Subspace-based blind channel estimation in nearly saturated down link multicarrier code division multiple access systems

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Abstract: A new subspace-based blind method for channel estimation in nearly saturated downlink multicarrier code division multiple access systems is proposed. In this method, the authors exploit the subspace structure of the received signal to estimate the channel directly in the frequency domain irrespective of the length of the FIR channel, thus allowing operation in nearly saturated systems (number of users can be one less than process gain). The method is shown to outperform some earlier blind channel estimation methods, whereas at the same time avoiding the limitations in the number of users that these methods suffer from.

1 Introduction

Multicarrier code division multiple access (MC-CDMA) systems are gaining increased interest as one of the promising technologies for present and future high-speed wireless multiuser communications [1–5]. Owing to their code structure, MC-CDMA systems can efficiently utilise the available spectrum of a multipath communication channel. However, the advantages of MC-CDMA systems come at the expense of additional requirements on the receiver system arising from the code structure and the resulting multiuser scenario. One of these requirements is the need for accurate channel estimation in MC-CDMA systems in order to be able to design efficient multiuser receivers [6, 7]. Although sending training sequences periodically can achieve this goal, the possibility of dealing with a continuously time-varying channel in some scenarios has made it necessary to look for blind methods [8, 9] that avoid the significant loss in data throughput associated with training.

In this paper, we focus on the problem of blind estimation of the downlink channel in MC-CDMA systems targeting the scenario of heavily loaded systems where the number of users can be very close to the process gain. Various methods have been proposed in the literature for blind channel estimation and detection in MC-CDMA systems. In [8], the channel is estimated blindly based on a max–min procedure where the channel is selected to maximise the constrained (constrained in the direction of the desired signal) minimum variance (CMV). In [9], the channel is estimated blindly based on the well-known multiple signal classification (MUSIC) principle where the channel is selected to maximise the orthogonality of a candidate spreading code, modified by the channel, with the noise subspace of the received signal. Both of these methods have excellent performance but they can operate only up to a maximum of (process gain-channel length) users. Instead of channel estimation, the authors in [10] propose to carry out blind time-domain equalisation directly by utilising the knowledge of non-active spreading codes in the channel and exploiting the guard interval redundancy, assuming use of orthogonal codes. However, finite-length time-domain equalisers cannot fully equalise the channel even in noise absence, thus giving rise to an unavoidable noise floor.

Unlike the proposal in [10], we here use the knowledge of the active spreading codes in the channel to blindly obtain an estimate of the channel directly in the frequency domain. In contrast to the CMV method [8] and MUSIC method [9], our goal here is to maximise channel use by providing a channel estimation method that can work properly for systems loaded by up to (process gain – 1) users. This is an important advantage when compared with some previous blind estimation methods [8, 9] that have limitations in the number of users that they can deal with. The proposed method here is based on utilising the subspace and code structure of the received signal in a two-step estimation process that yields the frequency-domain channel (Fourier transform of the FIR channel). Our procedure, however, has an additional requirement that is not difficult to fulfil in practice which is the need to know the spreading codes of the active users in the channel. Although the identification of active codes is not the topic of this paper, it is easy to see that such information can be passed to the receiver without much difficulty assuming that the receiver has knowledge of all spreading codes and needs only to know which of these codes is active in the channel.

This paper is organised as follows: In Section 2, we present the signal model. Section 3 describes the blind channel estimation methods.
estimation method. Computer simulation results are given in Section 4, followed by conclusions in Section 5.

2 System model

We consider the downlink of an MC-CDMA system with $K$ simultaneous users. Each symbol of each user is spread by a pseudorandom spreading sequence of length $N$. Hence after spreading, we obtain the sum vector $\sum_{k=1}^{K} d_k c_k$ where $d_k$ is the $k$th user symbol, and $c_k$ is the $k$th user spreading code vector given by $c_k = [c_k(1) \ c_k(2) \ \cdots \ c_k(N)]^T$.

Q2 Frequency-domain spreading is then achieved by performing an inverse fast Fourier transform (IFFT) of size $N$ to the sum vector yielding the vector

$$\sum_{k=1}^{K} d_k F c_k$$

where $F$ is an $(N \times N)$ inverse Fourier transform matrix whose $(m, n)$th entry is $(1/N)\exp(j \pi mn/N)$. This is followed by adding a cyclic prefix (CP) of suitable length (larger than the channel delay spread) to avoid inter symbol interference (ISI). The resulting MC-CDMA signal is then transmitted through the channel which is assumed to be constant over $M$ symbols, over which we will be carrying out blind channel estimation. The channel is modelled here by an $L$-tap FIR filter vector $h = [h(1) \ h(2) \ \cdots \ h(L)]^T$, which combines the effect of the channel and the transmit and receive shaping filters. At the receiver, the CP is first removed, followed by applying the fast Fourier transform (N-point FFT) to yield the vector

$$r = [r(1) \ r(2) \ \cdots \ r(N)]^T = \sum_{k=1}^{K} d_k C_k \tilde{F} h + n$$

where $C_k = \text{diag}[c_k]$ and $\tilde{F}$ is an $(N \times L)$ matrix formed from the first $L$ columns of an $(N \times N)$ FFT matrix whose $(u, v)$th entry is $\exp(-j \pi uv/N)$, and $n$ is the vector of noise, which is Gaussian noise with zero mean and variance $\sigma^2$. We can express the vector $r$ in (2) by

$$r = \sum_{k=1}^{K} d_k C_k h_f + n$$

where $h_f = \tilde{F} h$ denotes the frequency-domain channel vector, that is, vector of the complex gains at each $k$th subcarrier (or chip). Equation (3) can also be written as

$$r = \sum_{k=1}^{K} d_k \tilde{c}_k + n$$

where $\tilde{c}_k$ is termed the ‘effective’ code of the $k$th user at the receiver, which is given by

$$\tilde{c}_k = [c_k(1)h_f(1) \ c_k(2)h_f(2) \ \cdots \ c_k(N)h_f(N)]^T$$

The signal description above is useful in the following derivation of the estimation algorithm.

3 Blind channel estimation

To achieve the goal of blind estimation of the channel we will utilise the subspace and code structure of the received signal. Hence, we will first extract the noise subspace from the received signal. By constructing the autocorrelation matrix $R = E[r r^H]$ and performing eigenvalue decomposition, we obtain the matrix $U$ of size $(N \times (N-K))$ containing the eigenvectors corresponding to the $(N-K)$ smallest eigenvalues of $R$ and hence they span the noise subspace. Each column of $U$, denoted by $u_i$, is therefore orthogonal to the effective code vector $\tilde{c}_k$ of any of the active users in the channel. We can therefore express this orthogonality as

$$u_i^H \tilde{c}_k = 0$$

for $(k = 1, 2, \ldots, K)$ and $(i = 1, 2, \ldots, N-K)$. Using (5) we can rewrite (6) as

$$u_i^H [c_k(1)h_f(1) \ c_k(2)h_f(2) \ \cdots \ c_k(N)h_f(N)]^T = 0$$

After conjugation and rearranging terms, (7) can be rewritten as

$$h_f^H [c_k(1)u_i(1) \ c_k(2)u_i(2) \ \cdots \ c_k(N)u_i(N)]^T = 0$$

or also as

$$h_f^H v_{k,i} = 0$$

where $v_{k,i}$ is given by

$$v_{k,i} = [c_k(1)u_i(1) \ c_k(2)u_i(2) \ \cdots \ c_k(N)u_i(N)]^T$$

The same relation in (8) obviously applies to any $(k = 1, 2, \ldots, K)$ and $(i = 1, 2, \ldots, N-K)$. Thus, we can stack (8) for all $i, k$ values in one equation given as

$$h_f^H V = 0$$

where $V$ is an $(N \times (K(N-K))$ matrix given by

$$V = [v_{1,1} v_{1,2} \cdots v_{1,N-K} \ v_{2,1} v_{2,2} \cdots v_{2,N-K} \ \cdots \ v_{N-K,1} v_{N-K,2} \cdots v_{N-K,N-K}]$$

and ‘0’ is a $(1 \times (K(N-K))$ all zero vector. The matrix $V$ obviously has ‘at least one vector’ in its left null space and that is the channel vector $h_f$, and consequently we have rank($V$) $\leq N-1$. If $h_f$ was the only vector in the left null space of $V$ (i.e. rank($V$) $= N-1$) then the least squares estimate of the vector $h_f$ would be the least significant eigenvector of $(V V^H)$. However, rank($V$) in general may be less than $(N-1)$. Hence, it becomes necessary to investigate the rank of matrix $V$ and the parameters that affect its value. We therefore state the following theorem that is basic to the derivation of the blind estimation algorithm.
Proof: See Appendix.

This theorem provides a way to compute \( \text{rank}(V) \) for any set of active users in the channel using (12) irrespective of the channel vector \( h_t \). Assuming use of Walsh–Hadamard orthogonal codes, and using this theorem, we have investigated \( \text{rank}(V) \) for all possible \( K \)-values of each of the process gains \( N = 16, 32, 64 \) and 128. For each \( K \)-value we have generated 20,000 different randomly selected sets of active users codes and then computed \( \text{rank}(V) \) for each case. The results of this investigation showed that very few code combinations (less than 3%) resulted in \( \text{rank}(V) < N - 2 \). Hence, since this ratio is small, a reasonable choice (though obviously not the only choice) would be to develop a blind estimation procedure only for code sets with \( \text{rank}(V) \geq N - 2 \), leaving the task of avoiding the few code combinations with \( \text{rank}(V) < N - 2 \) to the code assignment procedure. In other words, the process of assigning codes to each user is designed in a manner that avoids sets of users whose \( \text{rank}(V) < N - 2 \) thus allowing a simpler receiver where the left null space of \( V \) would be of size 1 or 2. This is obviously possible because of the prior knowledge of the central station of the \( \text{rank}(V) \) for each combination of codes, which is the main result from Theorem 1, and because of the low percentage of code combinations with \( \text{rank}(V) < N - 2 \). It is also worthy of mention here that \( \text{rank}(V) \) basically depends on the numerical values of the spreading code elements and as an example if these codes were chosen as uniformly distributed random variable in \([0, 1]\) then \( \text{rank}(V) = N - 1 \) for all cases. However, the latter choice is of course not a good choice for spreading codes.

The blind estimation procedure, assuming code sets with \( \text{rank}(V) \geq N - 2 \), would therefore have an additional step based on the MUSIC approach. Since the matrix \( V \) in these cases has a rank of either \((N - 2)\) or \((N - 1)\), then the two least significant eigenvectors of \((VV^H)\), denoted by \( g_1 \) and \( g_2 \), obviously span the channel vector \( h_t \). Recalling that the vector \( h_t \) can also be expressed as \( h_t = FH_t \), then it is easy to see that the vector \( H_t \) is the intersection of the column spaces of the matrix \( F \) and the matrix \( G = [g_1 \ g_2] \). Hence, if we construct the matrix \( A = [G \ F] \) of size \((N \times (L + 2))\) then the only vector in its right null space, denoted by \( x \), would correspond to the desired intersection that yields \( h_t \). Hence, we can estimate \( h_t \), up to a complex scaling ambiguity, as \( h_t = Gx(1) \ x(2) \), where \( x \) is the least significant eigenvector of the matrix \((A^HA)\). The overall blind channel estimation procedure is summarised in Table 1.

As for the conditions on the channel length \( L \) for identifiability, it comes from the rank of the matrix \( A \), which has one vector in its right null space, and thus has a rank of \( L + 1 \). Thus, \( N \) should satisfy \( N \geq L + 1 \) in order to preserve a rank of \( L + 1 \). Thus, for proper operation we should have \( L \leq N - 1 \), irrespective of the number of users (up to \( K = N - 1 \), and that was clearly observed throughout simulations.

## 4 Computer simulations

In order to access the performance of the proposed blind channel estimation algorithm and verify its features, we have carried out several computer simulation experiments for various channel loadings including nearly saturated cases. In these experiments, we compare the performance of the proposed method to two blind channel estimation methods, which are the CMV [8] and the subspace MUSIC method [9], under various multiuser scenarios. The performance measure here is the normalised channel estimation error defined by the ratio \( |gh_t - h_t|/|h_t| \), where \( g \) is a complex scalar used to compensate for the complex scaling ambiguity that is usually present in blindly estimated channels, so as to obtain fair comparisons. The normalised channel estimation error in each experiment has been averaged over 100 different randomly generated channels. All experiments assume the use of Walsh–Hadamard codes, and we assume that the channel remains constant over a block of \( M \) symbols which will be used for estimation, whereas the block length itself will be varied in one of the experiments to investigate its effect on the performance of the three methods.

In the first experiment (Figs. 1 and 2), we have simulated an MC-CDMA system with a process gain of \((N = 64)\), and using a block length of \( M = 150 \), and a multipath channel of length \( L = 15 \). The elements of the FIR channel, for all experiments, are taken as independent complex Gaussian random variables with zero mean and unit variance. The number of users is taken to be \( K = 25 \) (Fig. 1) and \( K = 35 \) (Fig. 2). Figs. 1 and 2 show that the new method significantly outperforms the CMV and the MUSIC methods with the advantage increasing as channel loading increases. The high channel loading effect is investigated further in the second experiment where we have simulated

<table>
<thead>
<tr>
<th>Steps of the blind channel estimation procedure</th>
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<tbody>
<tr>
<td>1. Construct the autocorrelation matrix ( R = E[rr^H] ) and the noise subspace matrix ( U ) containing the ((N - K)) least significant eigenvectors of ( R )</td>
</tr>
<tr>
<td>2. Construct the matrix ( V ) as given by (9) and (11)</td>
</tr>
<tr>
<td>3. Construct the matrix ( G = [g_1 \ g_2] ), where ( g_1 ) and ( g_2 ) are the two least significant eigenvectors of ((VV^H))</td>
</tr>
<tr>
<td>4. Construct the matrix ( A = [G \ F] ) and compute ( x ) as the least significant eigenvector of ((A^HA))</td>
</tr>
<tr>
<td>5. The frequency-domain channel estimate, up to a complex scaling ambiguity, is given by ( \hat{h}_t = Gx(1) \ x(2) )</td>
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\[
P = \begin{bmatrix}
c_1(1)c_{K+1}(1) & c_1(1)c_{K+1}(2) & \cdots & c_1(1)c_N(1) \\
c_1(2)c_{K+1}(1) & c_1(2)c_{K+1}(2) & \cdots & c_1(2)c_N(2) \\
\vdots & \vdots & \ddots & \vdots \\
c_1(N)c_{K+1}(1) & c_1(N)c_{K+1}(2) & \cdots & c_1(N)c_N(N) \\
\end{bmatrix} \begin{bmatrix}
c_2(1)c_{K+1}(1) & c_2(1)c_{K+2}(1) & \cdots & c_2(1)c_N(1) \\
c_2(2)c_{K+1}(1) & c_2(2)c_{K+2}(2) & \cdots & c_2(2)c_N(2) \\
\vdots & \vdots & \ddots & \vdots \\
c_2(N)c_{K+1}(1) & c_2(N)c_{K+2}(2) & \cdots & c_2(N)c_N(N) \\
\end{bmatrix} = \begin{bmatrix}
c_K(1)c_{K+1}(1) & c_K(1)c_{K+2}(1) & \cdots & c_K(1)c_N(1) \\
c_K(2)c_{K+1}(1) & c_K(2)c_{K+2}(2) & \cdots & c_K(2)c_N(2) \\
\vdots & \vdots & \ddots & \vdots \\
c_K(N)c_{K+1}(1) & c_K(N)c_{K+2}(2) & \cdots & c_K(N)c_N(N) \\
\end{bmatrix}
\]
an MC-CDMA system with a process gain of \(N = 32\), and using a block length of \(M = 80\), and a multipath channel of length \(L = 7\) with an SNR of 30 dB.

The normalised channel estimation error is studied for varying number of users from \(K = 10\) to \(K = 31\) (i.e. \(N - 1\)). Fig. 3 shows that although the CMV and the MUSIC methods fail to work for \(K \geq 25\), the proposed method works fine up to \(K = N - 1\). This property is especially useful in practice for long FIR channels since the CMV and MUSIC cannot work for \(K > N - L\).

In the third experiment, we have investigated the effect of the block length (i.e. channel stationarity length) on the performance of the proposed method. We assume \(N = 32\), \(K = 16\), SNR of 30 dB, and using a block length of \(M = (20 - 200)\) symbols, and a multipath channel of length \(L = 7\). Fig. 4 clearly shows that the proposed method has significant performance advantage for the entire range of variation of block length (20–200). At the block length of 20 it is still significantly better than the MUSIC and the CMV methods. Proper operation using short data records is in fact another advantage of the proposed method that is very useful in practice.

The results in Figs. 1–4 clearly demonstrate a significant performance advantage for the proposed blind estimator especially in heavily loaded systems which are in fact the key to better utilisation of the spectrum by allowing full use of all spreading codes available.

5 Conclusions

In this paper, we have proposed a new blind channel estimation method for downlink MC-CDMA systems operating in highly loaded systems (i.e. up to \(K = N - 1\) users). The method estimates the channel directly in the frequency domain irrespective of the length of the FIR
channel. Computer simulation results have indicated that the proposed method performs significantly better than some earlier methods [8, 9] especially for moderately and highly loaded systems. Such a feature is very important in practice since it enables the efficient use of an MC-CDMA system by allowing the use of \( N - 1 \) spreading codes while blindly and efficiently estimating the channel and hence designing accurate multiuser receivers. The proposed blind estimator is therefore a very good candidate for practical systems.

6 References


7 Appendix: proof of Theorem 1

Define the matrix \( \bar{U} = [\bar{u}_1 \bar{u}_2 \cdots \bar{u}_{N-K}] \) to be the orthogonal subspace to the code subspace \( \{c_1, c_2, \ldots, c_{K+1}\} \), then consequently we have

\[
\bar{u}_i^T c_k = 0 \quad (13)
\]

for \( i = 1, 2, \ldots, N - K \) and \( k = 1, 2, \ldots, K \). Thus, from (4), (5) and (6), the noise subspace of the received signal \( U = [u_1 u_2 \cdots u_{N-K}] \) may be given by

\[
u_i = \left[ \frac{\bar{u}_i(1)}{h_1^T(1)} \frac{\bar{u}_i(2)}{h_2^T(2)} \cdots \frac{\bar{u}_i(N)}{h_N^T(N)} \right]^T \quad (14)
\]

This is justified by the fact that the vector \( \bar{c}_k \) given by (14) is orthogonal to \( \bar{c}_k \) (see (5)) since \( u^T_i \bar{c}_k = \bar{u}_i^T c_k = 0 \) from (13) above. Hence, the matrix \( V \) can be written as (see (15))

Since the channel gains \( h_1(1) \cdots h_N(N) \) have obviously only a row scaling effect on \( V \), the rank of \( V \) is independent of \( h_1(1) \cdots h_N(N) \) and may be given by rank(\( V \)) = rank(\( P \)), where (16)

Since the subspace \( \bar{U} \) is determined only by the active spreading codes, so does the rank of the matrix \( V \). Finally, for the choice of Walsh–Hadamard orthogonal spreading codes, the vectors \( [\bar{u}_1 \bar{u}_2 \cdots \bar{u}_{N-K}] \) in (16) can be obviously replaced by the code vectors \( \{c_{K+1}, c_{K+2}, \ldots, c_N\} \) yielding (12), thus proving the theorem. \( \square \)

\[
V = \begin{bmatrix}
c_1(1)\bar{u}_1(1) & c_1(1)\bar{u}_2(1) & \cdots & c_1(1)\bar{u}_{N-K}(1) \\
\vdots & \vdots & \ddots & \vdots \\
c_1(N)\bar{u}_1(N) & c_1(N)\bar{u}_2(N) & \cdots & c_1(N)\bar{u}_{N-K}(N)
\end{bmatrix}
\]

\[
P = \begin{bmatrix}
c_1(1)\bar{u}_1(1) & c_1(1)\bar{u}_2(1) & \cdots & c_1(1)\bar{u}_{N-K}(1) \\
\vdots & \vdots & \ddots & \vdots \\
c_1(N)\bar{u}_1(N) & c_1(N)\bar{u}_2(N) & \cdots & c_1(N)\bar{u}_{N-K}(N)
\end{bmatrix}
\]
Q1 Please provide the expansion for FIR, SNR.
Q2 IEE style for matrices and vectors is to use bold italics. Please check that we have identified all instances.