# Efficient Blind Equalization of BPSK signals

<table>
<thead>
<tr>
<th>Journal</th>
<th>Electronics Letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript ID</td>
<td>ELL-2010-0292</td>
</tr>
<tr>
<td>Manuscript Type</td>
<td>Letter</td>
</tr>
<tr>
<td>Date Submitted by the Author</td>
<td>30-Jan-2010</td>
</tr>
<tr>
<td>Complete List of Authors</td>
<td>Al-Bayati, A. K. S.; The Hashemite University / Faculty of Engineering, Department of Electrical and Computer Engineering Smadi, Mahmoud; The Hashemite University</td>
</tr>
<tr>
<td>Keywords</td>
<td>BLIND EQUALISATION, MULTIPATH CHANNELS, PHASE SHIFT KEYING</td>
</tr>
</tbody>
</table>
Efficient Blind Equalization of BPSK signals

A. K. S. Al-Bayati, and M. A. Smadi

Department of Electrical Engineering /The Hashemite University
Zarqa/Jordan

Abstract

We propose a new blind equalization approach for BPSK signals based on a certain phase encoding process. The embedded phase property will enable blind signal equalization without the need for any training data. The performance of the new receiver is equivalent to that of a non-blind trained receiver with 1/3 of its data reserved for training. The new receiver is therefore very suitable for dynamic channels, where it will save significant throughput without sacrificing performance.

1- Introduction

Equalization of digital communication signals is essential to combat the Inter-symbol Interference (ISI) problem resulting from multipath signal propagation. In particular, blind equalization techniques [1][2] are needed in time-varying channels in order to avoid the loss in data throughput due to training.

In this letter, we propose a new approach for blind equalization of Binary Phase-Shift Keying (BPSK) signals, through the application of a certain phase encoding process at the transmitter. Based on the well known similarity between the ISI problem and the multiuser interference (MUI) problem we propose to use the phase encoding scheme of [3], described for CDMA signals, but here in the context of blind equalization. The equalizer (receiver) is designed without any knowledge of data, based on exploiting the knowledge of the phase encoding process at the transmitter. The resulting blind equalizer is shown to have a performance that is equivalent to a trained receiver.
2- System Model and Equalizer Design

We consider a scenario where a BPSK signal is transmitted over a multipath channel. We use a complex base-band model. Let \( d(n) \) be a random and independent binary \((\pm 1)\) data sequence to be transmitted over a complex base-band channel. While channels are generally time-varying, it is reasonable to assume that a channel is constant over short time periods, taken here as \( N \)-bits. Let \( h(n) \) (of length \( L \)) be the overall channel impulse response (including the effect of the pulse shaping transmit/receive filters) sampled at the bit rate.

Applying a phase-encoding process similar to [3], but obviously using only one sequence, we define a pseudo-random (PN) sequence [4] whose elements are either \((1)\) or \((\exp(j\pi/2))\).

This sequence, denoted by \( p(n) \), is multiplied by \( d(n) \) to generate the modified data sequence \( b(n) = d(n)p(n) \) that will be transmitted through the channel. This process converts the original data sequence \( d(n) \) from binary \((\pm 1)\) to quaternary \((\pm 1, \pm j)\). The received signal is given by

\[
r(n) = \sum_{m=0}^{M-1} h(m)b(n-m) + v(n) \tag{1}
\]

where \( v(n) \) is a sample of additive white Gaussian noise with zero mean and variance \( \sigma_v^2 \).

To derive the equalizer we will first construct the data matrix

\[
\tilde{X} = \begin{bmatrix}
  r(n-M-1) & \cdots & r(n-1) & r(n) \\
  r(n+1-M-1) & \cdots & r(n) & r(n+1) \\
  \vdots & & \vdots & \vdots \\
  r(n+N-M-2) & \cdots & r(n+N-2) & r(n+N-1)
\end{bmatrix} \tag{2}
\]

An \((M\times 1)\) equalizer \( \omega \) (with a delay of \( D \) bits) in the least squares (LS) sense, should minimize the cost function

\[
J_i(\omega) = \| \tilde{X}\omega - \mathbf{b}_m \|^2,
\]

where \( \mathbf{b}_m = \begin{bmatrix} b(n-m) & b(n-m+1) & \cdots & b(n-m+N-1) \end{bmatrix}^T \). We then premultiply the matrix \( \tilde{X} \) by the diagonal \( N \)-by-\( N \) matrix

\[
\mathbf{P}_D = \text{diag} \begin{bmatrix} 1/l p(n-D) & 1/l p(n-D+1) & \cdots & 1/l p(n-D+N-1) \end{bmatrix}
\]
to get the matrix \( X = \mathbf{P}_D \hat{X} \). Here we are assuming synchronization of the \( p(n) \) sequence at the receiver, as is also needed usually for training sequences. Consequently we can re-write the cost function as 

\[
J_2(\omega) = \|X\omega - d_D\|^2,
\]

where 

\[
d_m = \begin{bmatrix} d(n-m) & d(n-m+1) & \cdots & d(n-m+N) \end{bmatrix}^T.
\]

In order to obtain the LS equalizer \( \omega \) blindly we will exploit the fact that the vector \( \mathbf{d}_D \) is a real vector. From (1) and (2), it is not difficult to see that in the absence of noise, each column of \( \hat{X} \) is a linear combination of the vectors \([b_0, b_1, b_2, \ldots, b_{M+L-1}]\). Consequently, each column of \( X \) is a linear combination of the vector set \([\mathbf{P}_D b_0, \mathbf{P}_D b_1, \ldots, \mathbf{P}_D b_{M+L-1}]\). Obviously \( \mathbf{P}_D \mathbf{b}_D = \mathbf{d}_D \), and hence this vector set can be written as 

\[
\begin{bmatrix} c_{D,0} & c_{D,1} & \cdots & c_{D,D-1} & \mathbf{d}_D & c_{D,D+1} & \cdots & c_{D,M+L-1} \end{bmatrix},
\]

where

\[
c_{k,m} = \begin{bmatrix} \frac{d(n-m)p(n-m)}{p(n-k)} & \frac{d(n-m+1)p(n-m+1)}{p(n-k+1)} & \cdots & \frac{d(n-m+N-1)p(n-m+N-1)}{p(n-k+N-1)} \end{bmatrix}^T.
\]

Except for the real vector \( \mathbf{d}_D \), each of the vectors \( c_{D,0} \) to \( c_{D,M+L-1} \) has obviously real and imaginary elements due to pseudorandom nature of the elements of \( p(n) \). Hence the equalizer can be obtained by attempting to force \( X\omega \) to have zero imaginary part since this can only be satisfied by extracting the real vector \( \mathbf{d}_D \). Any other combination of vectors from the above vector set cannot have zero imaginary part for all the \( N \) elements of the vector. It is important to note here that a finite length \( (M) \) equalizer cannot perfectly equalize the channel. Hence, the desired equalizer \( \omega \) cannot extract the vector \( \mathbf{d}_D \) with perfectly zero imaginary part, nevertheless, \( \omega \) can still be found as the vector that minimizes 

\[
\| (X\omega)_{(i)} \|^{2}
\]

while avoiding the trivial solution by constraining 

\[
\| (X\omega)_{(i)} \|^{2}
\]

to be a constant (\( (i) \) and \( (i) \) denote
real and imaginary parts). Following [3] we define \( g = \begin{bmatrix} w_T^{(r)} & w_T^{(i)} \end{bmatrix}^T \) so that the constrained minimization becomes

\[
\begin{align*}
\text{minimize} & \quad g^T A^T A g \\
\text{subject to} & \quad g^T B^T B g = \text{constant}
\end{align*}
\]

where \( A = \begin{bmatrix} X^{(r)} & X^{(r)} \end{bmatrix} \) and \( B = \begin{bmatrix} X^{(r)} & -X^{(r)} \end{bmatrix} \). Thus \( g \) can be found, up to a complex scaling ambiguity, as the generalized eigenvector associated with the minimum eigenvalue of the matrix pair \( (A^T A, B^T B) \), and obviously the blind equalizer \( w \) will be given as

\[
w = g_1 + j g_2 \quad \text{where} \quad g_1 = \begin{bmatrix} g(1) \cdots g(M) \end{bmatrix}^T \quad \text{and} \quad g_2 = \begin{bmatrix} g(M + 1) \cdots g(2M) \end{bmatrix}^T.
\]

3- Computer Simulations

We study performance via computer simulations. For fair comparisons we assume differential phase detection applied to both blind and trained receivers. The trained receiver (equalizer) here is a linear receiver that minimizes the equalization error in the least squares sense and hence minimizes the earlier cost function \( J_1(\omega) = \| X \omega - b_\rho \|^2 \) and is therefore given by \( \omega_{LS} = X^+ b_\rho \), where “+” here denotes the pseudo-inverse (Moore-Penrose generalized inverse) of a matrix, and the vector \( b_\rho \) obviously is the known training data vector. It is also important to mention here that while the matrix \( X \) in the cost function above is similar in structure to that in equation (2), it does not involve the phase encoding process which is employed only for the blind receiver (i.e., it uses normal BPSK). In the first experiment we simulated a BPSK signal transmitted through a 9-tap channel with channel vector \( h = [0.108+0.332i, 0.382+0.525i, 0.364-0.264i, -1.00i, 0.35i, -0.149+0.0094i, 0.037+0.297i, 0.142+0.046i, -0.146-0.202i] \). We assume that a block of \( N = 300 \) bits is used, over which the channel remains stationary, and we use an equalizer of length 20 and delay \( D = 10 \) bits. The delay \( D_\rho \) usually introduced to account for the equalizer and the channel delays, is taken here to be half the equalizer length \( M \). Although not optimal, but in the absence of
channel length knowledge, this delay may be a reasonable choice especially for long 
equalizers. The non-blind equalizer uses a training block of 50, 100, and 200 bits out of the 
300 bit block, corresponding to 16%, 33%, and 66% of the data block. Fig.1 shows that the 
performance of the blind receiver is equivalent to that of a 33% trained receiver while at the 
same time delivering 1.5 times its throughput. With 66% training, the non-blind receiver is 
slightly better, but obviously this high percentage of training reduces throughput significantly 
and is of no practical use. With 16% training, the performance of the non-blind receiver 
worsens significantly. The second experiment (Fig.2) demonstrates performance versus block 
size. The non-blind receiver is assumed to reserve 33% of its data for training. Here we 
assume $h=[0.205-0.283i, -0.278-0.856i, -0.139+0.428i, -0.161+0.117i]$ and $M=10$ and $D=5$.

Fig.2 shows that the blind receiver clearly outperforms the non-blind receiver, and as the 
block length is decreased to 80 bits the blind receiver shows a higher performance advantage 
especially at high SNR’s.

4- Conclusions

We have proposed a new approach for blind equalizer design for BPSK signals based 
on a phase encoding process at the transmitter. This encoding enabled obtaining a very 
efficient equalizer blindly. While the new blind equalizer enables utilizing 100% of the 
available throughput, it achieves a very good performance that is equivalent at least to a 33% 
trained equalizer. This makes the new equalizer ideal for channels with fast variations where 
training may be required at short periods. The use of the proposed blind equalizer here saves 
significant throughput while at the same time achieving the desired performance.

References

2- Dogancay K., and Kennedy R.A.,” Least squares approach to blind channel equalization”, 
*IEEE Transactions on Communications*, vol.47, No.11, pp.1678-1687, Nov. 1999.

3- Al-Bayati A.K.S., Prakriya S., and Prasad S., “ Block phase precoding for blind multiuser 
detection of BPSK/DS-CDMA signals”, *IEEE Transactions on Communications*, vol.52, 


FIGURE CAPTIONS

Fig.1- Performance of the blind equalizer (ooo) and the trained equalizer for a length 9 channel. 
N=300 bits, M=20, D=10. Training sequence length (in bits) = 50 (xxx), 100 (+++), 200 (**). 

Fig.2- Performance versus block length for blind (ooo) and 33% trained (+++ ) equalizers for 
a length 4 channel. M =10 bits and D = 5 bits. (Upper curves for SNR=10, lower curves for 
SNR=14 dB).
Fig.1- Performance of the blind equalizer (ooo) and the trained equalizer for a length 9 channel. N=300 bits, M=20, D=10. Training sequence length (in bits) = 50 (xxx), 100 (+++), 200 (***).

Fig.2- Performance versus block length for blind (ooo) and 33% trained (+++) equalizers for a length 4 channel. M =10 bits and D = 5 bits. (Upper curves for SNR=10, lower curves for SNR=14 dB).