New PWM switching technique for an optimum inverter operation

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Abstract: The main purpose of Pulse-Width–Modulation (PWM) technique in power inverters is to control the output voltage. The most familiar techniques: uniform-pulse-width-modulation and sinusoidal-pulse-width-modulation. A Small Boundary-Pulse-Width (SBPW) technique for improving the inverter performance is proposed. The new method is based on pulse-width technique. Varied pulse-widths for the middle and boundary pulses are produced, to obtain an improved inverter operation with less generation of harmonics. Simulation results show that the SBPW technique outperforms some of the traditional methods.

Keywords: Pulse-width–modulation (PWM) control, DC to AC inverters, Harmonic control, Energy conversion.

1 Introduction

The purpose of an inverter is to change a dc input voltage to an ac output voltage of a desired magnitude and frequency [1]. Practically, real-time switching control can be deduced using pulse width-modulation methods [2, 3]. The changeable inverter gain provides an efficient control drive [4, 5]. A variable inverter output voltage can be obtained by varying the gain of the inverter, which is achieved using pulse-width – modulation (PWM) [6, 7, 8]. This technique causes a discontinuity in the inverter output voltage and harmonics are produced [9, 10]. Therefore, many techniques are developed and tried to reduce the harmonic contents of inverter output voltage and getting acceptable inverter operation based on low distorted sinusoidal output voltage. The most common types of switching techniques are the single-pulse-width-modulation, uniform pulse-width-modulation and sinusoidal-pulse-width-modulation [11, 12, 13], all are tried to improve the performance of the inverter operation.

This paper describes new types of switching control that implement a form of pulse width-modulation. These two techniques are the Small Boundary-Pulse-Width (SBPW) and Large Boundary-Pulse-Width (LBPW). A control strategy based on variation of pulse-widths for the middle and boundary pulses is developed, to reduce the harmonic effect and obtaining an improved inverter operation. Based on minimum harmonic contents in the inverter output voltage, results are shown that the SBPW technique is more adequate and more effective.

2 Traditional methods

The main function of an inverter is to convert a dc input voltage to an ac output voltage. The output voltage of a practical inverter is non-sinusoidal waveform and contains harmonics, whereas a low distorted output voltage is needed. A variable inverter
gain can be obtained using the switching technique, which is accomplished by pulse-width modulation (PWM). Different switching techniques are developed to modify the gating angles of the inverter driver, and thereby to improve the inverter operation based on minimum harmonic contents in the inverter output voltage [14,15].

2.1 Single PWM Technique

The single-PWM technique produces only a one-pulse-width per half cycle [1]. The generation of different values of gating angles (\(\alpha\)) for the inverter driver, results in variable pulse-widths (\(\delta\)), leading to a variable inverter operation, as shown in Fig.(1).

Due to switching technique implemented by this method, the output voltage is discontinuous, and its Fourier series is given by the following equation:

\[
v_o(t) = \sum_{n=1}^{\infty} \left[ \frac{4 V_d}{n \pi} \sin \frac{n \delta}{2} \right] \sin n \omega t
\]

Where:
- \(v_o(t)\) the inverter output voltage.
- \(V_d\) the d.c supply voltage.
- \(n\) the harmonic order.
- \(\delta\) the pulse width.

Harmonic profile of the single PWM technique showed that the prevalent harmonics are the third and fifth components [1].

2.2 Uniform PWM Technique

In the uniform-pulse-width-modulation technique (UPWM), multiple pulses of equal widths per half cycle are produced [16], as shown in Fig.2. The Fourier series of the inverter output voltage is given by the following equation:

\[
v_o(t) = \sum_{n=1}^{\infty} \sum_{m=1}^{2p} \left[ \frac{4 V_d}{n \pi} \sin \frac{n \delta_p}{4} \right] \sin n \omega t
\]

Where:
- \(p\) the number of pulse per half cycle.
- \(m\) the pulse-number.
- \(\delta_p = \delta/p\).

The harmonic profile for this technique showed that the harmonic contents are less than that produced by the single-pulse-width-modulation technique [1].

2.3 Sinusoidal PWM Technique

Instead of keeping the pulse width fixed, the width of each pulse can be changed using the sinusoidal-pulse-width-modulation technique, as shown in Fig.3 [17, 18]. The width of each pulse depends on the amplitude of the sinusoidal reference signal. The Fourier series of the inverter output voltage is given by the following equation:
3 Boundary Pulse-Width Technique

In many applications, it is necessary to control the gain of inverters. A variable inverter gain is achieved by applying the switching control which implements the pulse-width-modulation techniques. The harmonic contents in the inverter output voltage are changeable and depend on the control strategy. This paper proposes two new switching techniques that are used to control the inverter gain. The two sections below summarize their control schemes.

3.1 Small Boundary - Pulse - Width Technique

In this technique, instead of generating a one pulse of width (δ) per half cycle of inverter output voltage, several pulses of number (p) are generated. The boundary pulses are given the smaller width and middle pulse is obtained the larger width, as shown in Fig.4. Assuming the number of pulses per half cycle is p.

The uniform-width of multiple-pulse per half cycle is given below:

\[ \delta_u = \frac{\delta}{p} \]  

Where: \( \delta \) is the width of the original single pulse.

The widths of the middle pulse and boundary pulses are determined from equations below. The sum of the middle pulse and boundary pulses must be maintained the same original pulse width (δ), to keep up the same condition for the inverter operation. This arrangement will modify the harmonic contents of the inverter output voltage and improve its operation.

The width of each boundary-pulse width is reduced to:

\[ \delta_b = \frac{2}{p} \delta_u \]  

These reductions in boundary-pulse widths are added to the middle-pulse (\( \delta_m \)) as below, to keep up the same inverter operation.

The middle-pulse adding (\( \delta_{mA} \)) is determined from the following equation:

\[ \delta_{mA} = \delta_u \times \left( \frac{p-1}{2} \right) \]  

This middle-pulse adding (\( \delta_{mA} \)) is added to the uniform middle-pulse width (\( \delta_u \)), results in:

\[ \delta_m = \delta_u + \delta_{mA} \]
3.2 Large Boundary – Pulse - Width Technique

In this technique, the width of the middle pulse is decreased by half of the uniform pulse-width ($\delta_u$). The reduction in middle pulse width is manipulated as below and equally added to boundary pulse widths. The modification in middle pulse and boundary pulses give the similar width of the original single-pulse ($\delta$) to keep up the same inverter operation.

The middle-pulse width ($\delta_m$) is reduced to:

$$\delta_m = \frac{\delta_u}{2}$$  \hspace{1cm} (8)

The boundary-pulse-adding ($\delta_{bA}$) is given by:

$$\delta_{bA} = \delta_u \times \left(\frac{1}{2(p-1)}\right)$$  \hspace{1cm} (9)

Then, the width of each boundary-pulse becomes:

$$\delta_b = \delta_u + \delta_{bA}$$  \hspace{1cm} (10)

4 Inverter Harmonic profile

The SBPW switching technique proposed in this paper has tried to improve the inverter operation. This technique modified the pulse width of the inverter driver to achieve minimum harmonic contents in the inverter output voltage. To determine the inverter harmonic performance using SBPW technique, it is necessary to deduce the coefficients for the Fourier series equation that is used to analyze the inverter performance. The general Fourier series for the inverter output voltage is[1]:

$$v_a(t) = a_o + \sum_{n=1}^{\infty} a_n \cos n(\omega t) + b_n \sin n(\omega t)$$  \hspace{1cm} (11)

Where: n is the harmonic order.

Due to the symmetry of the inverter output voltage, the Fourier coefficients ($a_o$) and ($a_n$) are zeros. In Fig.4, assume that the pulse of order (k) has a pulse width of ($\delta_k$) and gating angle of ($\alpha_k$). The Fourier coefficients of this pulse including positive and negative parts are derived in Appendix (A). For p-pulse per half cycle, the $b_n$ equation is given below:

$$b_n = \sum_{k=1}^{2p} \frac{2V_d}{\pi n} \left[ \cos n\alpha_k - \cos (\alpha_k + \delta_k) \right]$$  \hspace{1cm} (12)

5 Inverter Performance

Due to the switching control, the inverter output voltage contains harmonics, and the quality of a practical inverter is evaluated in terms of harmonic contents of its output voltage. The harmonic profile for the small boundary-pulse-width technique is shown in Fig.5, with the variation of conduction angle ($\delta$) of the inverter driver. Due to the symmetry of the output pulses for two half cycles, the even harmonics are absent. The prevalent harmonics are the third and fifth component.

6 Results

Instead of generating several pulses of a uniform pulse-width, multiple pulses of variable widths were presented in this paper. Figure.6 shows the third harmonic amplitude of inverter output voltage with the variation of the conduction angle ($\delta$). The results are
shown for uniform-pulse-width-modulation technique, small boundary-pulse-width and large boundary-pulse-width techniques. The third harmonic produced by the small boundary-pulse-width technique is at minimum and equals zero for the low range of the conduction angle. Figures 7 and 8 show the ninth and eleventh harmonic amplitudes of inverter output voltage for different control techniques. It can be seen that harmonic contents are reduced for the small boundary-pulse-width technique as the harmonic order is increased, and it is zero for the eleventh harmonic. The number of pulses (p) per half cycle plays a significant role on the harmonic amplitude of inverter output. Figure 9 shows the third harmonic amplitude as a function of the number of pulses per half cycle (p), using small boundary-pulse-width technique. It is clear that as the number of pulses (p) is increased as the harmonic amplitude is decreased, results in an optimum inverter operation of less harmonic contents.

7 Conclusions

The paper has represented a new pulse-width modulation technique for power inverter to control the output voltage and more important to improve the inverter performance. The technique was developed for determination of the variable-pulse durations of different pulses to achieve an optimum inverter operation based on minimum generation of harmonics. The small boundary-pulse-width technique has shown that the harmonic contents in the inverter output voltage were minimum compared with the other methods. Another advantage for the small boundary-pulse-width technique is that the number of pulses per half cycle can be modified to provide an optimum inverter operation.

Appendix A

Assume there is a pulse of order (k) has a pulse-width of $\delta_k$, triggered at angle $\alpha_k$. The Fourier coefficients $a_o$, $a_n$ and $b_n$ of this pulse including positive and negative parts are derived below. The integral limit is taken from $(\alpha_k+\delta_k/2)$ to $(\alpha_k+\delta_k/2+\pi)$ in order to include the calculations the positive and negative parts of the inverter output.

$$v_o(t) = a_o + \sum_{n=1}^{\infty} \left( a_n \cos(n \omega t) + b_n \sin(n \omega t) \right)$$  \hspace{1cm} (A-1)

The coefficients $a_o$:

$$a_o = \frac{1}{2\pi} \int_0^{2\pi} v(t) \, dt$$  \hspace{1cm} (A-2)

$$a_o = \frac{1}{2\pi} \left[ \int_{\alpha_k+\pi}^{\alpha_k+\delta_k/2} v(t) \, dt \right] - \left[ \int_{\alpha_k-\pi}^{\alpha_k+\delta_k/2} v(t) \, dt \right]$$

$$a_o = \frac{1}{2\pi} \left[ \alpha_k + \delta_k - \alpha_k - \frac{\delta_k}{2} \right]$$

$$a_o = \frac{1}{2\pi} \left[ -\frac{\delta_k-\pi+\alpha_k+\pi}{2} \right]$$

$$a_o = 0.$$  \hspace{1cm} (A-3)

The coefficients $a_n$:

$$a_n = \frac{2}{2\pi} \int_0^{2\pi} v(t) \cos(n \omega t) \, dt$$  \hspace{1cm} (A-4)
The Fourier coefficient \( b_n \) of \( p \)-pulses per half cycle is:
\[
b_n = \sum_{k=1}^{2p} \frac{2V_d}{\pi n} \left[ \cos n\alpha_k - \cos \left( \alpha_k + \delta_k \right) \right]
\] (A-7)

For \( p \)-pulses per half cycle:
\[
a_n = \sum_{k=1}^{2p} \frac{2V_d}{\pi n} \left[ \sin(\alpha_k + \delta_k) - \sin \alpha_k \right]
\] (A-5)

Because of the symmetry of inverter output, the coefficient \( (a_n) \) in equation (A-3) always equals zero.

The coefficients \( b_n \):
\[
b_n = \frac{2}{2\pi} \int_0^{2\pi} v(t) \sin n(\omega t) d\omega \quad (A-6)
\]
\[
b_n = \frac{2V_d}{\pi} \left[ \int_{\alpha_k + \frac{\delta_k}{2}}^{\alpha_k + \pi + \frac{\delta_k}{2}} \sin n(\omega t) d\omega - \int_{\alpha_k + \frac{\delta_k}{2}}^{\alpha_k + \pi} \sin n(\omega t) d\omega \right]
\]
\[
b_n = \frac{2V_d}{\pi} \left[ \int_{\alpha_k + \frac{\delta_k}{2}}^{\alpha_k + \pi + \frac{\delta_k}{2}} \left( \frac{1}{n} \cos n\alpha_k \right) \frac{1}{n} \left( \cos n\alpha_k + \frac{\delta_k}{2} \right) - \int_{\alpha_k + \frac{\delta_k}{2}}^{\alpha_k + \pi} \left( \frac{1}{n} \cos n\alpha_k \right) \left( \frac{1}{n} \cos n\alpha_k + \frac{\delta_k}{2} \right) \right]
\]
\[
b_n = \frac{2V_d}{\pi n} \left[ \cos n\left( \alpha_k + \frac{\delta_k}{2} \right) - \cos n \left( \alpha_k + \delta_k \right) \right]
\]
\[
b_n = \frac{2V_d}{\pi n} \left[ \cos n\alpha_k - \cos \left( \alpha_k + \delta_k \right) \right]
\]
References:

Figure 5  Harmonic profile for small-boundary pulse-width technique.

Figure 6  3rd harmonic amplitude for different techniques.
Figure 7  9th harmonic amplitude for different techniques.

Figure 8  11th harmonic amplitude for different techniques.
Figure 9: Small-boundary pulse-width technique for $p=3$, 5, 7..., 15.