Further Study and Analysis of the $^7$Li on $^{56}$Fe Reaction at 50 MeV Incident Energy

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Abstract. We present deuteron and triton spectra measured at 12 laboratory angles from the reaction of 50 MeV $^7$Li on $^{56}$Fe together with evaporative components as modeled by CASCADE and fitted to the backward angle data. The deuteron and triton “break-up” spectra obtained by subtracting the evaporative components from the measured spectra are also presented. The break-up of the $^7$Li projectile near the surface occurs with high probability and the major fraction of the break-up cross-section is taken by ($^7$Li,$\alpha$) transfer process. A crude estimate of the fraction of the total cross-section is found to be of the same order of magnitude as the overall spectroscopic factor determined by the diffraction model. The value of the estimated fraction of total cross section at 50 MeV incident energy is compared to that at 68 MeV incident energy for the same reaction. However, these values of estimated fraction of total cross section are found very much consistent with the measured yields at both incident energies. The importance of the level density parameter in locating the maximum of excitation energy is indicated in the diffraction model.

Keywords: low and intermediate energy heavy-ion reactions, transfer reactions
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1. Introduction

Attempts to study the reactions of $^7$Li on $^{56}$Fe have been made [1, 2]. Excitation functions and differential recoil range distributions for different radioactive products of the $^7$Li on $^{56}$Fe reaction together with inclusive proton, deuteron, triton and alpha spectra, have been measured up to $E(7Li) = 89$ MeV [1]. The excitation functions and locations of their peaks were reproduced by the CASCADE calculations while the complete fusion and incomplete fusion processes were found by using the recoil
range distributions [1]. The alpha, deuteron and triton spectra at an incident energy of 68 MeV and at 12 laboratory angles show a pronounced “break-up” component at forward angles. The alpha spectra measured at 50 MeV incident energy at different laboratory angles, were also presented [2]. The alpha-particle spectra at 50 and 68 MeV incident energies at different laboratory angles were analyzed using the diffraction model [1,2]. In such a model [1–3] the reactions are treated as surface transfer reactions populating continuum states. A modified version of this model was used [1,2], where the nuclear rainbow was introduced into the pure Coulomb deflection function. Also the level density of the continuum states in the residual nucleus was assumed to be similar to the usual statistical level density [1,2].

In the present work, the inclusive deuteron and triton spectra measured at 50 MeV beam energy at 12 laboratory angle is revealed. The measurement techniques are generally similar to those described previously [1,4]. The spectra also show a pronounced “break-up” component at forward angles. The energy-integrated cross section as a function of laboratory angles at 50 MeV for the measured “break-up” light particle spectra of alpha, triton and deuteron are presented and compared. The experiment and analysis show that the alpha particle break-up component has the highest exponential fall-off. Also the total yield of the transfer reaction $^7\text{Li}(^{56}\text{Fe},\alpha)^{59}\text{Co}^*$ at both incident energies of 50 and 68 MeV is estimated using simple qualitative treatments. The estimated results are found in agreement with the numerical values obtained using the full calculations. A simple approximate method of analysis is also adopted to show the role of the level density parameter in locating the maximum of excitation energy, in the diffraction model. We confirm that the fall-off of energy-integrated alpha-particle yield with laboratory angle is sensitive to the choice of level density parameter. However, $^7\text{Li}(^{56}\text{Fe},\alpha)^{59}\text{Co}^*$ incomplete fusion process is significantly more important than the corresponding process in which a triton or a deuteron is emitted.

2. Brief of Theory

The strong absorption analysis of the transfer reaction $^7\text{Li}(^{56}\text{Fe},\alpha)^{59}\text{Co}^*$ at 50 MeV has been based on the diffraction model [2]. In this model, for quasi-elastic transfer reactions to continuum states, the double-differential cross-section in the center of mass is given by [1–3, 5, 6]

$$\frac{d^2\sigma}{d\Omega dE_f} = \sum J \rho(0, E^*)(2J + 1) \exp \left( -\frac{J(J+1)}{2\sigma^2} \right) \sigma(\theta, E_f, J),$$  \hspace{1cm} (1)

where $\rho(0, E^*)$ is the spin-zero level density [2, 3, 8], given by

$$\rho(0, E^*) = \frac{\exp(\sqrt{2U})}{24\sqrt{2\pi^3}U^{1/4}},$$  \hspace{1cm} (2)

and $\sigma(\theta, E^*, J)$ is the no-recoil Distorted-Wave Born Approximation (DWBA) cross-section for a transfer reaction involving spinless particles which is taken in the center
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of mass [1–3] as

$$\sigma(\theta, E_f, J = L) = \tau \frac{\mu \mu_i}{(2\pi \hbar^2)^2} \frac{k_f}{k_i} \sum_{M} |T_{LM}^M|^2, \quad (3)$$

where

$$T_{LM}^M = \tau \frac{4\pi}{k_i k_f} \sum_{L \ell_f} \delta_{L-L-i} \beta_{\ell_i, \ell_f} (2\ell_f + 1)^{1/2} \exp\{i(\sigma_{\ell_i} + \sigma_{\ell_f})\}$$

$$\times \langle \ell_f L; 00 | \ell_f | -MM | \ell_f L; 00 \rangle Y_{\ell_f}^{-M}(\theta, 0). \quad (4)$$

Here $\ell$ is the partial wave, $\mu$ is the reduced mass, $k$ is the wave number, $\sigma$ is the Coulomb phase shift, and $\tau$ is a spectroscopic transfer parameter. The subscripts $i$ and $f$ refer to the entrance and exit channels, respectively. The form of the reduced matrix element $T_{LM}^M$, as indicated in Refs. [1] and [2], was developed by Austern and Blair [9] for inelastic scattering of strongly absorbed projectiles and modified by Hahne [10] for transfer reactions. The spin cutoff parameter, $\sigma$, as defined below, depends on the level density parameter $\alpha$ and the effective excitation energy $U$ [8].

3. Results and Analysis

The smooth solid curves in Fig. 1 (left-hand panel) give the predictions of the CASCADE code for the energy spectra of evaporated deuterons, which have been transformed to the laboratory frame. In a similar analysis to that for the alpha spectra, the angular distribution was assumed to be of the form $b\{1 + c / \sin \phi\}$ [1]. In this form, $b$ is a normalization constant and $c$ denotes the ratio of the coplanar/isotropic admixture. By adjusting these two parameters only, good fits were simultaneously obtained for all deuteron energy spectra at angles backward of 60°, as depicted in Fig. 1 (left-hand panel). The solid curves which represent the evaporation component are extrapolated from the calculations at backward angles. Subtraction of this component from the measured spectra yields the nearly symmetric broad peaks that presumably represent the break-up component, as shown in Fig. 1 (right-hand panel). The deuteron spectra exhibit features similar to those of alphas [1] with an evaporation component and a break-up component centered at about $E = 14.3$ MeV for the 50 MeV incident energy. This is compared to $E = 19.4$ MeV for the 68 MeV incident energy. (This is about 2/7 of the beam energy). On the other hand, triton measured spectra show a negligible evaporation component, which is in agreement with the CASCADE calculations. However, the break-up component of the triton spectra, shown in Fig. 2, is centered at 21.4 MeV for the 50 MeV incident energy, which is about 3/7 of the beam energy. This is also compared to 29 MeV for the 68 MeV incident energy. However, the observed total yield of break-up deuterons and tritons is less than 0.3 barn at 50 MeV. This is very small compared to the corresponding value of observed break-up alphas of 0.49 barn at the same energy.

Figure 3 depicts plots of energy-integrated cross-section as function of laboratory angles at $E( ^7\text{Li}) = 50$ MeV for the measured break-up components of alphas,
tritons and deuterons. The results of the α-particle component fall off the fastest, and the triton component is faster than the deuteron component. In each case, the angular distribution can be parameterized in the form $A \exp(-\theta/\theta_0)$. By extrapolating the exponential distributions to angles forward of $0^\circ$, estimates of the total yield of break-up alphas, triton and deuterons were obtained. The overall uncertainties in these estimates are about ±15%, which mainly originate from forward angle extrapolation and evaporation background subtraction. The values of these yield estimates together with those of $\theta_0$ were listed in Table 1 of Ref. [1]. The value of $\theta_0 = 6.8^\circ$ for the alpha particles at the 68 MeV incident energy is less than that of 10.1$^\circ$ at the 50 MeV indicates the faster fall-off of the angular distribution at the 68 MeV than that at the 50 MeV energy. Also the smallest value of 10.1$^\circ$ for alpha particles at the 50 MeV energy as compared to those of 14.1$^\circ$ and 11.8$^\circ$ for deuterons and tritons, respectively, at the same energy, indicates the fastest fall-off

Fig. 1. Left-hand panel: the deuteron spectra measured at 12 laboratory angles from the reaction of 50 MeV $^7$Li on $^{56}$Fe. The curves show the evaporative component, as modeled by CASCADE and fitted to the backward angle data. Right-hand panel: the break-up deuteron obtained by subtracting the curves in the left-hand panel from the data.
Fig. 2. The triton spectra measured at forward angles from the reaction of 50 MeV $^7\text{Li}$ on $^{56}\text{Fe}$.

Fig. 3. The values of the differential break-up cross-section for break-up deuteron, triton and $\alpha$ particles obtained by integrating the break-up energy spectra at 50 MeV incident energy. The solid lines show the best linear fits.
of the angular distribution of alpha particle component. By assuming a similar angular
distribution for the excess cross-section in the proton spectra, an upper limit
of 50 mb to the possible proton break-up yield is also estimated.

With respect to break-up into \( \alpha + t \), \(^7\text{Li}\) is bound by only 2.5 MeV. As a
consequence, reactions of \(^7\text{Li}\), even at low energy, are expected to involve a major
contribution of break-up process. The above results indicate that more break-up
alphas were possibly detected than deuterons and tritons combined. This implies
that these alphas are not simply fragments from pure projectile break-up but that,
in many cases, may be accompanied by complete mixing of the rest of the projectile
with the target. This is the reason for considering the \(^7\text{Li}(^{56}\text{Fe},\alpha)^{59}\text{Co}\) incomplete
fusion to be more important than any corresponding process in which a triton (or
possibly a deuteron) is emitted [1]. However, direct evidence of such incomplete
fusion was provided by the recoil range studies [1].

The steep fall-off of the integrated cross-section with angle at 50 MeV incident
energy is a prominent feature of transfer reactions involving strongly absorbed ions,
where the interaction is effectively confined to the surface region [2]. The corre-
sponding fall-off at the 68 MeV is faster than that at the 50 MeV. The normalizing
constant \( \tau \) obtained from matching the calculation to experiment was about 0.5
barn at the 50 MeV and 0.54 barn at the 68 MeV [1, 2]. This constant has the
dimension of area and represents an average strength for the reactions over the full
range of angular momentum transfer and excitation energies involved. So it may be
instructive to compare the value obtained for \( \tau \) with the calculated estimate of the
yield for peripheral reactions. The total absorption cross-section [11] is given by

\[
\sigma_{\text{abs}} = \pi \sum_{\ell=0}^{\infty} \frac{2\ell + 1}{\ell^2} (1 - \eta_\ell^2).
\]

Here the reflection coefficient \( \eta_\ell = |S\ell| \). Therefore, when the contribution of orbital
angular momentum is taken from zero up to the limit of grazing angular momentum
\( \ell_g \), then the expression \( \sigma_{\text{abs}} \) can be approximated as

\[
\sigma_{\text{abs}} \approx \pi R^2 \left( 1 - \frac{2n}{KR} \right).
\]

Here \( R = r_0(A_1^{1/3} + A_2^{1/3}) \), where \( A_1 \) and \( A_2 \) are the mass numbers of the pair
of nuclei in the entrance or exit channels of the reaction, and \( n = Z_1 Z_2 e^2/\hbar \nu \) is
the Sommerfeld parameter. The total cross-section of the present reaction can be
estimated as 1.92 barn.

A “crude” estimate of the fraction of the total cross-section \( \sigma_{\text{abs}} \), which is di-
verted to peripheral reactions may be obtained as the differential of \( \sigma_{\text{abs}} \) corre-
sponding to the peripheral range \( R \pm d \), thus \( \Delta R = 2d \), and

\[
\Delta \sigma_{\text{abs}} \approx (\sigma_{\text{abs}} + \pi R^2) \frac{\Delta R}{R}.
\]
The numerical value given by this relation for the 50 MeV \(^7\text{Li}\) on \(^{56}\text{Fe}\) is 0.47 barn while that for the 68 MeV is 0.493 barn. It is worth noting that these values have the same order of magnitude as the value of \(\tau = 0.5\) barn and 0.54 barn calculated in references [1,2]. It is also comparable to the angle- and energy-integrated cross-section or the total yield of the present transfer reaction. This latter quantity was estimated to be 0.49 barn and 0.55 barn for the 50 MeV and the 68 MeV incident energies, respectively [1,2].

It was seen that the shapes of the five energy alpha spectra are [2] well reproduced using the level density parameter, \(a\), as the only adjustable parameter. It is interesting that those fits, which reproduce well both the width and the peak energy for each spectrum were obtained without having to change any of the elastic scattering parameters. It is found that the positions of the energy spectra are quite sensitive to the value of parameter \(a\). This can be understood by examining Eq. (1) where the two functions \(\rho(0, E^*)\) and the \(J\)-weighted sum are plotted as functions of excitation energy [11]. It was noted that the \(J\)-weighted sum decreases rapidly with excitation energies while \(\rho(0, E^*)\) increases [11]. The decrease in the \(J\)-weighted sum with excitation energy is the result of the decrease in the relative availability of states with the minimum required spin as this minimum increases with excitation energy. The peak of the energy spectrum is determined by the balance of these two opposing effects. In fact, a closer examination of the expression for the double differential cross-section reveals the existence of four factors, which determine the shape and peak location of the energy spectra. This will be shown on the basis of a simplified version of Eq. (1).

The contribution of a particular angular momentum transfer \(J\) to the differential cross-section \(\sigma(\theta, E^*, J)\) is found significant [1-3] only for values of \(J\) equal or above \(|\ell_{1g} - \ell_{f_g}|\). The approximation will be made that the dependence of \(\sigma(\theta, E^*, J)\) on \(J\) is of the form of a step function which starts at \(J = |\ell_{1g} - \ell_{f_g}|\). Further, the \(J\)-weighted sum of the double differential cross-section is approximated by an integral over \(J\). Here \(J\) (renamed \(J_m\)) depends on the excitation energy \(E^*\). The dependence of \(J_m\) on \(E^*\) can be obtained from the values of \(|\ell_{1g} - \ell_{f_g}|\) for various excitation energies [2]. The differential cross-section \(\sigma(\theta, E^*)\) which depends on \(E^*\) mainly through the factor \((2\ell_{f_g} + 1)^2\) can be approximated by [11]:

\[
\sigma(\theta, E^*) = \sigma(\theta)(2\ell_{f_g} + 1)^2. \tag{8}
\]

By taking \(\ell_{1g} = 28\) for this reaction, using the effective excitation energy \(U = E^* - \Delta\) (where \(\Delta\) is the pairing energy and has a value of 1.3 MeV), \(\sigma^2 = 0.0888A^{2/3}/\sqrt{aU}\) and considering the above-mentioned approximations, a simplified version of the double differential cross-section is obtained as

\[
\ln \left( \frac{d^2\sigma}{d\Omega dE^*} \right) = 2\ln(57 - 2J_m) + (2\sqrt{aU} - 1.25\ln U) - \frac{J_m(J_m + 1)}{2\sigma^2} + C(\theta). \tag{9}
\]

Here \(C(\theta)\) is a function of \(\theta\) only, while \(\ell_{f_g}\) is expressed in terms of \(\ell_{1g}\) and \(J_m\).

The maximum of the energy spectrum is then determined by taking the derivative of Eq. (9) with respect to \(E^*\) such that \(d\ln(d^2\sigma/d\Omega dE^*)/dE^* = 0\). This will
give the equation
\[ \Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4, \] (10)
where
\[ \Gamma_1 = \left( \frac{a}{E^* - 1.3} \right)^{1/2} - \frac{1.25}{(E^* - 1.3)}, \]
\[ \Gamma_2 = -\frac{(2J_m + 1)}{2\sigma^2} \left( \frac{dJ_m}{dE^*} \right), \]
\[ \Gamma_3 = \frac{J_m(J_m + 1)}{2\sigma^2(E^* - 1.3)}, \]
\[ \Gamma_4 = -\frac{2}{(57 - 2J_m)} \left( \frac{dJ_m}{dE^*} \right). \]

The first term, \( \Gamma_1 \) gives the rate of the logarithmic increase of the cross-section with \( E^* \) as a result of the increase in the total number of available levels, i.e. increase in \( \rho(0, E^*) \). The second term, \( \Gamma_2 \) gives the rate of logarithmic decrease due to the decrease in the fraction of levels with spin \( J_m \) and above. The third term, \( \Gamma_3 \) represents the mild compensating effect of the increase of \( \sigma^2 \) with temperature. The last term, \( \Gamma_4 \) represents the decrease in phase space available with decreasing outgoing alpha energy.

Figure 4 shows a plot of the four terms \( \Gamma_1, \Gamma_2, \Gamma_3 \) and \( \Gamma_4 \) and their total \( \Gamma \) as functions of excitation energy. This is done for the same value of \( a = \frac{1}{13} A_f \) (where \( A_f \) is the mass number of the residual nucleus) for which the best fit was obtained.

![Figure 4](image-url)

**Fig. 4.** The logarithmic derivative \( \Gamma \) of the (c.m.) double differential cross-section against excitation energy. The components of \( \Gamma \) are depicted. The arrow indicates the root of the equation \( \Gamma = 0 \).
to the experimental data using the full calculation. It is seen that the maximum excitation energy of the spectra is very well reproduced by the root of $\Gamma = 0$ at $E^* = 34.5$ MeV. This indicates that the approximation used in the present analysis in obtaining Eq. (10) is a good one and that the physical factors determining the calculated energy spectra are well represented by the various terms in this equation.

The steep variation of $\Gamma_2$ with energy illustrates the strong combined effect of angular momentum matching and angular momentum spin cut-off on the calculated spectral shape.

It can be shown that $|\beta_i, \ell_f|$, in Eq. (4), has its maximum when both $\ell_i$ and $\ell_f$ are close to their respective grazing values, $\ell_{ig}$, and $\ell_{fg}$. However, the contribution of a given value of $J$ is highly significant when the excitation energy approaches a maximum value defined by $J = |\ell_{ig} - \ell_{fg}|$ [2]. The location of this maximum excitation energy $E_{\text{max}}^*$ is found to be sensitive to the chosen value of level density parameter $a$. This is expected since all three terms $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ depend on $a$. The numerical calculation shows that changing the parameter $a$ from $\frac{1}{16}A_f$ to $\frac{1}{10}A_f$ changes $E_{\text{max}}^*$ from 37 MeV to 31 MeV.

The energy-integrated alpha-particle yield with laboratory angle is sensitive to the choice of level density parameter $a$. A lesser value of level density parameter $a = 3.93$ MeV$^{-1}$ is required to fit the laboratory angular distribution of energy-integrated cross-section of alpha particles at 68 MeV than that of $a = 4.54$ MeV$^{-1}$ for the same particles at 50 MeV. This again indicates the faster slope at the 68 MeV than that at the 50 MeV. We confirm that the steep fall-off of the energy-integrated cross section is a prominent feature of transfer reactions involving strongly absorbed ions, where the interaction is effectively confined to the surface.

4. Conclusion

The simple analysis adopted here is very much consistent with the full calculation using the diffraction model for this transfer reaction which indicates that the break-up of $^7$Li projectile near the surface occurs with a high probability and that the major fraction of the break-up cross-section is taken by the ($^7$Li,\alpha) transfer process. This indicates that the break-up of $^7$Li is associated with the incomplete fusion of triton with $^{59}$Co. In fact, measurements and analysis showed that the yield of other particles such as tritons, deuterons or protons is significantly smaller than that of the alpha particles. Also, all the results of the yield for the reaction ($^7$Li,\alpha) using the full calculations in the diffraction model and the approximate treatments are compared with the experimental values obtained at the two different incident energies of 50 and 68 MeV. This comparison shows a good agreement among all results. It has been also shown that the level density parameter $a$ is the only sensitive parameter in the diffraction model which is able to locate the maximum of the excitation energy without the need to change any of the three McIntyre parameters in the entrance and exit channels.
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References