Effect of Darcy, Reynolds, and Prandtl Numbers on the Performance of Two-Phase Flow Heat Exchanger Filled with Porous Media

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Experimental investigation of two-phase laminar forced convection in a single porous tube heat exchanger is presented. The effect of Darcy, Reynolds, and Prandtl numbers on the performance of this heat exchanger during the condensation process of carbon dioxide at different test conditions were investigated. Gravel sand with different porosities is used as a porous medium. The flow in the porous medium is modeled using the Brinkman–Forchheimer-extended Darcy model. Parametric studies are also conducted to evaluate the effects of porosity and Reynolds and Prandtl numbers on the heat transfer coefficient and the friction factor. A dimensionless performance parameter is developed in order to be used in evaluating the porous tube heat exchanger based on both the heat transfer enhancement and the associated pressure drop. The study covers a wide range of inlet pressure ($P_{in}$), mass flow rate ($\dot{m}$), porosity of gravel sand ($\varepsilon$), and Darcy number ($Da$) which ranged: $34.5 \leq P_{in} \leq 43$ bars, $8 \times 10^{-5} \leq \dot{m} \leq 16 \times 10^{-5}$ kg/s, $34.9\% \leq \varepsilon \leq 44.5\%$, $1.6 \times 10^{-6} \leq Da \leq 5 \times 10^{-6}$, respectively. The study predicted the combined effect of the Reynolds number, Darcy number, porosity, and Prandtl number on the heat transfer and pressure drop of carbon dioxide during the condensation process in a porous medium. © 2013 Wiley Periodicals, Inc. Heat Trans Asian Res; Published online in Wiley Online Library (wileyonlinelibrary.com/journal/htj). DOI 10.1002/htj.21117

Key words: porous medium, forced convection, heat exchanger, two-phase flow, carbon dioxide

1. Introduction

Many researchers are interested in studying and understanding the fluid mechanics and heat transfer through porous media because the application of this complex phenomenon in industrial fields becomes extensive and important.

The prediction of the heat transfer coefficient and pressure drops in condensers and two-phase refrigerant transfer lines is important for the accurate design and optimization of refrigeration, air conditioning, and heat pump systems. Taking, for example, porous condensation heat exchangers, the © 2013 Wiley Periodicals, Inc.
optimal use of the two-phase pressure drop to obtain the maximum flow heat transfer performance is one of the primary design goals. There has been a growing interest in heat transfer enhancement using a porous medium and several studies have reported that the use of porous medium for heat transfer yields higher heat transfer performance than existing techniques such as pinned-fin array or twisted tape inserts. Alshqirate et al. [1] investigated the effect of heat exchanger size on a two-phase heat transfer coefficient by using carbon dioxide as the working fluid. They conducted a comparative study between porous tube, micropipe, and empty pipe heat exchangers. Al-Tarawneh [2] studied the characteristics of CO\textsubscript{2} during condensation and evaporation processes in porous media when used as refrigerant. The effect of the local inertial term on the fluid flow in channels partially filled with porous material was investigated by Abo-Hijleh and Al-Nimr [3] and Lee and Vafai [4] performed analytical characterization and conceptual assessment of solid and fluid temperature differentials in porous media. An investigation into the effect of porous medium on performance of a single-phase flow heat exchanger was adopted by Hadi Dehghan and Aliparast [5]. Jiang et al. [6] conducted experimental investigation on the convection heat transfer of CO\textsubscript{2} at super-critical pressures in vertical mini-tubes and in porous media. Alkam and Al-Nimr [7] studied the improvement of the performance of double-pipe heat exchangers by using porous substrates. Pettersen et al. [8] investigated the development of compact heat exchangers for CO\textsubscript{2} air-conditioning systems. Sheng-Chung et al. [9] studied the characteristics of CO\textsubscript{2} during condensation and evaporation processes in porous media. Alkam and Al-Nimr [7] studied the improvement of the performance of double-pipe heat exchangers by using porous substrates. Teamah et al. [10] performed a numerical simulation of laminar forced convection in a horizontal pipe partially filled with porous material. They investigated numerically the effect of Reynolds and Prandtl numbers on laminar forced convection in a horizontal pipe partially filled with porous material. The characteristics of evaporative heat transfer and pressure drop of carbon dioxide were studied by Yoon et al. [11]. Zilly et al. [12] studied experimentally the heat transfer and pressure drop of carbon dioxide during the condensation process in a horizontal, internally helical tube as compared to a smooth tube of similar diameter.

In the present work, the effect of porosity, Prandtl number, Darcy number, and Reynolds number on both heat transfer and pressure drop during condensation process of carbon dioxide in a single porous tube heat exchanger was studied. A forced convective flow exists inside the tube and a natural convection heat transfer mode on its walls. To perform a detailed study of the hydrodynamic and heat transfer of two-phase flow characteristics within a porous tube, the experimental results and the predicted empirical correlations for pressure drop and heat transfer of carbon dioxide during the condensation process in a single porous tube are developed. The flow in the porous medium is modeled using the Brinkman–Forchheimer-extended Darcy model as illustrated by Oosthuizen and Nayor [13].

**Nomenclature**

\[ A_i \]: heat exchanger internal surface area, m\textsuperscript{2}  
\[ A_p \]: cross-sectional area of the porous bed, m\textsuperscript{2}  
\[ A_T \]: total cross-sectional area of the porous tube, m\textsuperscript{2}  
\[ C \]: inertia coefficient  
\[ \text{CO}_2 \]: carbon dioxide  
\[ C_{p.m} \]: mean specific heat, kJ/kg\cdot k  
\[ \text{Da} \]: Darcy number  
\[ D_e \]: effective diameter, m  
\[ \text{Di} \]: internal diameter, m
2. Experimental Procedures

2.1 Experimental conditions

The condensation process of CO₂ gas is performed inside a selected single porous tube heat exchanger. The experimental conditions were determined and heat exchangers were fabricated according to the specifications listed in Table 1.
2.2 Experimental test rig

The experimental set-up of the test rig consists basically of the following main components:

1 - Carbon dioxide gas cylinder. 2 - High pressure regulating valve. 3 - Chest freezer 4 - Condenser. 5 - High pressure cutoff and isolating valves. 6 - Pressure gauges. 7 - Sight glasses. 8 - Evaporator. 9 - Data Acquisition System (DAS). 10 - Volume flow meter. The schematic diagram is shown in Fig. 1.

3. Pressure Drop During the Condensation Process

The inlet pressure ($P_{in}$) in kPa and the outlet test section pressure ($P_{out}$) in kPa were measured and recorded. Then the overall pressure drop ($\Delta P_{total}$) in kPa of the complete cooling tube can be calculated as follows:

![Schematic diagram of the test rig.](image-url)
The pressure drop in the vapor region of the tube (single phase), \( \Delta P_g \) in kPa, and the pressure drop in the liquid region (single phase), \( \Delta P_l \) in kPa, were calculated. Finally, the pressure drop of carbon dioxide in the the two-phase condensing region \( \Delta P_{\text{cond}} \) in kPa can be calculated according to the following equation:

\[
\Delta P_{\text{cond}} = \Delta P_{\text{total}} - (\Delta P_g + \Delta P_l)
\]

Then according to the Forchheimer extension of the Darcy model [13], which is valid for laminar flow in porous media, the cooling and the subcooling pressure drop can be calculated as follows:

\[
\Delta P_g = \frac{(\mu_g \rho_g \bar{V}_g)}{k} + C \rho_g \bar{V}_g^2
\]

\[
\Delta P_l = \frac{(\mu_l \rho_l \bar{V}_l)}{k} + C \rho_l \bar{V}_l^2
\]

where \( C \) is the inertia coefficient for spherical particles with mean diameter \( (d_m) \) [9] and is equal:

\[
C = \frac{1.75((1 - \varepsilon))}{\varepsilon^3 d_m}
\]

The permeability \( k \) of the porous media can be found according to the Forchheimer extension of the Darcy model, which is valid for laminar flow in porous media according to the following relation:

\[
k = \frac{(\bar{V}_l)}{A_p (\Delta P - C \rho_l \bar{V}_l^2)}
\]

where \( k \) is the permeability of the porous media (gravel sand), which is found experimentally according to the lab procedure of Dias and Fernands [14].

Here \( A_p \) is the cross-sectional area of the porous bed \( (A_p = \varepsilon A_T) \) in m\(^2\) where \( A_T \) is the total cross-sectional area of the condenser.

The system geometry is represented by the effective diameter \( (D_e) \), which was defined by Mills [15] and Yoon et al. [11] as

\[
D_e = d_m \left( \frac{\varepsilon}{1 - \varepsilon} \right)
\]

The apparent thermal conductivity, \( (K_a) \), of the porous bed was used to represent the thermal conductivity of the complex porous bed which consists of the working fluid and the porous material. It is given by the following equations [11]:

\[
K_a = \varepsilon K_m + (1 - \varepsilon) K_s
\]

where \( K_m \) is the mean thermal conductivity of the fluid and it is equal to

\[
K_m = \frac{(K_f + K_v)}{2}
\]
and $K_s$ is the thermal conductivity of the solid material (gravel).

The Reynolds number based on the effective diameter ($D_e$) is calculated as follows [15]:

$$\text{Re}_{D_e} = \rho_m \times u_m \times \frac{d_m \times \varepsilon}{\mu_m} (1 - \varepsilon)$$  \hspace{1cm} (10)

The Darcy number is calculated according to the following relation:

$$\text{Da} = \frac{k}{r_e^2}$$  \hspace{1cm} (11)

where $r_e = (D_e/2)$ is the effective radius of the porous bed.

The Prandtl number ($Pr$) is calculated according to the following relation:

$$Pr = \frac{C_p \mu_m}{k_a}$$  \hspace{1cm} (12)

The friction factor ($f$) is calculated according to pressure difference between inlet and outlet ports of the condenser ($\Delta p_{\text{cond}}$) of the porous tube as follows [5]:

$$f = -\frac{\Delta p_{\text{cond}}}{\rho_m u_m^2 l} \frac{D_e}{\mu_m}$$  \hspace{1cm} (13)

4. Results and Discussion

The results of the study of flow and heat transfer in a tube completely filled with porous material (gravel sand) will be introduced. The study covers a wide range of inlet pressure ($P_{\text{in}}$), mass flow rate ($\dot{m}$), porosity of sand ($\varepsilon$), and Darcy number ($Da$) which ranged: $34.5 \leq P_{\text{in}} \leq 43$ bars, $8 \times 10^{-5} \leq \dot{m} \leq 16 \times 10^{-5}$ kg/s, $34.9\% \leq \varepsilon \leq 44.5\%$, $1.6 \times 10^{-6} \leq Da \leq 5 \times 10^{-6}$, respectively. In addition, parametric studies are also conducted to evaluate the effects of porosity, Reynolds number, Prandtl number, and Darcy number on the heat transfer coefficient and the friction factor.

4.1 Effect of the Reynolds number on the average heat transfer coefficient

In order to study the influence of the Reynolds number on the average heat transfer coefficient, the experimental heat transfer coefficient is plotted against the Reynolds number for inlet pressure ($P_{\text{in}} = 3700$ kpa) and different porosities as shown in Fig. 2. As expected, the average heat transfer coefficient increases when Re increases. This is due to the fact that the increase in Re increases the

![Fig. 2. Experimental heat transfer coefficient versus Reynolds number for different porosities.](image-url)
thermal entrance length, and, as is known, the heat transfer in the entrance is high. Also it can be noticed that when the porosity of the sand decreases the heat transfer increases. This is due to the fact that as the porosity decreases the apparent thermal conductivity increases and accordingly the heat transfer rate increases.

4.2 Effect of Prandtl number on the average heat transfer coefficient

The experimental heat transfer coefficient is plotted against the Prandtl number for inlet pressure \(P_{in} = 3700\, \text{kPa}\) and different Darcy numbers as shown in Fig. 3. As expected, the average heat transfer coefficient increases when Pr increases. This is due to the fact that the increase in Pr increases the thermal entrance length, as is known, the heat transfer in the entrance is high. Also it can be noticed that when the porosity of the sand decreases (low Darcy number) the heat transfer increases. This is due to the fact that as the porosity decreases the apparent thermal conductivity increases and accordingly the heat transfer rate increases and as the permeability of the sand decreases the Darcy number decreases and the heat transfer coefficient increases.

4.3 Effect of the Reynolds number on the actual pressure drop

The influence of the Reynolds number on the actual pressure drop is investigated for two different Darcy numbers \((1.6\times10^{-6} \text{ and } 5\times10^{-6})\). The results are plotted for mass flow rate carbon dioxide of \((8\times10^{-5}\, \text{kg/s})\) and illustrated in Fig. 4. It can be noticed from this figure that the increase in Re elevates the actual pressure drop; this is due to the fact that the increase in Re increases the entrance hydrodynamic length, and it is known that the velocity gradient at the wall in the entrance region is higher (i.e., higher shear stress). Consequently, the pressure drop is higher in the entrance region, thus the actual pressure increases. Also, it is clear from the figure that for high permeability (high Darcy number) the pressure drop in the porous heat exchanger is low because of the small resistance resulting from the porous material in this case. The pressure drop increases as the Darcy number decreases (low permeability) and it increases steeply as the Reynolds number increases.

4.4 Effect of porosity on the average heat transfer coefficient

The experimental heat transfer coefficient is plotted against the gravel sand porosity for different inlet pressure \((P_{in} = 3700\, \text{kPa}, 4000\, \text{kPa}, \text{and } 4300\, \text{kPa})\) as shown in Fig. 5. As expected, the average heat transfer coefficient decreases when the porosity increases. This is due to the fact that the increase in porosity reduces the conduction heat transfer surface area between the fluid and the

![Fig. 3. Heat transfer coefficient versus Prandtl number for different Darcy numbers.](image)
gravel sand and between the sand particles themselves. Also it can be noticed that when the porosity of the sand decreases (low Darcy number) the heat transfer increases. This is due to the fact that as the porosity decreases the apparent thermal conductivity increases, and accordingly the heat transfer rate increases, and as the permeability of the sand decreases the Darcy number decreases and the heat transfer coefficient increases. Also it is clear from this figure that as the inlet pressure increases the heat transfer coefficient increases.

4.5 Effect of the Reynolds number and porosity on the performance parameter

In the present study and in the design of heat exchangers in which a porous medium is present, a dimensionless parameter is usually used to evaluate the performance based on both the heat transfer enhancement and the associated pressure drop. This parameter is defined according to Hadi Dehghan and Peiman Aliparast [5] as

\[
\varepsilon_p = \frac{\text{NuPr}^{\frac{1}{2}}}{f}
\]

where Nu is the Nusselt number and is calculated according to the following relation:

\[
\text{Nu} = \frac{h_i D_e}{k_a}
\]

Fig. 4. Pressure drop versus Reynolds for different Darcy numbers and \( \dot{m} = 8 \times 10^{-5} \text{ kg/s} \).

Fig. 5. Heat transfer coefficient versus porosity for different inlet pressures.
Figure 6 illustrates the relationship between the performance parameter and the porosity of the sand and Reynolds number. It can be noticed from this figure that as the Reynolds number increases and the porosity decreases the heat transfer enhancement increases and the performance parameter decreases. This is due to the fact that the increasing rate of the dimensionless pressure drop is much higher than that of the Nusselt number.

4.6 Effect of porosity on the friction factor in the porous tube

The effect of porosity on the friction factor is depicted in Fig. 7. It can be seen from this figure that as the porosity decreases the friction factor increases due to low permeability and the high resistance of the flow in the porous material.

5. Conclusions

- For different average particle diameters \(d_{mv}\) and for different Darcy numbers (Da), the insertion of porous material inside the pipe leads to a higher average heat transfer coefficient and correspondingly, a higher Nusselt number than that of fluid flow in a clear pipe.

- The average heat transfer coefficient increases when Pr and Re increase.

Fig. 7. Friction factor versus porosity for different inlet pressures.
As the porosity decreases the apparent thermal conductivity increases and accordingly the heat transfer rate increases.

The pressure drop increases as the Darcy number decreases (low permeability) and it increases steeply as the Reynolds number increases.

As the Reynolds number increases and the porosity decreases the heat transfer enhancement increases and the performance parameter decreases.

As the porosity decreases the friction factor increases due to low permeability and the high resistance of the flow in the porous material.

**Literature Cited**


