DDP: A Dynamic Dimensioning and Partitioning model of Virtual Private Networks resources

Abdallah Jarray, Ahmad Nahar Quttoum, Hadi Otrok, Zbigniew Dziong

A R T I C L E   I N F O

Article history:
Received 11 June 2011
Received in revised form 3 February 2012
Accepted 3 February 2012
Available online 14 February 2012

Keywords:
Virtual Private Networks
Network resource dimensioning
Bandwidth reservation
VCG pricing mechanism
Linear Programming

A B S T R A C T

In this paper, we consider the problem of optimizing the Virtual Private Network Provider's (VPNP) profit by providing a periodic Dynamic Dimensioning and Partitioning (DDP) model for utilizing network resources. In literature, Complete Sharing (CS), Complete Partitioning (CP), and Bandwidth Borrowing (BR) techniques have been proposed for resource allocation where the following limitations can be noticed: VPN operators can exaggerate their needs for resources so the resources might be underutilized and optimal bandwidth utilization is not guaranteed. Moreover, all the aforementioned techniques use a static dimensioning of network resources. Hence, it may result in a high blocking of VPN connections and resource underutilization resulting in reduced profit. To overcome the above limitations, we propose to dynamically partition the resources over different QoS classes and dynamically dimension the capacity of the required links. The DDP model will run periodically through auctions that can reduce the reasoning of exaggeration and maximize VNP's profit. Moreover, our solution guarantees the reservation of the previously allocated resources for the coming auction periods. Thus, we formulate our problem as Mixed-Integer Linear Program (MILP) that allows us to: (i) minimize the network dimensioning cost, (ii) maximize the VNP profit through the Profit Percentage Parameter (PPP), (iii) provide the optimal bandwidth division of each network link among Quality of Service (QoS) classes, and (iv) calculate the optimal routing of new VPN connections without disruption of the demands accepted in previous periods. To reduce the impact of resources' reservation, we propose a pricing approach based on Vickrey–Clarke–Groves (VCG) mechanism that is able to reduce: (i) the risk of profit loss due to reservation, and (ii) the risk of having unutilized resources.

© 2012 Elsevier B.V. All rights reserved.

1. Introduction

Virtual Private Networks (VPNs) are becoming widespread as an efficient data transport solution in today’s global network infrastructure. These VPNs use the Internet Service Providers (ISPs) public network infrastructure to establish secure and reliable services according to contracted Service Level Agreements (SLAs) [8]. In such networks, resource management is one of the main challenges facing the ISPs, where each VPN may have different requirements along with varying Quality Service (QoS) guarantees. The growing demands for such VPN services with certain QoS satisfaction conditions necessitate the ISPs to efficiently utilize their bandwidth resources. Models to provide an efficient and effective resource allocation schemes are extremely important. Relying on the current management models [3,5,11], that attempt direct interactions with the ISPs, is not satisfying anymore, as they have many limitations that can be summarized as follows: (1) These models increase the management operation expenses. (2) They provide slow response times. Such limitations can lead to high rates of users dissatisfaction. Thus, in recent years, there has been an increasing interest to provide efficient automated management models for resource allocation in VPNs. Complete Sharing (CS) [12,14] and Complete Partitioning (CP) [12,11,8] models have been proposed in literature to cope with such resources management problems by creating a framework for automated management. In CS, network resources are shared over all classes without any division. While, in CP, resources are statically divided among QoS classes where each class uses its own allocated resources. In CS approach, resources are allocated in a First Ask First Allocate (FAFA) scheme, where one QoS class can overwhelm all other classes in a way that reduces the VPN Provider (VPNP) profit. In CP approach, resources can be underutilized which reduces...
VPNP profit. Due to the long-term SLA agreement, CS and CP have the following common problem. VPN operators might exaggerate about their required resources to guarantee their QoS and to cope with any unpredicted variations in the network state. Such a behavior can affect provider's profit, and maximize VPN demands blocking rates. Last but not least, the CS and CP models assume the static dimensioning (fixed capacity) of network links. Accordingly, the blocking rates of VPN connections, due to a scarce of network resources, can be increased.

To enhance the resources utilization, Bandwidth Borrowing (BR) technique has been proposed, [6,13,20], to enable the VVPN to provide better resource utilization that guarantees the QoS for all VPNs. According to the dynamicity of the traffic, the BR technique allows the ISP to borrow the extra resources of the under loaded links and reallocate them to the overloaded ones. Note that the BR cannot provide an optimal bandwidth utilization solution when there are many under-loaded links and no overloaded ones.

To overcome the above problems, we propose deploying a Dynamic Dimensioning and Partitioning (DDP) approach for allocating the network bandwidth to VPN connections. In DDP, the dynamic class division process will take place in a periodic auction manner, where VPN operators will be competing to win the resource allocations through their submitted prices and connections' demands. The network planning time is divided into a set of short periods in order to reduce the reasoning of exaggeration and to eliminate the needs for any borrowing technique due to the absence of any under loaded links. At each new period, the profitable VPN connections are selected through an auction mechanism in order to maximize VVPN's profit. Moreover, the DDP allows the VVPN to dimension the network bandwidth capacity based on customers' requirements. Such a dimensioning of resources will reduce the network costs which maximizes the VVPN's revenue. The outcome of this approach will lead to the improvement of bandwidth utilization which will be beneficial for both the VVPN and customers. Furthermore, it will result into less blocking of VPN connections, a substantial reduction of the network cost, and becoming more competitive in the market.

Another important contribution of this approach that has not been addressed in previous work (e.g. [19,18]) is the time-reservation constraint. Most references cover independent auctions, where acceptance in one auction does not guarantee the acceptance in the next one. Accordingly, there is no bandwidth reservation for VPN connections that last for multi-periods. Future bidders are allowed to compete with incumbent ones. This is not practical for many VPN customers especially in the case when a VPN client wants to participate in the auction for video on-demand content for about 60 min long. Assuming that the auction will run every 50 min then he will not participate, if there is a risk of losing the resources close to the end of a Hockey game. In this work, we impose the condition that once allocated, resources are reserved for the service duration that typically lasts for more than one period of time. Future bids for new VPN connections are not allowed to displace incumbent users. There is no contention in future auctions. Thus, the VVPN has to assume the risk of loss in future auctions, where new VPN connections can offer better bids for the reserved resources.

To overcome the misuse of the periodical auction, especially during the periods where bandwidth unit cost is low, we propose using a dynamic pricing approach based on Vickrey–Clarke–Groves (VCG) mechanism. At the beginning of each new period, the reserving VPN operators are charged based on the inconvenience they cause to the VVPN profit. This inconvenience is measured as the profit losses occurred due to the existence of their connection on the network, where keeping their connections for multi periods requires reservation certain paths to hold them. This may increase the original cost per bandwidth-unit of the used links and consequently reduces the expected VVPN profits. Consequently, for each reserving demand, this inconvenience value to be added to a Profit Percentage Parameter (PPP) that is defined by the VVPN according to its revenue objectives.

To optimize VVPN's profit, we formulate the network dimensioning approach as a Mixed-Integer Linear Program (MILP) where the dynamic pricing is formulated based on VCG mechanism. Our approach is able to come-up with:

- The optimal VVPN profit through the maximization of the PPP, and
- The usage of a pricing VCG mechanism for reserved bandwidth.
- The minimum network dimensioning cost.
- The optimal bandwidth division of each network link among QoS classes.
- The optimal routing of the new VPN connections without making disruptions to the accepted demands in previous periods.
- A guaranteed reservation of resources, for users sessions, lasting for more than one period of time. Indeed, resources allocated in a given auction are reserved for the entire duration of the connection, not subject to future contention.

The remainder of the paper is organized as follows: Section 2 presents the related work. Section 3 defines the modeling of VPN demand. Section 4 illustrates the Dynamic Dimensioning and Partitioning model and compares it to benchmark approaches. Section 5 lists the proposed performance metrics, followed by the computational results in Section 6. Finally, Section 7 concludes the paper.

2. Related work

The Complete Sharing (CS) model has been proposed in [14] as a solution for the autonomic resource management issue, by enabling the users to self-managed the network resources. The CS model provides a solution to the challenging management loads at the VVPN's side, and delivers a satisfying resource utilization rates. However, it is proved that CS can lead to SLA violations. To overcome such a drawback, the Complete Sharing (CP) model has been proposed in [12]. The authors claimed that CP model may provide better QoS satisfaction rates than CS. However, in the other hand, CP may provide a less resources utilization ratio.

The Virtual Partitioning (VP) scheme has been proposed in [4,3,5] in order to handle the resource utilization problem of the CP model. The VP approach behaves as a CS at the light traffic case, while as a CP at the extreme one, [7]. The VP approach has been proposed also in [22] to handle the resources allocation in the wireless networks domain. Indeed, authors proposed using a balancing scheme that combines the open sharing and the static allocation properties of the CS and CP models, respectively. Accordingly, resources are allocated depending on the current network traffic load. In heavy-load situations, resources are allocated the same way in CP, but underutilized resources are borrowed from the under loaded classes to the overloaded ones. In the light-loaded situations, overloaded classes can use the nominal resources of all other classes. Similarly, in [9,13] the authors proposed a combination of bandwidth reservation and bandwidth sharing approaches in order to provide higher resources utilization rates and QoS guarantees in the context of multimedia wireless networks.

The VP approach overcomes all the aforementioned drawbacks of the CP model, but on the other hand it can lead to SLA violations. Indeed, the lender links (originally under-loaded) have no guarantees that they can return their resources back when they are needed. Consequently, this encourages malicious over-loading. Furthermore, the VP approach attempts a static design. Indeed, a pre-defined static configuration for the resource sharing process is applied at all possible traffic load conditions. To overcome such
3. VPN demand modeling

The dynamic VPN dimensioning can have a different meaning depending on the used management approach, and the network control plan. Clearly, in any VPN network, demand do not remain static, and the lower the network layer, the less frequent are the changes. In this work, VPN demand is represented in terms of periodic auctions. In each new period, VPN customers are asked to reveal their connection demands in terms of: source destination nodes, and bandwidth reservation for users sessions that last for more than one period of time. VPN resources are allocated to the bidders (VPN operators) who win the auction, i.e., who offer the lowest bandwidth cost. To win the auction, auctioneers (VPN operators) should avoid any extra payments in order to submit a competing bid. Deploying such periodic allocations will motivate the VPN operators not to exaggerate, since current allocations are valid for short times only, and so, there is no need to care about the future demand changes. However, PA model cannot solve the exaggeration issues for customers’ connections that last for more period of time. The assigned bandwidth for accepted VPN connections in given auction period can be a subject of future contention.

Moreover, PA and all the aforementioned approaches use a static dimensioning of network resources. It is assumed that all network links have fixed capacities. All network links are dimensioned to provide the same number of VPN connections every period, while traffic density changes. Furthermore, most often, it is assumed, that all network links are usually dimensioned with uniform capacity, while traffic pattern varies in time. Then, links may be under-loaded, while others over-loaded, which may cause end-to-end delays.

To overcome the limitations of the aforementioned approaches, we propose deploying the DDP approach. The VPNP provides required resources based on demand requirements. Such a resources dynamic dimensioning approach will result into a substantial reduction of the capital network cost, and an increasing of the VPNP competitiveness in the market. Furthermore, it will increase the customers’ satisfaction in terms of QoS, i.e., no more blocking of VPN connections due to a bandwidth scarce.

3.1. Single request demand

In the context of single request provisioning, new incoming connections are dealt with one at a time. A single routing path has to be established, one at a time, and independently of the other paths. Delay has usually to be kept at minimum between the connection request and the provisioning. The typical performance metric is the blocking VPN connection probability.

3.2. Small batch demand

In the context of a longer time frame made of hours up to a week, the objective evolves from minimum blocking rate to minimum cost of network design. Indeed, we are in an efficient network management or in network planning context, so the objective is often to serve all VPN connections at minimum bandwidth cost. In such a context, the traffic model can be described with a set of traffic matrices, one for each new period. From one period to the next, we assume that a significant fraction of the traffic demand remains the same, representing as an example the global steady state of VPN demand or the long term SLAs between the VPN and its customers. The variation of the demand corresponds to the add or the drop of some VPN connections. Each ending VPN connection releases an amount of bandwidth which can be reused to accept some new VPN connections. In more accurate manner, let $T$ be the set of network planning periods, indexed by $t \geq 1$ and $0^0$ the initial set of VPN connections, indexed by $k$. At the beginning of period $t$, the set of VPN connections is defined by:

$$K(t) = K(t - 1) + K_{ADD}(t) - K_{DROP}(t)$$

where $K(t - 1)$ is the set of accepted VPN connections at the beginning of period $t$, $K_{ADD}(t)$ ($K_{DROP}(t)$) is the set of new incoming (resp. ending) VPN connections at the outset of period $t$.

While the proposed network dimensioning scheme apply for the two illustrated classes, we will use, in this paper, the terminology of the second class of traffic modeling.

4. Dynamic Dimensioning and Partitioning approach

In this Section, we propose an efficient way to model the dynamic demand in VPN. Next, we present our Dynamic Dimensioning and Partitioning (DDP) model that improves the CS, CP, and BR techniques, and overcomes their limitations. We propose an approach that offers a dynamic partitioning of network resources with bandwidth reservation for users sessions that last for more than one period of time. VPN resources are allocated to the bidders based on periodic auctions. In each new period, VPN customers are asked to reveal their connection demands in terms of: source destination nodes, required QoS classes, and their offered prices (bids).

4.1. VPN Dimensioning scheme

The network dimensioning is done periodically, aware of the new VPN connections and the demands accepted in the previous periods. The VPNP adjusts the bandwidth capacity of network links in order to reach a minimum cost topology. At the beginning of each new period $t$, a set of VPN connections (collected during period $t - 1$) bids for an end-to-end fixed amount of bandwidth depending whether we deal with traffic engineering, or network planning (see [15] for definitions), a dynamic VPN demand has a different interpretation. We attempt, in the following, to clarify the various contexts of dynamic demand that have been distinguished in the literature studies and consequently, we make sure of the interpretation of the periodical VPN demand in this work.
providing a given QoS. The VPNP takes one-time decision and selects the best profitable routing scheme of new VPN connections.

The VPNP guarantees no disruption of demand accepted in previous periods and still active at the current period. The VPNP wants also to consider the following questions:

- How much bandwidth to put on each network link in order to satisfy all the VPN demand.
- How to assign the network bandwidth to the VPN connections in order to maximize the PPP? In more details:
  - Calculate the optimal routing path for each VPN connection,
  - Calculate the optimal PPP while accepting all the VPN connections.
- How to overcome the drawback of reservation? In other words: (i) minimize the risk of profit losses in future auctions due to the fact that new VPN connections can offer better bids for the reserved bandwidth, (ii) minimize the unused amount of reserved bandwidth, and (iii) reduce the abuse of reservation where VPN customers can bid only during the periods where bandwidth unit cost is low.

To do so, we propose to adopt the following model that consists of three phases. At each new period \( t \), the network dimensioning is done through the following phases:

**Phase I:** Calculate the unit cost to be used for the pricing of the available bandwidth. We use a set of discrete amounts of bandwidth at different prices, which can be modeled through a stepwise Capacity Cost Function as depicted in Fig. 1. This phase is presented in Section 4.2.1.

**Phase II:** The network links are dimensioned according to the optimal routing of VPN connections, and the optimal PPP. To do so, we use the Linear Programming methodology that will be illustrated in Section 4.2.1.

**Phase III:** To overcome the drawbacks of resources reservation, we propose a dynamic pricing based on the inconvenience caused to the new VPN connections.

Hence, the VPN operators’ utility function can be represented as follows:

\[
\text{utility} = \text{payoff} - \text{cost}
\]

where \( \text{payoff} \) is the expected gain from the granted connections and cost is the paid price of those connections. Assuming that these VPN operators are rational, users aiming to maximize their own utility function will avoid reservations unless the payoff gained is greater than the cost. The calculation of the cost is presented in SubSection Section 4.2.3 where we adopted the VCG mechanism\(^\dagger\) to calculate the profit losses value represented by the inconvenience value caused by reservation.

### 4.2. Mathematical modeling

The VPNP network topology is represented by a directed graph \( G = (V, L) \), where \( V \) denotes the set of nodes and \( L \) the set of directional links. We assume that each couple of network nodes are connected by two directional links, one in each direction. Each link \( l \) offers \( b_l \) bandwidth and has a cost unit \( c_l \). We denote by \( I = \{1, 2, \ldots \} \) the set of VPN customers. We denote by \( K(t) = \{\text{the set of all VPN connections at period } t, \text{where each connection has a source node } s_k, \text{a destination node } d_L, \text{a QoS class } j \in J, \text{and belongs to a VPN customer } i \in I\} \). Note that a VPN connection \( k \) belonging to a QoS class \( j \) uses bandwidth \( b_j \).

\( \chi(t) \): percentage of link \( l \) bandwidth reserved for the usage of VPN connections belonging to QoS class \( j \) at period \( t \).

- \( K_t \): maximum number of links forming each path used by a VPN connection \( k \) belonging to QoS class \( j \).
- \( P_t \): offered single bid for end-to-end bandwidth required to accept VPN connection \( k \) of QoS class \( j \) at period \( t \).
- \( L_t \): number of link/hop forming the path \( \pi \).
- \( T_t \): set of candidates paths that can be used to join destination node \( d_L \) from source node \( s_k \).

\[ P_{\text{thr}}(t) = b_j \times c_i(t) + b_j \times c_i(t) \times \alpha(t) \quad (1) \]

where \( c_i \) is the bandwidth original cost-unit over the link \( l \), and the parameter \( \alpha(t) \) refers to the profit percentage parameter (PPP) at period \( t \). This PPP reflects a tradeoff relationship between the VPNP profit objective and the blocking ratio of VPN demand. Consequently, as long as VPN connection bid \( p_k \) is greater than the target blocking threshold, optimal VPNP profit can be achieved by assigning a higher \( \alpha(t) \) value. The motivation behind this scheme is as follows:

- It reduces the likelihood of exaggeration. Indeed, with such a scheme, the VPN users will be motivated to use the lowest amount of bandwidth resources that satisfy their needs, especially is the QoS class that use a lot of resources.
- It limits situations where one QoS class overwhelms the others. This provides a kind of fairness between the classes. As presented in the aforementioned section, the network dimensioning is done through phases that will be executed sequentially at each new planning period \( t \).

#### 4.2.1. Calculation of the bandwidth unit cost

- From the previous history of period \( t \), find the average amount of bandwidth used over each link \( l \):

\[
\overline{B_l(t)} = \frac{\sum_{p=1}^{T_t} B_{l_p}^p}{T_t}
\]

Where \( B_{l_p}^p \) is the amount of bandwidth used on link \( l \) at period \( p \). \( T_t \) is the number of periods previous to period \( t \).

- Using the pricing curve of Fig. 1, find the dynamic cost \( C_l(t) \) of bandwidth \( B_l(T) \), and then calculate the bandwidth unit-cost for each link \( l \) as following:

\[
c_l(t) = \frac{C_l(t)}{B_l(t)}
\]

The pricing curve from Fig. 1 is used to:

1. Emulate the case of having upper bound limits for the bandwidth resources at the
providers side, for the reasons of load balancing at the network links. (2) Motivate the VPN operators to follow a desired behaviour and not to exaggerate. Note that this type of pricing is being used in several practical domains. In particular the "luxury tax" corresponds to such a pricing. The first or business class in the airplane industry are good examples where the user pays several times more for an incremental increase of used resources. Similar concept of pricing is applied by most cellular operators in North America where the monthly charge covers certain number of minutes but if user exceed this limit the price per minute rises drastically and the same applies to roaming charges.

4.2.2. Modeling with Linear Programming
We use the Linear Programming methodology to model the network dimensioning process. To do so, we define the following MILP:

The variables:
Before defining the MILP model we need to define the following decision variables.

- Selection of VPN connections:
  \[ z_k(t) = \begin{cases} 1 & \text{If VPN connection } k \text{ corresponding to bid } P_k(t) \text{ is accepted} \\ 0 & \text{Otherwise.} \end{cases} \]

- Selection of VPN connections routing paths:
  \[ x_{kl}^{p}(t) = \begin{cases} 1 & \text{If VPN connection } k \text{ uses path } \pi \text{ at period } t \\ 0 & \text{Otherwise.} \end{cases} \]

- Selection of routing path links:
  \[ y_{l}^{p}(t) = \begin{cases} 1 & \text{If path } \pi \text{ uses link } l \text{ at period } t \\ 0 & \text{Otherwise.} \end{cases} \]

- We define the variable \( b_{l}(t) \) in order to calculate the amount of bandwidth to be setup on a link \( l \) at period \( t \) for the usage of VPN connections belonging to QoS Class \( j \). We bound such a variable as following:
  \[ b_{l}(t) \leq b_{\text{MAX}} \]
  \( b_{\text{MAX}} \) is an upper bound on the bandwidth that can be setup on a given link in order to guarantee a certain load balancing of traffic among network links. \( b_{l}(t-1) \) is the amount of bandwidth used by VPN demand accepted in the previous period \( t - 1 \), and still active at the current period.

- We define the variable \( a(t) \in [0, 1] \) in order to calculate the PPP to be collected from all bidders at period \( t \).

The objective function:
\[
 f_{\text{obj}}(t) = \min A \times \left( \sum_{k \in K(t)} \sum_{l \in \pi(t)} c_l(t) \times b_{l}(t) \right) - B \times \left( \sum_{k \in K(t)} \sum_{l \in \pi(t)} a(t) \times b_{l}(t) \right)
\]

At each new period \( t \), the main objective is to maximize the VPNP profit. To do so, we adapt the bandwidth capacity of network links in order to serve all the VPN demand while minimizing the cost of required bandwidth (first term of Eq. (2)), and maximizing the PPP \( a(t) \) (second term of Eq. (2)).

The constraints:

- Link bandwidth division among QoS classes:
  \[ b_{l}(t-1) + \sum_{k \in K(t)} \sum_{\pi \in \pi(t)} \eta_{l}^{\pi} \times x_{kl}^{\pi}(t) \leq b_{l}(t); \quad l \in L, \quad j \in J \] (3)

The bandwidth assigned to each QoS class \( j \) is equal to the sum of:
- The bandwidth reserved for VPN connections of class \( j \) accepted in previous periods, and still active at the beginning of the current period.
- The added bandwidth required to accept the new VPN connections of class \( j \) \( (K_{\text{add}}(t)) \).

- Links capacity constraint:
  \[ \sum_{j \in J} b_{l}(t) \leq b_{\text{MAX}}; \quad l \in L \] (4)

The bandwidth capacity of a given link is upper bounded by \( b_{\text{MAX}} \) in order to guarantee a certain load balancing of the traffic on all the network links.

- Demand satisfaction constraint:
  \[ \sum_{\pi \in \pi(t)} x_{kl}^{\pi}(t) \leq 1; \quad k \in K(t) \] (5)

Only one routing path is selected for each accepted VPN connection.

- Linking variable constraint:
  \[ z_{k}(t) \leq \sum_{\pi \in \pi(t)} x_{kl}^{\pi}(t); \quad k \in K(t) \] (6)

If there is no way to route the connection, the connection is rejected.

- Calculation of the PPP:
  Which is the optimal PPP value that guarantees to accept all VPN connections?
  \[ \sum_{\pi \in \pi(t)} x_{kl}^{\pi}(t) \sum_{l \in \pi} P_{l}(t) \leq P_{k}(t); \quad k \in K(t), \quad j \in J \] (7)

We recall that the bid threshold for each couple of network link and QoS class is defined as follows:

- End-to-End routing delay constraint:
  \[ L_{\pi} \leq H_{j}; \quad k \in K(t), j \in J, \pi \in \pi(t) \] (8)

The upper bound on the number of hops per routing path is an indirect way to bound the delay authorized for each QoS class. Indeed, the end-to-end delay of each VPN connection is measured through the number of hops of the selected path.

- Demand serving capacity of VPNP:
  The dimensioning of the network links can be done with the respect to an overall demand serving capacity \( \gamma(t) \):
  \[ \sum_{k \in K(t)} z_{k} \geq (1 - \gamma(t))|K(t)| \] (9)

Quadratic terms and convexity problem:
We note that the objective function and constraints (7) include quadratic terms \( x_{kl}(t) a(t) \). Thus, our model is quadratic MILP and cannot be solved optimally using any MILP solver, unless the objective function is convex and constraints (7) define a convex region. We checked the convexity of our MILP using CPLEX. We concluded from CPLEX output that neither the objective function nor the quadratic matrix of constraints (7) are Positive Semi-Definite and this lack of convexity comes from the fact that the quadratic term \( x_{kl}^{p} a(t) \) is the product of one binary variable and one real variable.

However, as we use in our simulation, CPLEX solver, it is possible to solve this quadratic MILP in the special case where qua-
drastic terms (in the objective function and constraints) are product of binary variables. Therefore, we propose to discretize the real variables \( a(t) \) into binary variables. To do so, the solution is simple and consists of employing the standard method widely used by computers to represent numbers, which means using the sum of powers of two that is also known as binary base. Thus, the sum shown in the following equation represents the variable \( a(t) \).

\[
a(t) = \sum_{i=0}^{n-1} c^i \alpha_i(t)
\]

The coefficients \( c^i \) determine if the \( i \)-th power of two is part of the sum or not.

\[
\alpha_i = \begin{cases} 
1 & \text{if the element } i \text{ of } c^i \text{ is used to represent } a(t) \\
0 & \text{otherwise}
\end{cases}
\]

where \( c^i \) is the element \( i \) of the base vector that represents variable \( a(t) \). For simplicity, we have used as base vector, \( c^i \).

Or the profit percentage parameter \( a(t) \) is a value between 0 and 1 and we assume that the required accuracy of VPNP is equal to 0.01, i.e. 1% which is a practical value in terms of profit percentage. Thus, we choose \( A = 6 \) and we divide the unitary vector of base \( c^i \) by 100. Then any value of \( a(t) \) belonging to interval \([0, 1]\) can be represented in the base:

\[
c^i = (0.01, 0.02, 0.04, 0.08, 0.16, 0.32, 0.64)
\]

### 4.2.3. Pricing mechanism for reservation

In the proposed DDP approach, resources are allocated through periodical auction. At each new period of time the VPNP takes one-time decision and selects the best profitable routing scheme based on bids collected during each period of time. VPNP guarantee that once an amount of bandwidth has been allocated in a given period, the VPN connection has the reservation for the duration of the service. Accordingly, the VPNP must assume the risk of future auctions. Indeed, new VPN connections can offer better bids for the reserved resources. To deal with such a risk, and in order to minimize the amount of the unused reserved bandwidth, we propose a pricing scheme based on the well-known VCG incentive compatible mechanism [16].

In this approach, the inconvenience that each VPN operator causes to the whole network according to its reserved required bandwidth is defined in terms of utility drop that was caused to the whole network. The resulting value is denoted by the profit lost, where this profit lost value will be added to the original charge of the reserved bandwidth, which means less revenue rates achieved. Consequently, VPNPs’ operators will never tend to reserve unused bandwidth for长时间，as they know that such a behavior will decrease their revenue. Such an inconvenience is measured based on the variation of the PPP value between old and new VPN connections as the following. At each period \( t \):

- We denote by \( K(t) \) the set of old/accepted VPN connections at previous periods and still active at the current period \( t \), i.e., the set of VPN connections holding bandwidth reservation.
- For each VPN connection \( k \in K(t) \), find the new pricing \( P_k(t) \) of the current period, based on the inconvenience \( I_k(t) \) it caused to the new VPN connections as following:

\[
P_k(t) = P_k(t) + I_k(t)
\]

To measure the reservation inconvenience \( I_k(t) \) for each VPN connection \( k \in K(t) \) routed on path \( \pi_k \), we do as following.

- Calculate the original cost \( C(t) \) of the bandwidth \( B(t) \) used by new VPN connections \( K(t) \) at period \( t \) as following:

\[
C(t) = \sum_{k \in K(t)} \sum_{c \in cA} b_c c(t)
\]

Where \( \pi_k(t) \) is the routing path selected to route VPN connection \( k \) at period \( t 
- We denote by \( a(t) \) the PPP of old VPN connections \( K(t) \), and \( a(t) \) the PPP of new VPN connections \( K(t) \). We calculate the profit lost \( L(t) \) at period \( t 

\[
L(t) = C(t) \times (a(t) - a(t))
\]

- Calculate the profit lost per unit of bandwidth \( I_k(t) \) as following.

\[
I_k(t) = \frac{L(t)}{B(t)}
\]

- Charge each active VPN connection \( k \in K(t) \) as following:

\[
P_k(t) = P_k(t) + b_k \times L(t) \times I_k(t)
\]

where \( L(t) \) is the number of links in the routing path \( \pi_k \) used by a connection \( k \) in \( K(t) \).

### 4.3. Benchmark model

In DDP, the dynamic class division process will take place in a periodic auction manner, where VPN operators will be competing to win the resource allocations through their submitted prices and connections’ demands. The network planning time is divided into a set of periods in order to reduce the reasoning of exaggeration and to eliminate the needs for any borrowing technique. Thus, there is no need for such a BR technique since there is no underloaded links as a result of exaggeration. To compare our model, we selected the static dimensioning and partitioning CP [12] over the CS since the latter is a heuristic approach based on FAFA scheme. Moreover, we compare our model with the independent Periodical Auctions (PA) [18] approach, where PA can be seen as a variation of DDP approach that uses a static dimensioning of network resources without any inter-temporal considerations for VPN demands that last for more than one period of time (no reservation).

#### 4.3.1. PA Model

As mentioned previously PA uses a static dimensioning of network resources. Indeed, network links are dimensioned with fixed a uniform amount of bandwidth at the beginning of the network design. Network resources are allocated based on repeated auction, with the aim of selecting the best set of VPN operators from a competing environment. The process of selecting the winning set of VPN connections for resource allocation deploys an optimal selection algorithm presented in Table 1. In which, at each new period \( t \), the VPNP firstly collects the new VPN connection received in the period of \( t - 1 \) and forms the new VPN demand matrix. Next, PA approach formulates the bandwidth allocation problem as an Integer Linear Program (ILP).

In PA model, VPN operators own of VPN connections are assumed to be rational, and thus their aim is to have their connection demands admitted with the lowest possible prices, and at the same time acquire satisfactory QoS levels. On the other side, the VPNP aims to utilize its network bandwidth resources better, and maximize its profit by accepting the maximum number of VPN operators while providing them with satisfactory QoS levels by
competing prices. Accordingly, the VPNP profit \( P_B \) function can be expressed as:

\[
P_B = \max \left( \sum_{k=1}^{K} p_k - \sum_{k,J} c_k \right)
\]

(10)

where the \( P_B \) equals the sum of bids \( p_k \) collected from connection demands being admitted for allocation, reduced by the total cost of the bandwidth resources required to satisfy each connection demand \( c_k \). Accordingly, to maximize the \( P_B \), the VPNP has to choose the best set of VPN operators’ connections that maximizes the first term of the function, and also, deploy an efficient bandwidth utilization scheme in order to minimize the second term. By solving this ILP, the VPNP chooses the most profitable set of bidding VPN operators. We note that the assigned bandwidth for accepted VPN connections in a given auction period is available only for the duration of the current period. There is no bandwidth reservation for a VPN connection that lasts for more than one period of time.

4.3.2. CP Model

The CP model selects the winning set of VPN connections for resource allocations using the selection algorithm presented in Table 1 except we add the following variations. As aforementioned previously, the CP models use a static partitioning of network resources. Indeed, the bandwidth division of network links among QoS classes is done at the beginning of the network planning (period 0) based on the shortest path algorithm, and the traffic pattern statistics. The resulting link partitioning among QoS classes is used for the coming periods.

5. Performance metrics

As mentioned in the previous section, we propose to use PA and CP models as benchmark to evaluate the performance of the DDP approach. To do so, we are measuring the following metrics:

5.1. VPNP Profit

The VPNP profit is measured based on the bids collected from the admitted VPN connections, reduced by the cost-unit of the carrier network links. The bids and the cost-units are expressed in terms of \( \text{Sx} \), which represents the price of 1 Mb of bandwidth. This metric represents the profits collected from the whole bandwidth resources allocated over the network links.

5.2. VPNP profit-unit

The profit-unit value represents the gains collected per 1 Mb of bandwidth, which is measured as the ratio between the VPNP profit, and the amount of bandwidth over the whole network links, as:

\[
P_{\text{BU}} = \frac{P_B}{\sum_{i} b_i}
\]

(11)

5.3. VPN Demands’ blocking ratio

Demands’ blocking ratio represents the VPN operators’ satisfaction rates, where it is measured as the ratio between the numbers of admitted VPN connections to the number of the whole VPN demand participants in the allocation auction, as:

\[
B_B = \frac{\sum_{k,J} x_k z_k}{|K|}
\]

(12)

5.4. Available bandwidth

The available bandwidth is measured as the sum of links bandwidth capacity, as:

\[
U_a = \sum_{i} \sum_{j} b_i^j
\]

(13)

5.5. Bandwidth utilization

Bandwidth Utilization is measured as the ratio between the used and the total bandwidth amounts, as:

\[
U_b = \frac{\sum_{i} \sum_{j} b_i^j z_k}{\sum_{i} b_i}
\]

(14)

5.6. Routing scheme efficiency

To evaluate the routing scheme efficiency of the above-mentioned scenarios, we proposed measuring the End-to-End Delay. The average end-to-end delay per admitted VPN connection is measured through the average number of hops used to form a routing path, which is given by the ratio between the number of hops counted in composing the whole routing scheme to the number of admitted VPN connections. Only routing paths of the admitted connections are counted. This can be formulated as:

\[
H = \frac{\sum_{k,J} \sum_{i} z_k x_i^j}{\sum_{k,J} z_k}
\]

(15)

6. Computational results

6.1. Network and traffic instances

To assess the efficiency of the proposed dynamic dimensioning model, we carried out experimental assessments of a VPN network that consists of 10 nodes, connected by 40 bidirectional links. The network planning time is divided into 12 classes of 6 h periods per week [21], representing a new allocation auction every 6 h as illustrated in Table 2. We assume that the VPNP is selling certain service packages that stands for 6 h of time or more, where there is a dependency between two consecutive periods, and each period starts by a new auction. Accordingly, with such dependency scheme, we are emulating the scenario of having bandwidth reservations for those VPN connections that last for multi-periods. Hence, bidders for new connections are not allowed to compete with incumbent ones. Independent allocation scenarios (those with no dependency between the allocation periods) is not practiced for many VPN customers, especially in the case when VPN customers ask for services that stand for multi-periods. In such a case, VPN clients should have guarantees that their periodic reservations are guaranteed for the duration of the full service. As an example, assume that a VPNP runs an allocation auction every 50 min of time (the period length), customers of a video service (e.g. a movie) that stands for 90 min of time will not be motivated to participate

| Table 1 |
|-----------------------------|-----------------------------|
| PA Selection Algorithm. | |
| 1: **Input:** At each auction round \( f \), the VPNP does: | 2: Collect the VPN connections \( k \) \( (p_k, QoS_k, s_k, d_k) \) received through time \((t=1:t)\); |
| 3: Formulate the problem as an ILP; | 4: Solve the ILP, and find the optimal set of VPN connections; |
| 5: **Output:** the VPN connections that won the resource allocations; and the VPNP profit collected from these VPN connections; | |
in the auction, since they do not have the guarantees that they will be able to win the next period auction. The VPN demand is described with a set of demand matrices, one for each period of time. From one period to the next, we assume that a significant fraction of the VPN demand remains the same. At a new period $t \in T$, we have ADD new VPN connections and DROP dropped ones, where ADD and DROP belong to the set 10%, 15%, 20%, 25%, 30%, 40%, giving us a range of cases from slowly fluctuating dynamic traffic instances (10%) to fast changing dynamic traffic instances (40%). At the beginning of the network dimensioning cycle, we consider an initial instance of VPN demand randomly generated and distributed among all the network node pairs.

6.2. Numerical results

Through this Section, we study the performance of the proposed DDP model compared to CP and PA in terms of: 1) Bandwidth usage measured through the available and used bandwidth. 2) VPNP total profits. 3) The blocking ratios of VPN demand. 4) VPNP profit-unit, and 5) Routing efficiency measured through the average end-to-end delay. Moreover, we are also presenting some results that show the performance of the pricing mechanism deployed in order to reduce the risk of profit lost in future auctions, and to minimize the unused amount of reserved bandwidth.

6.2.1. Analysis of QoS parameters

We are expecting that DDP model will serve all the VPN demand with a minimum bandwidth cost. There is no-blocking of VPN connections due to a bandwidth scarce. Consequently, we are expecting that the DDP will deliver the highest VPNP profits. Indeed, compared to the static dimensioning approaches, i.e., PA and CP, the DDP model is expected to manage better the tradeoff between minimizing the used bandwidth cost and maximizing the profit while accepting all the VPN connections.

Fig. 2 plots the resulting VPNP total profits vs. the allocation time periods. In this Figure, we are comparing the profit for the three aforementioned scenarios.

We are showing that the DDP model provides the highest profit, followed by the PA, and lastly the CP providing the lowest rates. The profit gap between the dynamic and the two static network dimensioning approaches solution varies from 9% to 58%. Fig. 3 shows the blocking ratios of the VPN connections vs. the allocation time periods. Notice that for the presented three models, blocking due to the offered bids is not considered. It is assumed that the whole receiving bids are higher than or equal to the resources original cost. The DDP accepts all demands, where there is no blocking due to the bandwidth availability. This is one of guidelines of the dynamic dimensioning approach. The DDP model provides bandwidth where is needed due to the bandwidth capacity adaptation. Similarly, the same performances as in the Figs. 2 and 3 can be noticed for the two static dimensioning approaches. Indeed, PA provided non-zero blocking ratios, followed by CP approach which shows the highest blocking ratios.

Fig. 4 plots the percentage of bandwidth utilization vs. the allocation time periods. In this Figure, the DDP model provides a utilization of 100% of the networks' bandwidth resources through all the planning period of time. The PA used an average of 40%, while it is around the average of 30% for the CP. This tendency is also confirmed through the results depicted in Fig. 5. Indeed, it is shown that the DDP model uses clearly the lowest amount of bandwidth, while accepting all the demand. The PA and CP approaches use more bandwidth, but on the contrary, they are unable to accept all the VPN demands.

<table>
<thead>
<tr>
<th>Period index</th>
<th>Corresponding time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Monday to Friday: Morning</td>
</tr>
<tr>
<td>2</td>
<td>Monday to Friday: Afternoon</td>
</tr>
<tr>
<td>3</td>
<td>Monday to Friday: Evening</td>
</tr>
<tr>
<td>4</td>
<td>Monday to Friday: Night</td>
</tr>
<tr>
<td>5</td>
<td>Saturday: Morning</td>
</tr>
<tr>
<td>6</td>
<td>Saturday: Afternoon</td>
</tr>
<tr>
<td>7</td>
<td>Saturday: Evening</td>
</tr>
<tr>
<td>8</td>
<td>Saturday: Night</td>
</tr>
<tr>
<td>9</td>
<td>Sunday: Morning</td>
</tr>
<tr>
<td>10</td>
<td>Sunday: Afternoon</td>
</tr>
<tr>
<td>11</td>
<td>Sunday: Evening</td>
</tr>
<tr>
<td>12</td>
<td>Sunday: Night</td>
</tr>
</tbody>
</table>

Fig. 2. VPNP profit $P_t$.

Fig. 3. VPN demand blocking $B_k$.

Fig. 4. Bandwidth utilization $U_b$. 

Table 2

<table>
<thead>
<tr>
<th>Period index</th>
<th>Corresponding time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Monday to Friday: Morning</td>
</tr>
<tr>
<td>2</td>
<td>Monday to Friday: Afternoon</td>
</tr>
<tr>
<td>3</td>
<td>Monday to Friday: Evening</td>
</tr>
<tr>
<td>4</td>
<td>Monday to Friday: Night</td>
</tr>
<tr>
<td>5</td>
<td>Saturday: Morning</td>
</tr>
<tr>
<td>6</td>
<td>Saturday: Afternoon</td>
</tr>
<tr>
<td>7</td>
<td>Saturday: Evening</td>
</tr>
<tr>
<td>8</td>
<td>Saturday: Night</td>
</tr>
<tr>
<td>9</td>
<td>Sunday: Morning</td>
</tr>
<tr>
<td>10</td>
<td>Sunday: Afternoon</td>
</tr>
<tr>
<td>11</td>
<td>Sunday: Evening</td>
</tr>
<tr>
<td>12</td>
<td>Sunday: Night</td>
</tr>
</tbody>
</table>
Thus, we can conclude that the DDP provides the best utilization ratio of bandwidth resources. Based on that, one can conclude the following. The DDP approach used the lowest amount of bandwidth resources, and at the same time, it provided the highest profits while accepting all VPN connections. PA model used more bandwidth compared to DDP, but on the contrary, it is unable to serve all the demand. Consequently, less profit and more blocking is noticed. The CP scheme used on average the same amount of resources as PA, but still, it provided the lowest profits along with the highest blocking ratios. Accordingly, the results confirm the aforementioned expectation that the static dimensioning approaches may result in high blocking of VPN connections, and the lack of profits to gain due to bandwidth scarce.

Fig. 6 plots the VPNP profit-units to the allocation time periods. Clearly, we can remark that the highest profit-units are always provided by the dynamic dimensioning approach, i.e., DDP, giving an average of $10.90 profit-unit compared with $7.80 and $3.25 provided by the PA and the CP, respectively. Accordingly, the DDP provides an average of 4 times to what the CP provides. Fig. 7, plots the average number of hops per path used to route the accepted VPN connections along the planning time periods. The results of this Figure confirms that although the dynamic dimensioning approach uses less bandwidth, it is possible to serve all the demand with a routing scheme that uses on average the same number of hops as PA and CP. Consequently, we can state that DDP calculates the most efficient routing scheme.

6.2.2. Simulation CPU time

The CPU time (calculation or convergence) for the considered simulation scenario varies from few seconds to few minutes (< 5 minutes) depending on the number of VPN requests. More specifically, we present in Table 3 the variation of CPU time (run-time) according to the number of requests [K] that varies from 10 to 200 VPN requests.

<table>
<thead>
<tr>
<th># Requests</th>
<th># variables</th>
<th># constraints</th>
<th>CPU time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>400</td>
<td>10</td>
</tr>
<tr>
<td>30</td>
<td>900</td>
<td>900</td>
<td>10</td>
</tr>
<tr>
<td>40</td>
<td>1600</td>
<td>1600</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>2500</td>
<td>2500</td>
<td>20</td>
</tr>
<tr>
<td>60</td>
<td>3600</td>
<td>3600</td>
<td>30</td>
</tr>
<tr>
<td>70</td>
<td>4900</td>
<td>4900</td>
<td>30</td>
</tr>
<tr>
<td>80</td>
<td>6400</td>
<td>6400</td>
<td>50</td>
</tr>
<tr>
<td>90</td>
<td>8100</td>
<td>8100</td>
<td>300</td>
</tr>
<tr>
<td>100</td>
<td>10 000</td>
<td>10 000</td>
<td>300</td>
</tr>
<tr>
<td>110</td>
<td>12 100</td>
<td>12 100</td>
<td>300</td>
</tr>
<tr>
<td>120</td>
<td>14 400</td>
<td>14 400</td>
<td>300</td>
</tr>
<tr>
<td>130</td>
<td>16 900</td>
<td>16 900</td>
<td>600</td>
</tr>
<tr>
<td>140</td>
<td>19 600</td>
<td>19 600</td>
<td>600</td>
</tr>
<tr>
<td>150</td>
<td>22 500</td>
<td>22 500</td>
<td>600</td>
</tr>
<tr>
<td>160</td>
<td>25 600</td>
<td>25 600</td>
<td>600</td>
</tr>
<tr>
<td>170</td>
<td>28 900</td>
<td>28 900</td>
<td>600</td>
</tr>
<tr>
<td>180</td>
<td>32 400</td>
<td>32 400</td>
<td>600</td>
</tr>
<tr>
<td>190</td>
<td>36 100</td>
<td>36 100</td>
<td>600</td>
</tr>
<tr>
<td>200</td>
<td>40 000</td>
<td>40 000</td>
<td>600</td>
</tr>
</tbody>
</table>

It is clear from the table that the larger the number of requests is the longer the simulation CPU time is. We found that the gap in performance compared to other heuristic models worth the runtime which is less than 10 min that is an appropriate time (much less than the provisioning period length of 6 h) to find the optimum. However, since we are not addressing the computational complexity in this paper, we are unable to give a clear relationship between CPU time and the number of VPN requests.

6.2.3. Pricing of reserved bandwidth

As mentioned previously in Section 4.2.3, we impose in the proposed DDP model the condition that once allocated bandwidth resources are reserved for the service duration that typically lasts for multi-period of time. There is no contention in future auctions. Accordingly, the VPNP have to assume the risk of loss in future auctions, where new VPN connections can offer better bids for
the reserved resources. To overcome such a limitation, we propose to charge the reserved bandwidth based on the inconvenience defined in terms of utility drop that was caused to the whole network (VPNP and new VPN connections). The resulting value is denoted by the profit losses and it is added to the original charge of the reserved bandwidth. Consequently, VPNNs’ operators will never tend to reserve unused bandwidth for long-time, as they know that such a behavior will decrease their revenue. We illustrate this pricing mechanism through the simulation results explained in the following paragraph.

In Table 4, we are showing the inconvenience of reserved bandwidth represented by the difference in the PPP values of the old and the new set of connections. We measured the total profit lost, and the profit per unit of bandwidth before and after pricing depending on the variation of the PPP values.

Through the periods from 2 to 12-period 1 is not considered since there is no bandwidth reservation—we are showing the PPP values with and without the reserving connections. At each period, the PPP value of the reserving connections will represent an upper-bound to any possible PPP value in the whole network. Accordingly, profit losses are expected. These losses represent the penalty to be paid by the reserving connection, each according to the bandwidth amount used.

Hence, for each reserving connection, the reservation penalty is calculated based on the profit loss per unit of bandwidth. This profit-loss unit to be added to the original cost-unit (per 1 Mb of bandwidth) used for the considered connection.

7. Conclusion

Using Linear Programming and Mechanism Design, we proposed the DDP approach for an optimal network design. The network links are dimensioned periodically in order to accept the new VPN demands, without disruption of the ones accepted in previous periods. Such a bandwidth reservation can reduce the reason of the exaggeration. The advantages of our model lies in its ability of: (i) finding the optimal tradeoff point of the used bandwidth amounts, and the VPNN profit by the calculation of the optimal PPP, (ii) providing the optimal routing scheme of new demand while guarantee no-disruption of demand routed in previous periods, and (iii) the optimal partitioning of links bandwidth among QoS classes. In addition, we proposed a pricing approach based on VCG mechanism in order to minimize: (i) the risk of profit lost in future auctions, and (ii) the unused amount of reserved bandwidth. The time-reservation of bandwidth is charged according to the inconvenience caused to the new VPN connections. Experiments were conducted using Concert Technology environment. We showed that the DDP model outperforms the benchmark approaches CP and PA. On average, the VPNN profit is increased up to 58%. Blocking of VPN connections due to bandwidth scarce is reduced. Finally, we showed that the DDP model is able to calculate the optimal PPP that maximize the VPNN profit while accepting more VPN connections compared to other models.

References