Some Empirical Evidence on the Cyclical Behavior of GDP and S&P500 in the U.S Economy

Arqam AL-Rabbaie
Corresponding Author, Assistant Professor of Economics
Department of Economics, Hashemite University
Zarqa 13133, Jordan
Tel: +962-53903333, Fax: +962-53826613
E-mail: rabbaie@hu.edu.jo

Samer A. M. Al Rjoub
Associate Professor of Finance, Department of Banking and Finance
Hashemite University, Zarqa 13133, Jordan
Tel: +962-53903333, Fax: +962-53826613
E-mail: salrjoub@hu.edu.jo

Abstract
Business cycles research in developed and emerging markets has gained considerable interest over the last decade. In this paper we study the dynamic behavior of two important cycle variables; the business cycle and stock market cycle. We link the real sector with the financial sector using multivariate unobserved components approach that accounts for long-term and short-term cyclical patterns. In particular we are interested in testing whether cycles in the S&P 500 coincides with business cycle as measured by real GDP. Our main finding shows a co-movement between the stock market index and GDP for short and medium cycles, and a counter-cyclical movement for long cycle.

1. Introduction
One of the well-known stylized facts is the existence of cycles in macroeconomic and financial series. Therefore, business cycles research in developed and emerging markets has gained considerable interest over the last decade; see for example Aguiar and Gopinath (2007) and Stock and Watson (2005) for an overview. The relationship between economic activity and financial market is a debatable topic; hence the question emerges of whether economic activity and S&P500 indicators move in procyclical or countercyclical fashion. Traditionally the cyclical components of time series are modeled using univariate filters. The Hodrick-Prescott H-P (Hodrick & Prescott, 1997), the approximate band-pass (Baxter and King (1999) and the Christiano and Fitzgerald (1999) filters are widely used to model business cycles. Moreover, decomposing the nonstationary time series received an interest during the eighties when the seminal work of Beveridge and Nelson (1981) is published. However, Harvey and Jager (1993) criticized the traditional approach on the grounds that is provides a spurious cyclical behavior.

As an opposed approach, Harvey (1989) and Harvey and Jager (1993) argue that “structural time series models provides the most useful framework with which to present the stylized facts on time series. These models are explicitly based on the stochastic properties of the data”. P. 231. In addition, Carvalho and Harvey (2005) argue that using multivariate unobserved component (structural) time series model can help to establish stylized facts about the cycle and the movements in the series.
Furthermore, for recent developments in modeling trend and cycles in multivariate times series, see Runstler (2004) and Harvey and Trimbur (2003).

In this paper we study the dynamic behavior of two important cycle variables; the business cycle and stock market cycle. Our contribution comes from linking the real sector with the financial sector using multivariate unobserved components approach that accounts for long-term and short-term cyclical patterns. We are particularly interested in testing whether cycles in the S&P 500 coincides with business cycle as measured by real GDP.

We build a multivariate time-series model for share price index and real GDP. Empirical models that link stock market indices to macroeconomic variables are rare and can be found in Bolten and Weigand (1998) who present a theoretical framework that shows how stock prices and fundamental factors based on the dividend discount model interact over the business cycle.

Rigobon and Sack (2003) also found that movements in the stock market can have a significant impact on the macroeconomy and are therefore likely to be an important factor in the determination of monetary policy. Actually they specify the magnitude of the Federal Reserve's reaction to the stock market; a 5 percent rise (fall) in the S&P 500 index will increase the likelihood of a 25 basis point tightening (easing) by about a half.

This article proceeds as follows. We discuss the data in Section 2. Section 3 presents the modeling approach and the assumptions that characterize movements of stock prices and the business cycle. The empirical results are contained in Section 4. Finally, conclusions are drawn in Section 5.

2. Data

We use two data series in our analysis, real GDP and S&P 500. The first series, real GDP, is taken from the data base of the Federal Reserve Bank of St. Louis (FRED). The GDP frequency is annual over the period from 1929 to 2007 in Billions of chained 2000 dollars. The second series is the S&P 500 value weighted index over the same period.

3. Modeling Approach

We use the modeling approaches of Harvey (1989) and Durbin and Koopman (2001) to describe the dynamic behavior of the two series as well their interdependencies. The model specification is as follows:

\[ y_t = \mu_t + A\gamma_t + B\psi_t + C\Omega_t + e_t \]  

(1)

Where \( e_t \) is independently and identically distributed with mean zero and covariance matrix \( \Sigma_e \), \( t = 1, \ldots, n \) with \( t=1 \) for 1929 and \( t=n=78 \) for 2007, \( y_t \) represents a 2×1 column vector of S&P500 and RGDP and given by:

\[ y_t = \begin{bmatrix} RGDP \\ S & P500 \end{bmatrix}, t = 1, \ldots, n \]

(2)

The trend component is represented by the vector \( \mu_t \) and the cycle components by the vectors \( \gamma_t \) (short cycle), \( \psi_t \) (medium cycle) and \( \Omega_t \) (long cycle). Whereas, A, B and C are the loading matrices for short, medium and long cycles respectively.

The trend is assumed to be:

\[ \mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \]

(3)

\[ \beta_t = \beta_{t-1} + \zeta_t \]

(4)

1 We followed the same model format of that of Koopman and Lucas, 2005, Journal of Applied Econometrics 20,311-323.
Where $\eta_t$ and $\zeta_t$ are mutually uncorrelated multivariate white noises with covariance matrices $\Sigma_\eta$ and $\Sigma_\zeta$, $\mu_t$ is supposed to filter out low frequency or long-term dynamics from the data compared to (for details see Koopman and Lucas, 2005).

The specification of the cycles can be written as

$$\begin{align*}
\begin{bmatrix}
\gamma_t \\
\gamma_t^*
\end{bmatrix}
&= \phi_\gamma
\begin{bmatrix}
\cos(\lambda_\gamma)I & \sin(\lambda_\gamma)I \\
-\sin(\lambda_\gamma)I & \cos(\lambda_\gamma)I
\end{bmatrix}
\begin{bmatrix}
\gamma_{t-1} \\
\gamma_{t-1}^*
\end{bmatrix}
+ \begin{bmatrix}
\kappa_t \\
\kappa_t^*
\end{bmatrix},

\begin{bmatrix}
\omega_t \\
\omega_t^*
\end{bmatrix}
\sim iid \ N(0, I_2 \otimes \Sigma_\gamma),

(5)
\end{align*}$$

$$\begin{align*}
\begin{bmatrix}
\psi_t \\
\psi_t^*
\end{bmatrix}
&= \phi_\psi
\begin{bmatrix}
\cos(\lambda_\psi)I & \sin(\lambda_\psi)I \\
-\sin(\lambda_\psi)I & \cos(\lambda_\psi)I
\end{bmatrix}
\begin{bmatrix}
\psi_{t-1} \\
\psi_{t-1}^*
\end{bmatrix}
+ \begin{bmatrix}
\omega_t \\
\omega_t^*
\end{bmatrix},

\begin{bmatrix}
\omega_t \\
\omega_t^*
\end{bmatrix}
\sim iid \ N(0, I_2 \otimes \Sigma_\psi),

(6)
\end{align*}$$

$$\begin{align*}
\begin{bmatrix}
\Omega_t \\
\Omega_t^*
\end{bmatrix}
&= \phi_\Omega
\begin{bmatrix}
\cos(\lambda_\Omega)I & \sin(\lambda_\Omega)I \\
-\sin(\lambda_\Omega)I & \cos(\lambda_\Omega)I
\end{bmatrix}
\begin{bmatrix}
\Omega_{t-1} \\
\Omega_{t-1}^*
\end{bmatrix}
+ \begin{bmatrix}
\nu_t \\
\nu_t^*
\end{bmatrix},

\begin{bmatrix}
\nu_t \\
\nu_t^*
\end{bmatrix}
\sim iid \ N(0, I_2 \otimes \Sigma_\Omega),

(7)
\end{align*}$$

The specification of the cycles through the equations (5-7) represents the stochastic specification of short, medium and long cycles respectively. These equations show the cycles frequency in radians, for short, medium and long cycles, which are $\lambda_\gamma$, $\lambda_\psi$ and $\lambda_\Omega$ respectively. In addition, the elements $\phi_\gamma$, $\phi_\psi$ and $\phi_\Omega$ are the damping factors of the three different cycles that should be less than one. The disturbances in the equations are serially and mutually uncorrelated and normally distributed, where the elements $D_\gamma$, $D_\psi$ and $D_\Omega$ represent the variance of the cycle, hence the strength of a cycle rely on the variance of its disturbances. The factor loading matrices $A$, $B$ and $C$ scale the cycles $\gamma_t$, $\psi_t$ and $\Omega_t$ for each individual series. The loading matrix is restricted to a lower triangular matrix with unity as diagonal elements and its size is 2 by 2. These matrices are represented by A, B and C in Table (1).

This stochastic cycle specification generates three stationary cyclical processes with a common period and damping factor.\(^2\)

### 4. Empirical Results

Table 1 reports the estimation results of three common cycles: short, medium and long cycles as specified in the previous section. The columns (2 & 3) labeled lagged are just performed on the GDP. For the sake of comparing the results we consider the results in column 1 as reference results.

Diagnostic statistics are in column 1. The diagnostic tests show no evidence of autocorrelation, heteroscedasticity, and no-normality despite the limited number of observations. The short cycle $\lambda_\gamma$ is bivariate, such that there are two cycles given the values of $D_\gamma$ are not zero. This common cycle with a damping factor $\phi_\gamma$ of 0.699 has a period roughly of 4 year.

In order to determine whether the two series are moving together or not we calculate the loading matrix $A$. The evidence shows that there is a common short cycle between GDP and S & P500, which is co-cyclical as indicating by the loading of 2.896. This loading means that if economic activity increases S & P 500 increases over this high frequency cycle.\(^3\)

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\(^2\) It is worth noting that STAMP software allows up to three cycles in each series, but it imposes the restriction that for a given cycle, the damping factor and the frequency are the same for all series.

\(^3\) Hence, the definition of the cycle is the deviation form a deterministic or stochastic trend
The medium cycle $\lambda_\psi$ length is estimated with 9.981 years with a damping factor of 0.948. In addition, the $D_\psi$ element indicates the existence of the two cycles. The loading matrix B shows that there is co-cyclicality between the two series at this medium frequency as indicating by the loading element of 0.569.

The longer cycle ($\lambda_\Omega$) estimation gives a damping factor of 0.989 and a period of 23.286 years. The loading matrix C has a negative sign of 0.998, which intuitively indicates there is counter-cyclical relationship between the series of interests.

To sum up, we conclude, from the discussion of our reference results reported in column 1, the existence of co-movement among the series of interest for short and medium cycles, and the existence of counter-cyclical movement in the long cycle.

The estimation result in column 2 and 3 are implemented on the first lagged GDP and second lagged GDP respectively. The diagnostic tests are almost the same as the reference model in column 1. The elements $D_\gamma$ are almost the same under different speciation on GDP. In addition, the length of the common cycle in column 2 & 3 almost similar to reference model in column 1. However, there is a significant difference that the two series show countercyclical relationship as indicating by loading matrix A. The relationship may exist under different specification for the short cycle $\lambda_\gamma$ despite the cycle's length are similar. This might be ambiguous.

However, the results for the $\lambda_\psi$ seem to carry over. Hence the length of the cycles in column 2 & 3 are similar and the co-cyclical relationship between the series exist under different specification the GDP as indicating by the loading matrix B.

For the longer cycle, the counter-cyclicality exists as indicating by loading matrix C and for the three specifications on GDP. The length of the cycle in column 3 is 39.312 years compared to all other specifications.

5. Conclusion
Business cycles research in developed and emerging markets has gained considerable interest over the last decade. In this paper we study the dynamic behavior of two important cycle variables; the business cycle and stock market cycle. We link the real sector with the financial sector using multivariate unobserved components approach that accounts for long-term and short-term cyclical patterns. In particular we are interested in testing whether cycles in the S&P 500 coincides with business cycle as measured by real GDP. Our main results support the existence of co-movement among the series of interest for short and medium cycles, and the existence of counter-cyclical movement in the long cycle.

Table 1: The estimated parameters for the model

<table>
<thead>
<tr>
<th>Period</th>
<th>No lags 1929-2004</th>
<th>gdp, Lag 1 1929-2004</th>
<th>gdp, lag 2 1929-2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\pi/\lambda_\gamma$</td>
<td>3.997</td>
<td>4.1000</td>
<td>3.862</td>
</tr>
<tr>
<td>$\phi_\gamma$</td>
<td>0.797</td>
<td>0.81</td>
<td>0.852</td>
</tr>
<tr>
<td>$D_\gamma$</td>
<td>0.0014</td>
<td>0.0198</td>
<td>0.002</td>
</tr>
<tr>
<td>Load Matrix A</td>
<td>$s$ &amp; $P500$</td>
<td>$gdp$</td>
<td>$s$ &amp; $P500$</td>
</tr>
<tr>
<td>gdp</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>2.896</td>
<td>1.000</td>
<td>-11.201</td>
</tr>
<tr>
<td>$2\pi/\lambda_\psi$</td>
<td>9.981</td>
<td>9.602</td>
<td>9.518</td>
</tr>
</tbody>
</table>
### Table 1: The estimated parameters for the model - continued

<table>
<thead>
<tr>
<th>φψ</th>
<th>0.989</th>
<th>0.953</th>
<th>0.864</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dψ</td>
<td>0.008</td>
<td>0.016</td>
<td>0.0082</td>
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**Load Matrix B**

<table>
<thead>
<tr>
<th>S&amp;P500</th>
<th>gdp</th>
<th>S&amp;P500</th>
<th>gdp</th>
<th>S&amp;P500</th>
<th>gdp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.00</td>
<td>1.000</td>
<td>0.00</td>
<td>1.000</td>
<td>0.00</td>
</tr>
<tr>
<td>0.569</td>
<td>1.00</td>
<td>1.023</td>
<td>1.00</td>
<td>0.574</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\( 2\pi / \lambda \omega \) Period

| 23.286 | 23.135 | 39.312 |

| φω | 0.989 | 0.953 | 0.998 |

**Load Matrix C**

<table>
<thead>
<tr>
<th>Ωgdp</th>
<th>ΩS&amp;P500</th>
<th>Ωgdp</th>
<th>ΩS&amp;P500</th>
<th>Ωgdp</th>
<th>ΩS&amp;P500</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>-0.998</td>
<td>1.000</td>
<td>-0.910</td>
<td>1.000</td>
<td>-7.986</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\( \sum e \) diagonal

| 0.000007 | 0.00038 | 0.000 | 0.005 | 0.000003 | 0.00043 |

\( \sum \eta \) diagonal

| 0.000011 | 0.00014 | 0.000013 | 0.0004 | 0.00000 | 0.00011 |

**Diagnostic tests**

<table>
<thead>
<tr>
<th>Standard error</th>
<th>0.020</th>
<th>0.025</th>
<th>0.010</th>
<th>0.058</th>
<th>0.011</th>
<th>0.051</th>
</tr>
</thead>
<tbody>
<tr>
<td>R^2</td>
<td>0.75</td>
<td>0.61</td>
<td>0.72</td>
<td>0.47</td>
<td>0.76</td>
<td>0.51</td>
</tr>
<tr>
<td>N</td>
<td>3.11</td>
<td>4.25</td>
<td>1.11</td>
<td>7.125</td>
<td>6.8</td>
<td>2.39</td>
</tr>
<tr>
<td>r^2</td>
<td>0.29</td>
<td>-0.85</td>
<td>-0.008</td>
<td>-0.019</td>
<td>0.06</td>
<td>-0.034</td>
</tr>
<tr>
<td>Q((10))</td>
<td>6.12</td>
<td>4.31</td>
<td>7.09</td>
<td>2.78</td>
<td>8.10</td>
<td>4.20</td>
</tr>
<tr>
<td>Failure test</td>
<td>1.05</td>
<td>0.90</td>
<td>0.76</td>
<td>0.68</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>Cusmt</td>
<td>-0.96</td>
<td>-0.64</td>
<td>-0.84</td>
<td>0.10</td>
<td>-1.10</td>
<td>2.53</td>
</tr>
</tbody>
</table>

**References**


