Improved image watermarking scheme using fast Hadamard and discrete wavelet transforms

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Abstract. We propose a robust image watermarking scheme by applying the fast Hadamard transform (FHT) to small blocks computed from the four discrete wavelet transform (DWT) subbands. Different transforms have different properties that can effectively match various aspects of the signal's frequencies. Our approach consists of four main steps: (1) we decomposed the original image into four subbands, (2) the four subbands are divided into blocks; (3) FHT is applied to each block; and (4) the singular-value decomposition (SVD) is applied to the watermark image prior to distributing the singular values over the DC components of the transformed blocks. The proposed technique improves the data embedding system effectively, the watermark imperceptibility, and its resistance to a wide range of intentional attacks. The experimental results demonstrate the improved performance of the proposed method in comparison with existing techniques in terms of the watermark imperceptibility and the robustness against attacks. © 2007 SPIE and IS&T. [DOI: 10.1117/1.2764466]

1 Introduction

The problem of multimedia protection for digital images has attracted a great deal of attention in research in the past few years. Its importance is increasing rapidly because of the growing problem of unauthorized replication. Watermarking refers to the process of adding a hidden structure, called a ‘watermark’, which carries information about either the owner of the cover or the recipient of the original data object. Such a watermark can be used for several purposes including copyright protection and transactional watermarks.

A variety of watermarking techniques has been proposed for digital images. These techniques can be divided into two main categories according to the embedding domain of the cover image: spatial domain methods and transform domain methods. The spatial domain methods are the earliest and simplest watermarking techniques but have a low information hiding capacity, and the watermark can be easily erased by lossy image compression. On the other hand, the transform domain approaches, insert the watermark into the transform coefficients of the image cover, yielding more information embedding and more robustness against watermarking attacks.

Recently, the unitary transformations have been widely used for data embedding including the discrete cosine transform (DCT), the discrete Fourier transform (DFT), fast Hadamard transform (FHT), and discrete wavelet transform (DWT). Watermarking using singular-value decomposition (SVD) and its variants has been proposed. The main idea of these approaches is to find the SVD of a cover image and then modify its singular values to embed the watermark. In Ref. 9, a hybrid non-blind watermarking scheme based on the SVD and the DWT was proposed. This method consists of decomposing the cover image into four transformed subbands, then the SVD is applied to each band, followed by modifying the singular values of the transformed subbands with the singular values of the visual watermark. This modification in all frequencies provides more robustness to different attacks. Another SVD-block-based watermarking scheme was proposed in Ref. 11, where the watermark embedding is done in two layers. In the first layer, the cover image is divided into small blocks and the singular values of the watermark are embedded in those blocks. In the second layer, the cover image is used as a single block to embed the whole watermark. In addition to the limited robustness to some attacks, the main weakness of the SVD-based techniques is that the SVD produces low-rank basis functions that do not respect the nonnegativity of the cover image.

Motivated by the need for more robustness against attacks, better visual imperceptibility, and the computational simplicity of the FHT, we propose a robust, imperceptible watermarking method. Our technique uses the DWT, FHT, and SVD. Experiments are performed to demonstrate the potential and the much-improved performance of the proposed method in comparison to other methods.

The remainder of this paper is organized as follows. Section 2 is devoted to the background material. In Section 3, we briefly review some works that are closely related to our proposed scheme. In Section 4, we introduce the proposed watermark embedding and extraction algorithms. In Section 5, we present some experimental results to demonstrate the much-improved performance of the proposed method in comparison with existing techniques, and also to show its robustness against the most common attacks. Finally, we conclude in Section 6.
2 Background

2.1 Discrete Wavelet Transform (DWT)

The DWT provides a number of powerful image processing algorithms including noise reduction, edge detection, and compression. The DWT is computed by successive low-pass and high-pass filtering of the discrete time-domain signal. Its significance is in the manner it connects the continuous-time multiresolution to discrete-time filters. At each level, the high-pass filter produces detailed information, while the low-pass filter associated with scaling function produces coarse approximations. To use DWT for image processing, we use a 2-D version of the analysis and synthesis filter banks. In the 2-D case, the 1-D analysis filter bank is first applied to the columns of the image and then applied to the rows. If the image has \( m \) rows and \( m \) columns, then, after applying the 2-D analysis filter bank, we obtain four subband images (LL, LH, HL, and HH), each having \( m/2 \) rows and \( m/2 \) columns.

2.2 Fast Hadamard Transform (FHT)

The 2-D Hadamard transform has been used with great success for image compression and image watermarking. Unlike the other well-known transforms, such as the DFT and the DCT, the elements of the basis vectors of the Hadamard transform take only the binary values +1 and −1. Hence, the FHT is well suited for digital image processing applications where computational simplicity is required.

Let \( C \) be the original image of size \( m \times m \). The 2-D Hadamard transform of \( C \) is given by \( \hat{C} = (1/m) H C H \), where \( H \) is a Hadamard matrix of order \( m = 2^k \) (\( k \) is an integer), and with entries \{−1, +1\}. The Hadamard matrix \( H \) has mutually orthogonal rows or columns and satisfies \( H H^T = m I_m \), where \( I_m \) is the identity matrix. Hence, the original image may be recovered using \( C = (1/m) H \hat{C} H \).

Figure 1 shows an example of a 2-D image with its FHT result. Furthermore, the Hadamard matrix of order \( m \) may be generated from the Hadamard matrix of order \( m/2 \) using the Kronecker product property \( H_m = H_2 \otimes H_{m/2} \), where

\[
H_2 = \begin{bmatrix}
+1 & +1 \\
+1 & -1
\end{bmatrix}
\]

is the Hadamard matrix of order \( m=2 \). Consequently \( H_4 \) becomes

\[
H_4 = \begin{bmatrix}
+1 & +1 & +1 & 1 \\
+1 & -1 & +1 & -1 \\
+1 & +1 & -1 & -1 \\
+1 & -1 & -1 & +1
\end{bmatrix}
\]

2.3 Singular-Value Decomposition (SVD)

The SVD of an image \( C \) of size \( m \times m \) is given by \( C = U \Sigma V^T \), where \( U \) is an orthogonal matrix \( (U^T U = I) \), \( \Sigma = \text{diag}(\lambda_i) \) is a diagonal matrix of singular values \( \lambda_i \), \( 1 \leq i \leq m \), arranged in decreasing order, and \( V \) is an orthogonal matrix \( (V^T V = I) \), as depicted in Fig. 2. The columns of \( U \) are the left singular vectors, whereas the columns of \( V \) are the right singular vectors of the image \( C \). The matrix \( V^T \) denotes the transpose of \( V \).

3 Related Work

In this section, we will review four representative methods for digital image watermarking that are closely related to our proposed approach. We briefly discuss their mathematical foundations and algorithmic methodologies as well as their limitations.

3.1 SVD Watermarking Scheme

Let \( C \) be a cover image of size \( m \times m \) and \( W \) be a watermark image of size \( n \times n \) with \( n \leq m \). The SVD of the cover image and the watermark image are given by \( C = U \Sigma V^T \) and \( W = U_w \Sigma_w V_w^T \), respectively, where \( \Sigma_w = \text{diag}(\lambda_w) \) is a diagonal matrix of singular values (SVs) of the visual watermark. The SVD watermark embedding algorithm is given by

\[
\lambda^\beta_i = \lambda_i + \alpha \lambda_w, \quad 1 \leq i \leq n,
\]

where \( \lambda^\beta_i \) denotes the distorted SVs of the watermarked image, and \( \alpha \) is a constant scaling factor. Hence, the watermarked image is given by \( M = U \Sigma^\beta V^T \), where \( \Sigma^\beta = \text{diag}(\lambda^\beta_i) \). We may extract the SVs of the visual watermark using \( \hat{\lambda}_w = (\lambda^\beta_i - \lambda_i) / \alpha \). Consequently, the extracted watermark \( \hat{W} \) is given by \( \hat{W} = U_w \hat{\Sigma}_w V_w^T \), where \( \hat{\Sigma}_w = \text{diag}(\hat{\lambda}_w) \).

In the SVD-block-based watermarking scheme, \( \sum \) the cover image \( C \) is divided into blocks of size \( \ell \times \ell \) and the SVD of each block \( L \) is given by \( L = U_s \Sigma_s V_s^T \). The SVs of the visual watermark \( W \) are embedded into each block of the cover image by modifying the largest singular value of each block.

3.2 Block-Based FHT-SVD Scheme

The cover image \( C \) is divided into blocks of size \( \ell \times \ell \), and the FHT of each block \( L \) is given by \( \hat{L} = (1/\ell) H \ell H \), where \( H \) is a Hadamard matrix of order \( \ell = 2^k \) (\( k \) is an integer). The SVs of the visual watermark \( W \) are embedded into each
block of the cover image by modifying the DC components of the transformed blocks. The block-based FHT-SVD watermark embedding algorithm is given by

\[ d'_w = d'_0 + \beta \lambda'_w, \]

where \( i \) ranges from 1 to the number of block, and \( d'_i \) and \( d'_w \) are the original and the modified DC components, respectively. The scaling factor \( \beta \) is chosen in relation to the DC components where \( \beta = \alpha b/n \), where \( \alpha \) is a constant, \( b \) is the block number, and \( n \) is the watermark image width. Hence, we may extract the SVs of the visual watermark using \( \lambda'_w = (d'_w - d'_0)/\beta \). Therefore, the extracted watermark \( \hat{W} \) is given by \( \hat{W} = U_w \Sigma_w V'_w \), where \( \Sigma_w = \text{diag}(\lambda'_w) \).

### 3.3 DWT-SVD Watermarking Scheme

The cover image \( C \) is decomposed into four subbands: the approximation coefficient \( LL \), and the detailed coefficients \( HL, LH, \) and \( HH \). The SVD is applied to each subband \( C^k \in \{LL, HL, LH, HH\} \) of the cover image \( C^k = U^k \Sigma^k V'^k \), \( k = 1, 2, 3, 4 \), and \( \lambda^k_l, 1 \leq i \leq m/2 \), are the SVs of \( \Sigma^k \). The SVD of the watermark image of size \( m/2 \times m/2 \) is applied to \( W = U_w \Sigma_w V'_w \), where \( \Sigma_w = \text{diag}(\lambda'_w) \). The SVs of the cover image in each subband are modified with the SVs of the watermark as follows:

\[ \lambda^k_{i} = c_b \lambda^k_{i} + \alpha_b \lambda_w, \quad 1 \leq i \leq m/2, \quad 1 \leq k \leq 4, \]

where \( \lambda^k_{i} \) denotes the distorted SVs of a subband of the watermarked image. Hence, the four modified subbands are obtained by \( M^k = U^k \Sigma^k V'^k \), where \( \Sigma^k = \text{diag}(\lambda^k) \). Then, the inverse DWT is applied using the four sets of the modified DWT coefficients to produce the watermarked image. The algorithm is invertible, and the watermark can be extracted from the watermarked image.

### 4 Proposed Watermarking Method

In this section, we provide the main steps of the proposed watermark embedding and extraction algorithms, which are also illustrated in the block diagrams shown in Figs. 3 and 4.

In Refs. 9 and 13, the authors prove experimentally that embedding the watermark in the low- and high-frequency components increases the robustness against attacks. For example, embedding in low-frequency components increases the robustness to the attacks that have low-frequency characteristics like filtering, lossy compression, and geometric distortions. However, embedding in the middle- and high-frequency components is typically less robust against low-pass filtering and small geometric deformations of the image, but are extremely robust to noise adding, contrast adjustment, gamma correction, and histogram manipulations.

Therefore, our goal of the proposed approach may be described as applying multiple transforms to the cover image to embed the watermark many times in all the frequencies, which provides better robustness against attacks, amplifies the difficulty of destroying the watermark from all the frequencies, and provides a high visual quality of the watermarked image. The use of these multiple transforms is motivated by the facts that the DWT is a powerful analysis tool for multiresolution image representation in scalable lossless coding and the FHT has significant advantage in shorter processing time and ease of hardware implementation. Denote by \( C \) the cover image of size \( m \times m \) and by \( W \) the watermark image of size \( n \times n \) with \( 2n = m \).

#### 4.1 Watermark Embedding Algorithm

1. Apply DWT to the cover image \( C \) to obtain 4 subbands \( LL, LH, HL, HH \).
2. Divide each subband into blocks of size \( \ell \times \ell \).
3. To each block \( L \), apply the FHT: \( \hat{L} = (1/\ell) HBH \),

![Fig. 3 Watermark embedding algorithm.](image-url)
4. Apply SVD to the watermark image: 
\[ W = U_w \Sigma_w V_w' \]
where \( \Sigma_w \) is a diagonal matrix of SVs.

5. Modify the DC components of the transformed blocks \( \hat{L} \) using 
\[ \hat{d}_w = d_0 + \beta \lambda_w \]
where \( d_0 \) and \( d_w \) are the original and the modified DC components, respectively, \( \lambda_w \) are the SVs of \( W \), and \( \beta = ab/n \), where \( a \) is a constant and \( b \) is the block number. Add the original DC components to the secret key if the original covers are not available for the extraction algorithm.

6. Apply the inverse fast Hadamard transform (IFHT) to all the blocks to produce the watermarked transformed subbands.

7. Apply the inverse discrete wavelet transform (IDWT) using the four watermarked subbands from Step 6 to produce the watermarked image.

4.2 Watermark Extraction Algorithm

1. Apply the first three steps of the embedding algorithm to the watermarked image.

2. For each subband, extract the SVs from each block using 
\[ \lambda_w = (d_0 - d'_w) / (\beta) \]
where \( d_0 \) and \( d_w \) are the original and the modified DC components, respectively, and \( \beta = ab/n \) where \( a \) is a constant and \( b \) is the block number. Note that the \( d_0 \) are saved in the secret key during the embedding stage.

3. Construct the four watermarks images using the SVs extracted from the four subbands: 
\[ \hat{W}_k = U_w \Sigma_k V_w' \]
where \( U_w \) and \( V_w \) are the left and right singular vectors of \( W \), respectively, and \( \Sigma_k \) is the extracted matrix of SVs for each subband \( k \) (1 ≤ k ≤ 4).

5 Experimental Results

In this section, we perform a number of experiments using a variety of gray-scale images to show the effectiveness of our proposed scheme. We conducted the experiments to test the imperceptibility of the watermark and robustness against attacks. Comparisons between our proposed scheme and other transformed watermarking techniques were also conducted. The images used in the experiments are of size 512 × 512 for the cover images and of size 256 × 256 for the watermark images. The LL subband has the lowest-frequency components of the cover image and the highest wavelet coefficients (highest magnitude). The scaling factor is chosen according to the wavelet coefficients of each subband. The HL, LH, and HH subbands have very similar wavelet coefficients values; for simplicity, we used one scaling factor for all middle- and high-frequency subbands. Figure 5 shows the wavelet coefficients values for all four subbands. In particular, the wavelet coefficients of the LL subband are the highest among all the coefficients of the other bands.

We used a strength factor \( \beta = ab/n \) defined in terms of the block position \( b \), the subband width \( n \), and the constant scaling factor \( a \) set to 0.7 for the LL subband and 0.2 for all the other subbands. The scaling factors \( a \) are chosen ex-
perimentally to repel as much attack as possible and also to obtain a watermarked image with invisible degradation to the human observer.

We split the subbands of size 256×256 into blocks of size 16×16. In this case, the resulting number of blocks is 256. Hence, we may embed the watermark image once in each subband. Figures 6(a) and 6(c) depict an example of the cover image Guy and the watermark image Peppers, respectively. The watermarked image Guy and one of the extracted watermarks from the four subbands are shown in Figs. 6(b) and 6(d), respectively.

5.1 Robustness

To verify the robustness of our proposed method, we applied different attacks to the watermarked image. The attacks include JPEG compression, Gaussian noise, multiplicative noise, Gaussian filter, deblurring with undersized point-spread function (PSF), deblurring with oversized PSF, gamma correction, histogram equalization, cropping, rescaling, sharpening, contrast adjustment, brightness change, motion blurring, and morphological opening. It is worth pointing out that the morphological opening may be

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**Fig. 5** DWT coefficients of all the four subbands of the guy image: (a) LL subband; (b) LH subband; (c) HL subband; and (d) HH subband.

**Fig. 6** (a) Original image; (b) watermarked image; (c) visual watermark; and (d) one of the four extracted watermarks from the four subbands.
used as an attack to destroy the watermark by separating 
the touching objects in an image. Figure 7 shows the wa-
termarked images with different kinds of attacks, and Fig. 8 
depicts their corresponding best extracted watermarks. For 
each attack, we extracted four watermarks from the four 
subbands, and then we selected the best watermark that had 
the highest correlation coefficient with the original water-
mark. The correlation coefficient $\rho$ between the original 
watermark image $W$ and the extracted watermark $\hat{W}$ is de-
fined as 
\[
\rho = \frac{\sum_{i,j=1}^{n} (W_{ij} - \overline{W})(\hat{W}_{ij} - \overline{\hat{W}})}{\sqrt{\left(\sum_{i,j=1}^{n} (W_{ij} - \overline{W})^2\right)\left(\sum_{i,j=1}^{n} (\hat{W}_{ij} - \overline{\hat{W}})^2\right)}},
\]
where $\overline{W}$ and $\overline{\hat{W}}$ are the mean values of $W$ and $\hat{W}$, respectively. The label below each image in Fig. 8 shows the best 
extracted watermark subband and the correlation coefficient 
between the original and the best extracted watermark.

5.2 Invisibility
To measure the perceptual quality of the watermarked im-
ages, we calculate the peak signal-to-noise ratio (PSNR), 
which is used to estimate the quality of the watermarked 
images in comparison with the original ones. The PSNR $^{15}$ 
is defined as follows:
\[
\text{PSNR} = 20 \log_{10} \left( \frac{\text{MAX}_i}{\text{MSE}} \right),
\]
where $\text{MAX}_i = \max \{C_{ij}, 1 \leq i, j \leq m\}$, and the MSE is the 
mean-squared error between the cover image $C$ and the 
watermarked image $\hat{C}$:
\[
\text{MSE} = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} \|C_{ij} - \hat{C}_{ij}\|^2.
\]
The PSNR experimental results as shown in Figs. 9 and 10 
and Table 1 clearly indicate that the proposed method gives 
high visual quality of the reconstructed image, and hence it 
guarantees the watermark’s imperceptibility.

5.3 Comparisons with Existing Techniques
We conducted several experiments to compare the robust-
ness of the proposed method with related existing tech-
niques, in particular with the pure SVD watermarking scheme, $^{10}$ DWT with SVD scheme, $^{9}$ block-based 
FHT-SVD, $^{6}$ and block-based SVD. $^{11}$ Figure 11 depicts the 
correlation coefficient comparisons between our proposed 
method and four different schemes with different attacks. In 
these comparisons, we used the Peppers, Guy, Lena, and 
Liftingbody as a cover images and the Cameraman, Peppers, Letter G, and MRI as watermark images. The results 
obtained for all the attacks clearly indicate that our pro-
posed method performs the best in terms of robustness against the attacks.
The negative correlation coefficients indicate that the extracted watermark image looks like the picture in the negative film—the lighter areas appear dark, and vice versa. For some watermark images like the letter G, the extracted watermark could be considered as detected in both cases: a white letter G on a uniform black background (high positive correlation) or a black letter G on a white background (high negative correlation).

Table 1 lists the PSNRs of the proposed method and the other existing methods for the same test images. Our algorithm clearly outperforms all other methods in terms of the visual quality of the reconstructed watermarked image. This better performance is, in fact, consistent with a variety of images used for experimentation.

![Fig. 8 Best extracted watermarks for different attacks.](image)

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![Fig. 9 PSNR between different cover images and their corresponding watermarked images with different strength factors.](image)

![Fig. 10 PSNR between different cover images and their corresponding watermarked images with different strength factors.](image)

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Fig. 11 Correlation coefficient comparison results between the proposed approach and other methods for different cover and watermark images. (a) Gaussian noise $\sigma=0.3$; (b) multiplicative uniform noise $\sigma=0.4$; (c) additive uniform noise $\sigma=0.4$; (d) salt and pepper’s 4%; (e) JPEG compression $Q=30$%; (f) low-pass filter $5\times 5$; (g) gamma correction 0.6; (h) histogram equalization; (i) sharpening; (j) rescaling 512–256–512; (k) corroding 50% right; (l) brightness, −128; (m) motion blurring 45°; (n) morphological opening; (o) deburring with undersized PSF; (p) deburring with oversized PSF; (q) deburring with initial PSF.
5.4 Computational Complexity

The computational complexity of embedding a watermark of size $m/2 \times m/2$ into a cover image of size $m \times m$ is obtained by the calculation of the DWT applied to the entire cover image and also by the calculation of the FHT to small blocks of size $(\ell \times \ell)$, where $\ell \ll m$. The DWT and FHT costs are given by $O(m \log m)$ and $O(\ell \log \ell)$, respectively. Therefore, the most expensive process of our proposed scheme is the computation of the SVD of the watermarked image, which is given by $O((m/2)^3)$.

6 Conclusion

In this paper, we proposed a simple and robust image watermarking methodology for embedding a watermarked in the transform domain. The key idea is to encode the SVs of the watermarked image after applying the FHT to small blocks computed from the four DWT subbands. The performance of the proposed method was evaluated through extensive experiments that clearly showed a better visual imperceptibility and an excellent resiliency against a wide range of attacks.

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References