Technical Note
Numerical procedures for deformation calculations in the reinforced soil walls

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Abstract
This study presents a membrane analogy method to evaluate the deflection of fabric-reinforced earth walls. The resulting equations were solved using a finite difference scheme to obtain the deflection. The numerical results were compared with a full-scale study. The comparisons show good performance of the model.

Keywords: Geosynthetic; Soil wall; Membrane analogy; Numerical analysis; Case study

1. Background

Lower cost, lightweight, improved durability, high frictional characteristics, and the relative ease associated with handling and transportation have contributed to the rapid growth of polymeric fabrics in reinforced earth wall technology over the last few decades. However, because of their lower moduli much greater strains are induced in fabric-reinforced earth walls. Therefore, deformations need to be considered to ensure that the deflections at the face of the wall are tolerable.

Field measurements of tensile force distribution have shown it to be nonlinear with the maximum occurring at a distance away from the face of the wall (Floss and Thamm, 1979). Several researchers have accounted for the nonlinear tensile stress distribution using a membrane analogy (Love et al., 1987; Bourdeau, 1989; Espinoza, 1994; Shukla and Chandra, 1994, 1995; Yin, 1997a, b). The resulting governing equations have been solved using finite difference as well as finite element numerical schemes. These methods have focused on the applications on the sub-grade deflection.

This study used finite difference scheme of a membrane analogy to calculate the short-range deflections in the fabric and in the face of the reinforced earth wall. The results are compared with a full-scale test results reported by Public Works Research Institute in Japan (PWRI), 1997 and those predicted by Rowe and Skinner (2001) using finite element method.

2. Deformation of the fabric based on membrane analogy

The flexibility of the fabric along with the conditions of the load and support conditions leads to the development of a membrane type behavior. This had prompted a number of researchers to apply the membrane analogy to study the deflection of highway subgrades reinforced with fabric (Love et al., 1987; Bourdeau, 1989, Espinoza, 1994; Shukla and Chandra, 1994, 1995; Yin, 1997a, b). The membrane analogy is applied here to the case of wall deflection.

Fig. 1 shows a two-dimensional plane strain of the static equilibrium of an elastic membrane. Using the stochastic stress diffusion theory (Sergeev, 1969; Harr, 1977) for two-dimensional plane strain conditions, the expected vertical stress \( \bar{\sigma}_z \) at a point (defined by the coordinate \( x \) and \( z \)) given as (Bourdeau, 1989)

\[
\frac{\partial \bar{\sigma}_z}{\partial z} = D \frac{\partial^2 \bar{\sigma}_z}{\partial x^2},
\]

where \( D \) is the diffusion coefficient.
where $D$ is the coefficient of diffusion, which governs the rate at which the upper soil layer spreads the applied surface load. It can be assumed to be related to, $K_0$, the coefficient of earth pressure at rest and the depth as (Bourdeau, 1989)

$$D = K_0 z.$$  \hspace{1cm} (2)

Using Gaussian distribution (Harr, 1977), the expected vertical stress under an applied pressure, $p$, uniformly distributed over a strip of width $2a$, can be evaluated as

$$\sigma_z = \psi \left( \frac{x + a}{\sqrt{K_0}} \right) - \psi \left( \frac{x - a}{\sqrt{K_0}} \right),$$  \hspace{1cm} (3)

where $\psi$ is the cumulative Gaussian distribution function:

$$\psi(x) = \int_{0}^{x} e^{-t^2/2} \, dt. \hspace{1cm} (4)$$

The compressible soil is assumed to offer a reaction to the loading pressure proportional to its deflection as in Winkler model as

$$\bar{\sigma}_{z,2} = k_s \omega(x),$$  \hspace{1cm} (5)

where $\bar{\sigma}_{z,2}$ is the vertical stress at the fabric–lower soil layer interface, $\omega(x)$ the membrane deflection, and $k_s$ the coefficient of subgrade reaction.

The interface frictional stress at the sand fabric interface is given by the Mohr Coulomb as:

$$\tau(x) = \mu(\sigma_{z,1}(x) + \gamma H_1). \hspace{1cm} (6)$$

where $\sigma_{z,1}(x)$ is the vertical stress at the backfill underneath fabric interface, $\gamma$ the unit weight of the backfill upper layer, $H_1$ the thickness of the soil column above the fabric, and $\mu$ the interface friction coefficient.

The force acting on the deflected membrane is as shown in Fig. 2. The equilibrium of forces in the horizontal direction results in (Fig. 2):

$$T_H(x) + \int_{0}^{x} \tau_H(x) \, dx = T_0, \hspace{1cm} (7)$$

where $\tau_H(x)$ is the horizontal component of the frictional stress at the interface, $T_H(x)$ the horizontal component of the tensile force in the membrane, and $T_0$ the horizontal tensile force, at the origin of coordinate (i.e., section A–A in Fig. 1).

Equilibrium in vertical direction is written as

$$T_V(x) - \int_{0}^{x} (\sigma_{z,1} - \sigma_{z,2}) \, dx + \int_{0}^{x} \tau_V(x) \, dx = T_0, \hspace{1cm} (8)$$

where $\tau_V(x)$ the vertical component of the frictional stress.

Taking the derivative of Eq. (8) and using Eq. (5) results in

$$\frac{dT_V}{dx} + \tau_V(x) + k_s \omega(x) = \bar{\sigma}_{z,1}.$$  \hspace{1cm} (9)

From the geometry of forces and deflection (Fig. 2):

$$\frac{T_V}{T_H} = \frac{d\omega}{dx}. \hspace{1cm} (10)$$

Rewriting Eq. (10) and taking the implicit derivative with respect to $x$

$$\frac{dT_V}{dx} = \frac{d(T_H d\omega)}{dx} = \frac{dT_H}{dx} \frac{d\omega}{dx} + T_H \frac{d^2\omega}{dx^2}.$$  \hspace{1cm} (11)
Substitution of Eq. (11) into Eq. (9) results in

\[
\frac{d^2T_H}{dx^2} + T_H \frac{d^2\omega}{dx^2} + \tau_H \frac{d\omega}{dx} + k_s \omega = \sigma_{z,1}.
\]  

(12)

Finally, substitution of the derivative of Eq. (7) into Eq. (12) results in

\[
T_H(x) \frac{d^2\omega(x)}{dx^2} + k_s \omega(x) = \sigma_{z,1}(x).
\]  

(13)

The total deformation condition of the reinforcement fabric can be given as (Bourdeau 1989):

\[
\delta_L = \int_0^L \left[1 + \left(\frac{d\omega(x)}{dx}\right)^2\right] dx - L = \frac{1}{E} \int_0^L T_H(x) \left[1 + \left(\frac{d\omega(x)}{dx}\right)^2\right] dx,
\]  

(14)

where \(E\) is the elongation modulus of the fabric of total length \(2L\).

The boundary conditions are

\[
\frac{d\omega}{dx} = 0 \text{ for } x = 0 (x = 0 \text{ located in section A-A (Fig. 1)),}
\]  

(15a)

\[
T_H = 0 \text{ for } x = L.
\]  

(15b)

The membrane equation (Eq. (13)) can be solved using a finite difference scheme. Assuming that the reinforcement is discretized in intervals of length \(\Delta x\), Eq. (13) becomes

\[
T_{H,i}\omega_{i-1} + 2(\Delta x^2 k_s - T_{H,i})\omega_i + T_{H,i}\omega_{i+1} = 2\Delta x^2 \sigma_{z,1},
\]  

(16)

where the subscript \(i\) denotes the node number ranging from 1 to \(N\).

The above equation can be written in a matrix form as

\[
[B] \cdot (\omega) = (\bar{\sigma})
\]  

(17)

where \([B]\) is a tridiagonal matrix, and \((\omega)\) and \((\bar{\sigma})\) are the nodal vectors of displacement and tension, respectively. This finite difference scheme can be solved subject to the constraints imposed by Eqs. (7) and (12) and the boundary conditions (Eqs. (15a) and (15b)). The flow chart in Fig. 3 depicts the iterative procedure to solve the linear system of equation presented in Eq. (17). It is worthy to note that the minimum number of node was 40 along the geosynthetic grid. Refine beyond this discretization gave the same result.

3. Case study

The developed finite difference scheme for the membrane analogy model is examined using the results of a full-scale test study reported by the Public Works and Research Institute (PWRI) in Japan. It involves a test wall constructed in the Kanto region of Japan by PWRI (Ochiai and Fukuda, 1996; Nakajima et al., 1996; PWRI, 1997; Tsukada et al., 1998; Miyata, 1996).

Fig. 4 shows the cross-section details of the wall, reinforced fabric, and the foundation soil layers. The wall is 8 m in high, concrete facing block and reinforced with 16 layers of geosynthetic grid reinforcement. Eleven are 6.0 m long and five are short, 1.0 m long, used to improve the local stability of the facing blocks. The backfill used in the construction was sand with about 30% of silty clay as fine materials. The detailed description of the foundation soil layers can be found in the Rowe and Skinner (2001).

4. Model parameters

The model parameters required are deduced from both measured data reported by PWRI (1997) and those used by Rowe and Skinner (2001). The backfill material friction angle is taken to be 29°, which will affect both the subgrade reaction and lateral earth pressure moduli. The subgrade reaction modulus is given by Scott (1981) as

\[
k_{s0} = 18N_{cor}.
\]  

(18)
where \( k_{30} \) is the 0.3 m \( \times \) 0.3 m square plate subgrade reaction and \( N_{\text{cor}} \) is the corrected standard penetrations resistance. The corrected standard penetrations resistance \( N_{\text{cor}} \) is correlated to soil friction angle as (Hatanaka and Uchida, 1996):

\[
\phi = \sqrt{20N_{\text{cor}} + 20}.
\]

The subgrade reaction \( k_{30} \) for backfill soil based on Eq. (18) will be 73.0 MN/m\(^3\). Das (1999) gave the following correction factor for the subgrade reaction of length \( L \) and width \( B \):

\[
k = \frac{k_{B \times B}(1 + 0.5 \phi)}{1.5}.
\]

Eq. (20) indicates that the value of \( k \) of a very long foundation with a width of \( B \) is approximately 0.67 \( k_{B \times B} \), which will be the case here, since we calculated the deformation along the length \( L \) and strip width of 1.0 m. Therefore, \( k \) will be 48.6 MN/m\(^3\). The reported young modulus for the used geosynthetic grid materials and the allowable tension strength are 980.0 and 29.4 kN/m, respectively.

5. Comparison of observed and calculated strain in reinforcement

The measurement of strain on the reinforcement was taken in five levels above the base. The strain measured showed an increase trend toward the wall face. Similar trends were reported by several investigators, Floss and Thamm (1979), Bathurst and Simac (1994). All measured strains were below 1.5%, therefore, they are below 3.0% calculated directly from the reported allowable design strength and young modulus. The observed and calculated results are shown in Fig. 5. The calculated strains show good agreement especially along the bottom three layers. The predictions appear to be slightly on the higher side for the top two layers.
deformation in the face of the wall and those predicted by Rowe and Skinner (2001) using finite element are shown in Fig. 6. The results show a reasonable agreement in the lower part of the wall. However, the predicted deformations in the upper part are slightly a way from the reported values. This might be due to the fact that the membrane affect is not functioning for the short reinforcement and thus the reported finite element by Rowe and Skinner (2001) show superiority in this regard.

6. Conclusions

The deflection in the face of the reinforced soil and in the geosynthetic fabric was predicted using a finite difference scheme adopt a membrane analogy. The calculated deformations from this scheme were compared with full-scale test data. The comparison was found to be reasonably good.

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References


