chosen such that it contains a number of Linguistic Terms appropriate to represent the overall group of FCMs being assembled. This process is called the normalization of the linguistic sets; and for each FCM, before combining with other FCMs, its fuzzy subsets are converted into fuzzy subsets in the BLTS. The objective of this process is to manage the imprecise connection values of FCMs in a way that prevents any loss of information during the aggregation process by normalizing the fuzzy subsets representing these imprecise values into a standard set. BLTS [24]. To do so, we convert the \(\beta\) connection weight values of FCMs into the interval \([-1, 1]\); the procedure for this step was explained previously. Then, we transform the linguistic set of FCM into the BLTS. Let \(S = \{s_1, s_2, \ldots, s_p\}\) be the linguistic set of an FCM and BLTS = \(\{t_{-1}, \ldots, t_1\}\) be the linguistic set of the BLTS, such that \(p \leq g\); then the function \(T_{S\_BLTS}\) defines each fuzzy subset of \(S\) in the BLTS as follows [24]:

\[
T_{S\_BLTS}(S_k) = \{(t_k, \alpha_k^y), y \in \{0, \ldots, g\}, k \in S\}
\]  
(29)

\[
\alpha_k^y = \max(y \in \{0, \ldots, g\}) \left(\min(\mu_{s_k}(y), \mu_{t_k}(y))\right)
\]  
(30)

where max and min are the usual maximum and minimum operations, respectively, \(y\) is a connection weight value on \([-1, 1]\) universe of discourse, and \(\mu_{s_k}(y)\) and \(\mu_{t_k}(y)\) are the membership functions of the fuzzy subsets associated with the terms \(s_k\) and \(t_k\), respectively.

As the next step in the process, using the transformed uniform linguistic set of FCM, a conversion of connection values in the interval \([-1, 1]\) back to \(\beta\) values is carried out. Once the above steps are performed on all FCMs in the system, the next step of the FCM aggregation is to initialize an adjacency matrix (Soc) for the group FCM to include all possible nodes and connection weights between them that could result from combining the FCMs into the group and fill its elements by zero values as follows:

\[
\text{Soc}_{ij} = \begin{cases} 
0 & i, j = 1 \ldots NC \\
1 & i, j \text{ are connected nodes in the group graph} \\
\end{cases}
\]  
(31)

where \(\text{Soc}_{ij}\) represents the connection weight between node \(c_i\) and node \(c_j\) in the group or social FCM and NC is the number of distinct nodes in all FCMs to be aggregated in the system.

This step is followed by an FCM augmentation process; here, the adjacency matrix of each FCM (i.e., \(FCM_k\), where \(k = 1\) to the number of FCMs in the system) is augmented to include all nodes in all FCMs. The column and row of each new node added to the
matrix are filled with zero values. The augmented matrix is then multiplied by the FCM credibility weight as follows:

\[ FCM_{wk} = FCM_k \times CW_k \]  \hspace{1cm} (32)

where \( FCM_{wk} \) is the resulting weighted matrix of FCM obtained from multiplying the FCM connection weight values \( FCM_k \) by its credibility weight value \( CW_k \).

Finally, the weighted augmented matrix is added to the adjacency matrix of group FCM as follows:

\[ Soc = Soc + FCM_{wk} \]  \hspace{1cm} (33)

The above steps of the FCM aggregation process are repeated until the last FCM in the group has been added to the group FCM.

To aggregate the example ‘expert’ and ‘local people’ FCMs presented in Fig. 4 into a group FCM, consider their \( \beta \) values in Table 1. Suppose the results from calculating the credibility weights (see Section 3.2.5) of the ‘expert’ and ‘local people’ FCMs are 0.6 and 0.4, respectively, and suppose the linguistic term set \( S \) used to represent their connection weights versus BLTS itself. Therefore, there is no need for the normalization process. Applying the stated steps of the fuzzy aggregation (Eqs. (31)–(33)), the \( \beta \) weight values of the resulting group FCM are presented in Table 8. This table shows that the group FCM includes all distinct nodes and all possible connections between these nodes that existed in both FCMs. Fig. 6 shows ‘expert’ and ‘local people’ and resulting group FCMs. It shows the connection weights of these FCMs represented by 2-tuple values (equivalent of \( \beta \) obtained from Eq. (4)).

In addition to individual (original) FCMs, the condensation and aggregation processes create a number of diverse FCMs (group FCMs and a social group FCM) to be analysed and simulated. This allows us to reach towards a comprehensive knowledge and understanding of the problem and allows access to a wide range of individual and group perceptions such that analysing and simulating these perceptions can lead to many reliable suggestions or potential solutions to the problem.

### 4.7 FCM analysis process (step 7)

After the completion of the various FCM processes described in the previous sections, the developer of the system becomes familiar with the details/issues of the problem and has the understanding and experience to initiate FCM analysis and simulation processes. Analysis and reflection on the original, condensed and aggregated perceptions/FCMs lead to a greater understanding of the problem domain and examination of different perceptions of the stakeholders and their interests. Therefore, the seventh process of the proposed FCM model is to analyse the structures of different FCMs such as those for individual stakeholders, stakeholder group FCMs and the social FCM and then make comparisons among them. The proposed FCM model can benefit from graph theory indices to analyse the structures of the perceptions/FCMs and compare them accordingly. It can also use different statistical measures to find similarities and dissimilarities between nodes in individual FCMs, and between FCMs in FCM groups. In addition, the CCM index identifies the importance of FCMs and nodes in FCMs, and based on this, the most important (central) nodes are determined. These nodes make great contributions to their FCMs and could be used as influential instruments to change the behaviour and outcome of the FCMs (system) when these FCMs are simulated.

### 4.8. FCM simulation process (step 8)

Because systems representing complex and dynamic real-life problems include many feedback loops, system simulation and policy scenario simulations are required to explore the system behaviour and reach better outcomes. The last process of the proposed FCM model is the FCM simulation. Different FCMs can be simulated and used as simulation models to conduct different what-if policy scenarios. The first step of simulating any FCM is to simulate the FCM from the current (initial states of nodes) until steady state is reached. This steady state is the status quo outcome of the FCM. For this purpose, the FCM is treated as a neural network with feedback (a recurrent network) where each neuron represents a concept [47]. Initial state (or initial input) of each neuron is the state of a variable (or a group of variables) in the range of \([0, 1]\) specified by stakeholders or determined from existing information. Each neuron or node computes its output as the weighted sum of inputs coming from the concepts connected to it processed through a sigmoid function. Weights are the fuzzy connection values assigned as discussed previously. As there are feedback loops, the simulation will run many cycles before the states of all nodes reach equilibrium, which represents the steady state of the system.

After the status quo outcome is obtained, different policy scenarios (what-if questions) can be applied to test whether the new scenario leads to a desirable outcome. These what-if scenarios constitute proportional changes or perturbations to one or several most central variables in an FCM identified previously. Typically, the altered states of the chosen variables are clamped (fixed) throughout the iterations until equilibrium (new steady state) is reached. Then, a comparison between this new state and the status quo steady state of the FCM reveals the influence of the new policy on the system outcomes. With a good understanding of the domain, what-if scenarios can be designed such that the simulated scenarios are practically reasonable and effective in producing desirable policy outcomes. Finally, the analysis and comparison between the results from simulating multiple policy scenarios would help explore and rank potential solutions to the problem. These solutions are introduced to decision makers as recommendations that could help them in making effective decisions (for example, actions to be taken in the form of changes to be made to the influential factors (variables) in order to alleviate an acute water scarcity problem). A brief summary of a case study using the proposed method is given below to demonstrate its application and usefulness. A larger and more comprehensive study for alleviating water scarcity situation in a country using the proposed method is planned for a forthcoming article.
5. *An example from reality: mitigating groundwater degradation and depletion in Jordan*

Water is critical to survival and a significant matter for the safety and wellbeing of any society. Jordan faces severe water crises and there are signs of increasing water problems in Jordan. Government in Jordan is paying a great deal of attention to address the water issues [38,28]. Here, we present the application of the proposed model to this real-life problem—‘Groundwater Degradation and Depletion in Jordan’. We address challenges of Jordan groundwater problem and suggest some meaningful recommendations that could help decision makers in managing and improving the groundwater situation in Jordan to mitigate its degradation and depletion. This study was carried out according to the steps of the proposed model as summarised below.

Firstly, we collected knowledge and perceptions on the groundwater problem in Jordan from the perspective of different relevant stakeholder groups using in-depth semi-structured interviews. The idea was to get as much information as possible on the problem including factors affecting ground water degradation/depletion, strength of their interactions and any other helpful information with a view to improving the situation through the proposed method. 15 interviews were conducted with 3 stakeholder groups: local people, farmers and managers. Then, 15 stakeholder FCMs were developed. 9 stakeholders preferred to draw the connection weights between variables using linguistic values, while the rest used the interval [0, 1]. These FCMs were then thoroughly reviewed to ensure that all variables and connections that were mentioned in the recorded interviews are present in the FCMs. The stakeholders defined 67 different original variables in total.

The connection weights were then represented in a unified format using the proposed 2-tuple fuzzy linguistic model. We used a linguistic term set, $S$, consisting of 11 linguistic terms (membership functions) as follows: $S = \{−5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5\}$. Then we calculated $\beta$ and 2-tuple fuzzy linguistic values $(s_i, \alpha)$ for these weights. The next step was calculating Consensus Centrality Measure (CCM) and credibility weights for variables and FCMs. We present here the CCM and credibility weights only for the three condensed group FCMs and the social group FCM in Table 9 which could be clearer if read in conjunction with the following discussion on FCM condensation.

The next stage was using one level of FCM condensation to simplify the 67 variables into following 12 subjectively defined concepts at a higher level of abstraction: groundwater improvement, causes of degradation, modern technology, law enforcement, groundwater basins, groundwater uses, stakeholder awareness, financial support, other water resources, highland water extraction, private sector, and pollution. The indirect connections between variables were manipulated using the proposed refinement process during the FCM condensation. The individual condensed FCMs were combined into 3 stakeholder group FCMs using the obtained credibility weights of these FCMs and the proposed FCM aggregation process. Then, we combined the group FCMs into one social FCM using the above process and credibility weights of these FCMs are shown in Table 9. We analysed the condensed individual, group and social FCMs using CCM of concepts, most mentioned concepts and map density measures. Table 9 (bottom row) shows the calcu-
Table 9
Some graph theory indices of individual, group and social condensed FCMs at the level of the 12 concepts: frequency of concept (i.e., how many of the 15 individual condensed FCMs contain the concept), the CCM and credibility weight values of concepts in groups and social FCMs, and the map density, CCM and credibility weight values of group and social FCMs.

<table>
<thead>
<tr>
<th>Concept name</th>
<th>No. of condensed individual FCMs containing concept</th>
<th>CCM and credibility weight (CW) values of concepts in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Local people group FCM</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CCM</td>
</tr>
<tr>
<td>Groundwater improvement</td>
<td>15</td>
<td>0.479</td>
</tr>
<tr>
<td>Causes of degradation</td>
<td>11</td>
<td>0.202</td>
</tr>
<tr>
<td>Modern technology</td>
<td>13</td>
<td>0.442</td>
</tr>
<tr>
<td>Law enforcement</td>
<td>12</td>
<td>0.302</td>
</tr>
<tr>
<td>Groundwater basins</td>
<td>11</td>
<td>0.099</td>
</tr>
<tr>
<td>Groundwater uses</td>
<td>10</td>
<td>0.187</td>
</tr>
<tr>
<td>Stakeholder awareness</td>
<td>11</td>
<td>0.207</td>
</tr>
<tr>
<td>Financial support</td>
<td>12</td>
<td>0.245</td>
</tr>
<tr>
<td>Other water resources</td>
<td>10</td>
<td>0.078</td>
</tr>
<tr>
<td>Highland water extraction</td>
<td>10</td>
<td>0.253</td>
</tr>
<tr>
<td>Private sector</td>
<td>10</td>
<td>0.286</td>
</tr>
<tr>
<td>Pollution</td>
<td>9</td>
<td>0.057</td>
</tr>
<tr>
<td>FCM density</td>
<td></td>
<td>0.318</td>
</tr>
<tr>
<td>FCM CCM</td>
<td></td>
<td>0.592</td>
</tr>
<tr>
<td>FCM credibility weight (CW)</td>
<td></td>
<td>0.302</td>
</tr>
</tbody>
</table>

Table 10
Outcomes of the applied policy scenario simulations in group and social FCM systems by fixing the influential concepts at high values to show their impacts on Groundwater Improvement concept. The impact is shown in %.

<table>
<thead>
<tr>
<th>Influential concept</th>
<th>The impact of the influential concepts on groundwater improvement in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local people group FCM %</td>
</tr>
<tr>
<td>Modern technology</td>
<td>30%</td>
</tr>
<tr>
<td>Law enforcement</td>
<td>33%</td>
</tr>
<tr>
<td>Highland water extraction</td>
<td>15%</td>
</tr>
<tr>
<td>Stakeholder awareness</td>
<td>11%</td>
</tr>
<tr>
<td>Financial support</td>
<td>11%</td>
</tr>
</tbody>
</table>

related values of these measures. The map density ($D$) (extent of FCM connectivity) is calculated as follows:

$$D = \frac{C}{(N \times (N - 1))} \quad (34)$$

where $C$ is the number of connections and $N$ is the number of concepts in FCM. Density of a fully connected map is 1.0.

The analysis process revealed that the stakeholders in different groups defined many similar concepts and their perceptions were approximately similar to each other with regard to the ground water situation/improvement. For example, Table 9 shows that the density of group FCMs is very close to each other. Also, the density is relatively high meaning that the stakeholders defined a large number of connections among the concepts. According to the CCM values, Modern Technology and Law Enforcement concepts have high values in all groups and social FCMs. Highland Water Extraction and Stakeholder Awareness concepts have high CCM values in manager group FCM, and Financial Support concept has high CCM value in farmer and manager FCMs. Therefore, these concepts are the most important (influential) concepts in FCMs and could be considered as the most appropriate and effective concepts for policy scenario simulations.

Several policy scenarios were simulated in the condensed group and social FCMs to provide useful recommendations for “Mitigating Groundwater Degradation and Depletion”. The objective is to find policies (concepts) that most strongly influence the concept Groundwater Improvement. As an interconnected systems, all concepts that significantly impact Groundwater Improvement will find their steady states impacted in the simulation. Initially, the systems consisting of group FCMs and the social FCM at the level of concepts were simulated to find the steady state of concepts under the present conditions. These outcomes indicate the eventual state of systems if the present conditions persist, according to the perceptions of the groups of stakeholders. Then, several policy scenarios were applied by fixing the above mentioned influential concepts at high values, for example 1, throughout the simulation process to see if the system reaches desirable states towards mitigating the problem. Specifically, these policy scenario simulations were implemented to test the effectiveness of each of the selected influential concepts. Table 10 summarises the outcomes of the applied policy scenario simulations in group and social FCM systems. It shows the contributions of the selected influential concepts to Groundwater Improvement. Finally, we analysed the outcomes of these scenario simulations in search of appropriate recommendations based on how much the scenarios contribute to Groundwater Improvement.

As a result, we proposed the following three useful recommendations to the decision makers of water management in Jordan: (a) use modern technologies to recharge groundwater basins in Jordan; (b) prevent the overuse of highland water that lead to over-exploitation of groundwater; and (c) reinforce and activate deterrent and effective laws to reduce theft and exploitation of unlicensed wells and protect groundwater resources from attacks and illegal use.

6. Conclusions and future work

This article proposes a robust semi-quantitative FCM model that can address and model various complex contemporary real-life problems. The advantages of this model over other previous FCM models lie in its ability to make the FCM model more com-
putationally robust, consistent, functional and efficient. This is achieved by the enhancement of the existing processes of the FCM approach or adding new ones. The proposed FCM model consists of eight coherent and consistent processes. These processes model the problem from the phase of data collection in the form of ambiguous perceptions to the phase of knowledge extraction in the form of solutions or recommendations. The model collects comprehensive data (FCMs/perceptions) using qualitative interviews with a sufficient number of relevant stakeholders. It allows, represents and deals with multiple types (symbolic and numeric) of imprecise human perceptions throughout all computational processes without information loss or inconsistency using a 2-tuple fuzzy representation method. It introduces a novel Consensus Centrality Measure and credibility weight for nodes and FCMS to identify important nodes and FCMS as well as to properly incorporate differences in nodes and FCMS during FCM condensation and aggregation. It includes a novel semi-quantitative condensation method for FCM which consists of multi-level condensation to simplify a complex and large FCM into a smaller and simpler one for easier understanding, traceability and predictability in a way that information is not lost between levels of condensation. It provides a fuzzy weighted aggregation method to obtain a reasonable consensus FCM from many relevant FCMS/stakeholders. It analyses the structure of the FCMS to probe into the reasonableness of representation and understanding of the problem. Finally, it allows simulation of FCM systems to study their steady states (status quo) and examines them for several policy scenarios based on identified influential variables to select scenarios that could shift the system behaviour towards a more desirable outcome than the status quo. Accordingly, appropriate recommendations or solutions could be defined and introduced to decision makers for addressing the problem concerned. To test the applicability of the proposed FCM model, we applied it in a real life problem—Mitigating Groundwater Degradation and Depletion in Jordan. Accordingly, useful outcomes were acquired in the form of recommendations that could help decision makers toward mitigating the problem. In a future publication, the authors would report on the detailed results from a successful application of this model to a more comprehensive problem and one of the acutely stressful real-life problems, namely “Mitigating the Water Scarcity Problem in Jordan”. Finally, we have noticed that there could be room for improvement in the process of obtaining credibility weights for variables and FCMS using other methods besides CCM measure. In addition, the qualitative process of the selection of group variables at the higher level of condensation should be replaced by quantitative methods such as intelligent computing methods. These limitations are the focus of our future work.

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References


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