Optimal Real-Time Digital Control for a DC-Motor Proposed for Minimum Generation of Harmonics

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Abstract: The separately excited dc-motors give a pliable control, in which the armature and field circuits built with separate sources. For a sudden change in applied torque, there is a change in the motor speed. This speed change goes into transient period for some time before settling to the new value. One method of obtaining a variable-speed dc-motor drive is the dc chopper. The dc chopper converts the fixed-voltage dc source to variable-voltage dc source. The variable output voltage of the chopper depends on the chopping frequency. This chopping frequency can be provided using semiconductor switches. The switching technique that is used with chopper drive causes a discontinuity for armature current of dc motor and results in generation of harmonics.

In this work, a real-time discrete control for dc motor drive is proposed, implementing a dc chopper of a constant chopping frequency, with multi-chopping periods ($\sigma_i$). To reduce the discontinuity of the armature current caused by the switching technique, the single-conduction angle ($\sigma$) of the dc chopper is replaced by multi-internal conduction angles ($\sigma_i$) spreading on the chopping period. These multi-internal conduction angles will reduce the wideness of the increasing and decaying of armature current and forcing it to come near the continuity, and very less generation of harmonics. The total period of internal-conduction angles was maintaining same as the period of the corresponding single-conduction angle to guarantee that the same amount of average power was supplied to the dc motor.

Keywords: Chopper Drive, DC-Dynamic Response, Pulse-Modulation Control, Digital Control, Transient Response, Real-time Control, Sampled-data Control, Optimum Control, Proportional Control.

1 Introduction

Both series and separately excited dc motors are normally used in variable-speed drives [1]. With the help of their variable characteristics, it is possible to obtain speed control over a wide range. The most flexible control is obtained by means of a separately excited dc motor in which the armature and field circuits are provided with separate sources. This arrangement produces speed-torque characteristics approximately close to the ideal characteristics. For armature voltage, a controlled rectifier or a chopper is required [2]. The field current can be also controlled by the same provisions. Controlled rectifiers provide a variable dc output voltage form a fixed ac voltage, whereas choppers can provide a variable dc output voltage from a dc voltage.

In this work, the continuous dynamic response of the dc motor for a sudden change in load torque is sampled. The speed is measured at each sampling instant and the action for proportional control of armature voltage is taken at discrete manner. The method that was suggested in this work for determining the sampling period gives a faster and optimum speed response for the dc motor.

A real-time discrete control for the dc-motor speed is proposed. Instead of generating a single-conduction angle for the dc-chopper ($\sigma$), multi-conduction angles ($\sigma_i$) are generated, spreading on the chopping period ($T$). By this method, the expansion of rising and falling of dc motor-armature current is reduced, results in high continuity and low discontinuity, leading to minimum generation of harmonics.
2 DC Motor-Speed Control

The steady-state speed of a dc-motor reacts to the change in load torque, armature voltage and field flux. A separately excited motor with constant values of armature voltage and field current develops a speed that is decreased as the load torque is increased.

During the transient period for a sudden change in load torque, the addition load torque demand is met initially by a sudden retardation of the rotating masses which contribute to the moment of inertia of the motor shaft. Some studies ignore the mechanical system transient by considering rotor speed to be constant over the short time, or ignoring machine field and armature winding inductance [3].

The dc/dc converter or chopper and phase controlled ac/dc converter are using exclusively on the control of the armature voltage supply [4]. The chopper is involving semiconductor switches to chop the constant supply voltage at a fairly high repetition rate, enabling the average value of armature voltage to be variable between zero and the supply voltage. In the essential circuits of the chopper, the constant voltage source is feeding to the thyristor switch. This thyristor switch chops the voltage source before applied to the armature circuit. As the conducting period of the thyristor switch is changed, the average value of armature voltage is varied which result in speed control.

3 DC Motor- Model for Transient Operation

The equivalent circuit of a separately excited dc-motor is shown in Fig.1. The field current is assumed constant. The control is applied on terminal voltage \(v_a\) which equals:

\[ v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \quad V \]  

(1)

and:

\[ e_a = K \Phi \omega \quad V \]  

(2)

where:

- \(e_a\): armature e.m.f (volts).
- \(\Phi\): flux per pole (web).
- \(\omega\): motor speed (r/s).

The air-gap or electromechanical torque \(T_e\) equals:

\[ T_e = J \frac{d\omega}{dt} + B \omega + T_w \quad N.m \]  

(3)

Also:

\[ T_e = K \Phi i_a \quad N.m \]  

(4)

Where:

- \(J\): Inertia of both motor and driven mechanism (kg.m²).
- \(B\): Viscous friction of both motor and driven mechanism (N.m.s/r).
- \(T_w\): loading torque (N.m).

![Fig.1 A separately excited dc-motor.](image)

Transient behavior of dc motor can be analyzed by transforming the above equations to s-domain. By taking the Laplace transform of (1) and after some arrangements becomes:

\[ I_a(S) = \frac{V_a(S) - K \Phi \Omega(S)}{R_a (1 + \tau_a . S)} \]  

(5)

\[ \tau_a = L_a / R_a \quad (sec) \] , is the armature-circuit time constant.

Also by taking the Laplace transform of (3) and (4) we get:

\[ \Omega(S) = \frac{T_e(S) - T_w(S)}{B (1 + \tau_m . S)} \]  

(6)

And:

\[ T_e(S) = K \Phi I_a(S) \]  

(7)

\[ \tau_m = J / B \quad (sec) \] , is the mechanical time constant.
4 Single-Conduction Angle dc-Chopper

Real-time discrete control of the dc-motor is proposed using a dc-chopper of a constant frequency, as shown in Fig.2. With the pulse-width-modulation method, the chopping period of the chopper can be defined which is the same sampling period (T) of the digital controller. Figure 2-b explains the way of generating switching signals for the chopper. The triangular carrier signal (c) has a frequency of (1/T Hz). The amplitude (p_r) of the rectangular reference signal (r) depends on the speed error signal caused by the change of the loading torque. The conduction angle (σ) of the chopper is then determined by comparing the carrier signal amplitude (p_c) with the reference signal amplitude (p_r). If the difference between the carrier signal (c) and reference signal (r) becomes zero, the chopper will stop conducting and the reference signal sets to zero. Figure 3 shows the way of generating different conduction angles at different speed error signals.

4.1 Multiple-Conduction Angle dc-Chopper

The average voltage at the terminals of the armature dc-motor circuit can be controlled by generating different values of thyristor-firing angles which results in different values for chopper-conduction angles. By keeping the period (T) of the dc chopper at constant value and dividing the duration of the conduction angle (σ), multiple conduction angles (σ_i) can be generated for the same load-torque condition, as shown in Fig.4. These multiple conduction angles are spreading on the period (T) and must be determined in the way that to keep the same average power supplied to the dc-motor circuit. By this method, the expansion of increasing and decaying of dc motor-armature current is reduced; causing the armature current to come near the continuity and the generation of harmonics is very minimum.
The harmonics are greatest at:

\[ \sigma = T/2 \quad (8) \]

The number of internal conduction angles \( N_\sigma \) is proposed equals:

\[ N_\sigma = T/\sigma \quad (9) \]

Then the width if a one internal conduction angle \( \sigma_i \) is:

\[ \sigma_i = \sigma / N_\sigma \quad (10) \]

Where:
\( \sigma \): single conduction angle of the dc chopper.
\( \sigma_i \): internal conduction angle of the dc chopper.

The discontinuity (DIS) is a measure of harmonics generated by the dc chopper:

\[ \%\text{DIS} = 100 \times (I_{ax} - I_{an}) / I_{ax} \quad (11) \]

\( I_{ax} \): the maximum armature current at increasing period.
\( I_{an} \): the minimum armature current at decaying period.

These two initial current values are obtained from Appendix (A) and given below:

\[ I_{ax} = \left[ V_a (1 - e^{-\sigma/\tau_a}) / R_a (1 - e^{-T/\tau_a}) \right] - e_{ao} / R_a \quad (12) \]

\[ I_{an} = \left[ V_a (e^{\sigma/\tau_a} - 1) / R_a (e^{T/\tau_a} - 1) \right] - e_{ao} / R_a \quad (12) \]

5 Transient Response With Discrete Control

The motor-mechanism model is determined from (5), (6) and (7) and given in Fig.5. For a sudden change in applied torque \( T_w \), there is a change in the motor speed. This speed takes some time before settling to new value. To keep a constant motor speed for a sudden change in applied torque, a proportional voltage control is added. In this work, the response time is sampled with sampling period of \( T \)-sec, and the action of proportional control is taken at each instant of the discrete period. This discrete manipulation will show a faster speed response with less oscillation.

The transient response of motor speed to an unit-step change in load torque with no control of armature voltage is determined and given in Fig.6. The final value of speed is changed and settled on new low value because there is no action of armature control.

The control of armature voltage can be activated using discrete manner. This is done by sampling the time response of speed at sampling time of \( T \)-sec and the load change is manipulated at each sampling instant. The manipulation is done at off-time, i.e.; the calculations for new action of armature control is taken at each sample.

It is important to select a proper value for the sampling frequency to have a satisfactory transient response. The transient response of the continuous-data control system is investigated by [11] and a proper equation is developed for the determination of sampling period \( T_p \).

Speed response to an unit-step change of load torque using discrete control for the armature voltage with different values of sampling period is given in Fig.7. The best value of sampling period is calculated by [11] which equals 0.06 sec and its response shown in Fig.7-a. The response in Fig.7-b

Fig.5 DC- motor block diagram.

![DC- motor block diagram](image)

Fig.6 Speed response to an unit-step change in \( T_w \)
is for a larger sampling period of 0.1 sec, which has
more overshoots and become larger.
In the above results, the calculations and action
for new speed were done simultaneously. But
practically, the controller needs some time for
calculations before taking the new action for the
armature voltage. Therefore, the first sample after
the change is left for calculations, and the action
of controller is taken in the second sample. The
speed response to an unit-step change of load
torque and for sampling period equals to that one
calculated by [11] but with off-on controller is
shown in Fig.8. The response for a half value of
the calculated sampling period is shown in Fig.9.
Results show that the first overshoot with T= 0.03
sec is less than that with T = 0.06 sec but with
more speed perturbation.

Fig.7 Speed response to an unit-step change of
load torque

Fig.8 Real-time speed response with discrete control
of \( V_a \) at \( T = 0.06 \) sec.

Fig.9 Real-time speed response with discrete control
of \( V_a \) at \( T = 0.03 \) sec.

6 Optimal Discrete Control for
Minimum Generation of Harmonics
For a sudden change of load-torque, the period (T)
of the dc chopper was kept constant and its
conduction angle was divided into multiple-
internal conduction angles. For an unit-step
change of load torque on dc motor, the
discontinuity amplitude for different sets of
multiple-internal conduction angles of dc chopper
is given in Fig.10. The results show that as the
number of internal conduction angles is increased
as the discontinuity amplitude is decreased.
Figure.11 shows the discontinuity amplitude for
double step-change of load torque. The chopping
frequencies for maximum internal conduction
angle in Figs.10 and .11 are 5.3 kHz and 5 kHz
respectively. These values of chopping frequency
are more suitable for thyristor-switching used in
the driving circuit.
Figure.12 shows the armature current of dc motor
at unit-step change of load torque with different
sets of multiple-conduction angle. The results show that as the number of the internal conduction angles for dc chopper is increased as the discontinuity of armature is decreased results in minimum harmonics.

Fig.10 Discontinuity amplitude for an unit-step change of load torque

Fig.11 Discontinuity amplitude for a double-step change of load torque

**7 Conclusions**

A real-time discrete controller for the dc-motor drive is proposed, using a dc-chopper of a constant frequency to control the armature voltage for a constant speed at a sudden change in load torque.

For a sudden change in load torque and instead of generating one conduction angle for the dc-chopper, multiple-conduction angles are generated, spreading on the chopping period (T) to reduce the wideness of rising and falling of dc motor-armature current. The results were shown that the high continuity and low discontinuity, leading to minimum generation of harmonics was obtained at high number of multiple conduction angles for dc chopper.

Fig.12 Armature current of dc motor
References:


Appendix-A:

By rearranging Eq.1 for dc-motor armature current, we get:

\[
\frac{di_a}{dt} + R_a i_a / L_a = \left( \frac{v_a - e_a}{L_a} \right) \text{ A/sec} \quad (A.1)
\]

Assuming the dc copper starts conducting at t =0, with armature current equals Ian:

\[
e_a = K \varphi \omega = E_b \quad (A.2)
\]

\[
v_a = Cons = V_a \quad (A.3)
\]

Solving Eq.A.1 with above conditions the armature current for a conduction angle \( \sigma \) is:

\[
i_a = \left[ \frac{(V_a - E_b)}{R_a} \right] (1 - e^{-t/\tau_a}) + Ian e^{-t/\tau_a} \quad (A.4)
\]

For non-conducting period, Eq.A.1 becomes:

\[
\frac{di_a}{dt} + R_a i_a / L_a = -e_a / L_a \text{ A/sec} \quad (A.5)
\]

and the armature current decays through \( R_a \) at initial armature current equals Iax:

\[
i_a = \left( - \frac{E_b}{R_a} \right) (1 - e^{-t/\tau_a}) + Iax e^{-t/\tau_a} \quad (A.6)
\]