Synthesis of Four-bar Motion Generation Considering Worst Case Joint Tolerances

Samer Mutawe, Rajpal.S. Sodhi , Yahia M. Al-Smadi , and Ashish Bhargava

***Abstract*—** Four-bar motion generation is used to synthesize a mechanism which passes through or approximate prescribed rigid-body positions. This work will discuss the motion generation of four-bar mechanism with position tolerance variations due to joint running clearance. The tolerance variations study will be based on the standards of American National Standard Institute (ANSI). The new design constraint introduced in this paper will consider the joint tolerances and incorporate it into the displacement position matrix of coupler points described in the conventional planar four-bar motion generation models. The synthesized planar four-bar mechanism will produce tolerance limits for moving pivots and link length from which any mechanism can be synthesized to satisfy the prescribed coupler points with their prescribed tolerances. The included example demonstrates the synthesis of a four-bar mechanism with joint tolerances. [1]

***Keywords*—**Motion generation, joint tolerances, worst case tolerance, mechanism tolerance

I. INTRODUCTION

THE objective of four-bar motion generation is to calculate the mechanism parameters required to achieve or approximate a set of prescribed rigid-body poses. This mechanism design objective is particularly useful when the rigid-body must achieve a specific displacement sequence for effective operation (e.g., specific tool paths and/or orientations for accurate fabrication operations). In Fig. 1 four prescribed rigid-body poses are defined by the coordinates of variables **p**, **q**, and **r** (motion generation model input), and the model outputs are the calculated coordinates of the fixed pivots $\mathbf{a}_0$ and $\mathbf{b}_0$ and moving pivot variables $\mathbf{a}_1$ and $\mathbf{c}_1$.

Planar mechanism synthesis with tolerances is a well-established field. These tolerances can be found in joints and linkages due to many factors such as manufacturing processes, loading and unloading of the mechanism which increases joint clearance after service period and causes impulsive forces. This paper investigates the effect of the required joint assembly tolerances on the synthesis of four-bar mechanism. Several methods and analyses have been used to include the error caused by joint clearances and link geometry tolerances in the mechanism synthesis. Graphical and mathematical approaches to investigate the efficiency of planar mechanisms to approximate the coupler poses considering the errors/tolerances in mechanism linkages were developed by [3] and [4]. Recent contributions performed by [5, 6 and 7] modeled the joint clearance as a massless virtual link (clearance link) and investigated the joint clearance effect on the mechanism performance to achieve the prescribed coupler curves/points. A method to predict the limits of the tolerance region by choosing the clearance value was also proposed by [7].

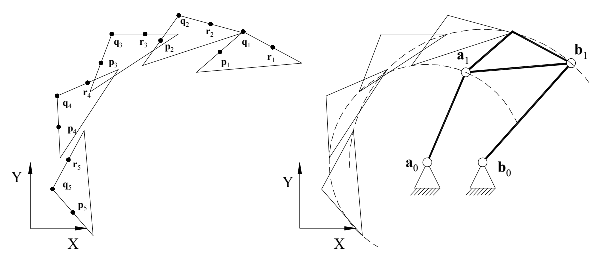

Fig. 1 Prescribed rigid-body poses and calculated planar four-bar mechanism

II. CONVENTIONAL PLANAR FOUR-BAR MOTION GENERATION ANALYSIS

Equations (1) through (3) encompass a conventional planar four-bar motion generation model presented by [1]

$$\left(\left[\mathbf{D}_{1j}\right]\mathbf{a}_1-\mathbf{a}_0\right)^T\left(\left[\mathbf{D}_{1j}\right]\mathbf{a}_1-\mathbf{a}_0\right)-L_1^2=0\ , \tag{1}$$

$$\left(\left[\mathbf{D}_{1j}\right]\mathbf{b}_1-\mathbf{b}_0\right)^T\left(\left[\mathbf{D}_{1j}\right]\mathbf{b}_1-\mathbf{b}_0\right)-L_2^2=0\ , \tag{2}$$

$$\left[\mathbf{D}_{1j}\right]=\begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} p_{1x} & q_{1x} & r_{1x} \\ p_{1y} & q_{1y} & r_{1y} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \tag{3}$$

where j=1,2,3,4

These equations are "constant length" constraints and ensure the fixed length of links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_0$-$\mathbf{b}_1$ throughout the prescribed rigid-body displacements. Variables L_1 and L_2 in (1) and (2) are the prescribed scalar lengths of links $\mathbf{a}_0$-$\mathbf{a}_1$ and $\mathbf{b}_0$-$\mathbf{b}_1$, respectively.

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Equation (3) is a rigid-body planar displacement matrix. From this conventional planar four-bar motion generator model, 6 of the 10 unknown variables $\mathbf{a}_0$, $\mathbf{a}_1$ L_1, $\mathbf{b}_0$, $\mathbf{b}_1$, and L_2 are calculated with two arbitrary choice of parameter (where $\mathbf{a}_0=[a_{0x}, a_{0y}, 1]$, $\mathbf{a}_1=[a_{1x}, a_{1y}, 1]$, $\mathbf{b}_0=[b_{0x}, b_{0y}, 1]$, and $\mathbf{b}_1=[b_{1x}, b_{1y}, 1]$.

III. TOLERANCE ANALYSIS.

This paper presents a technique of synthesizing planar four-bar mechanism considering joint assembly tolerances. Insensitive analyses for optimal coupler points trajectory have been performed in mechanism synthesis, many goal functions have been formulated in an effort to define the linkages with a tolerance to approximate the desired coupler points trajectory with tolerable accuracy. Investigations carried out by [20] and [21] on RRCC mechanism and multi-phase four-bar mechanism respectively show the mechanism motion synthesis with a prescribed tolerance for one position, while work presented in [8] shows analytical solutions for the kinematic analysis of position, velocity, acceleration and transmission angle of geared linkage mechanisms. Several optimization algorithms, objective/goal functions and techniques on the shape of coupler curve and points have been presented in [15-18]. Al-Smadi et al [12] developed a nonlinear optimization to investigate four-bar with structural constraints. This research adds the consideration of the calculated gear train tolerances to the synthesis of the four-bar mechanisms.

The bilateral tolerance system selected for this study is in accordance with [13] which specify bilateral tolerance for pin-hole clearance fit, the main required tolerance for operation of four-bar mechanism was selected and presented herein. For any prescribed rigid-body pose to be achieved, the plus or minus deviation from the specified value would be the allowable tolerance limits. The variation required is the running clearance which is specified when mating parts are assembled, description of clearance fits can be found in [13]. The clearance adopted in this investigation will be medium running fits RC5 or RC6, as shown in Fig. 2. These fits are suitable for high running speed or heavy journal pressures.

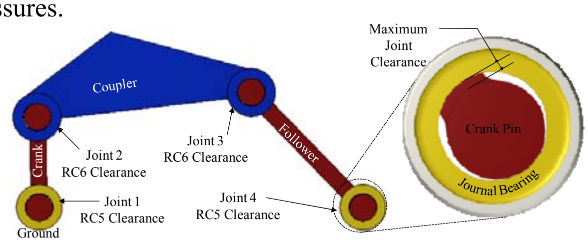

Fig. 2 Joint running clearance required for planar four-bar mechanism

IV. WORST CASE TOLERANCE SYNTHESIS

Worst case tolerance model is widely used in the field of tolerance analysis. It measures tolerance stack-up of the mechanism in a simple form by summing the extreme hence absolute limits of the tolerances. The utilization of worst case tolerance will determine the allowances for coupler points **p**, **q** and **r**. this simplified tolerance model (5) is concluded in [25] which is based on the tolerance accumulation model (4) estimated by [23] and [24].

$$\Delta\Phi_1 = \left|\frac{\partial F_1}{\partial a}\right| \mathrm{tol}_a + \left|\frac{\partial F_1}{\partial b}\right| \mathrm{tol}_b + \left|\frac{\partial F_1}{\partial c}\right| \mathrm{tol}_c \qquad (4)$$

where $\Delta\Phi_1$ is the worst case variation.

The predicted assembly tolerance is

$$\delta_{Assembly} = \sum_{i=1}^{n} |T_i| \qquad (5)$$

where n is number of parts considered in the tolerance analysis.

Worst case tolerance analysis converts the dimension with tolerance limits to a mean dimension with symmetrical tolerance limits. The stack-up tolerance is the summation of tolerance limit variation to the mean dimension for each part. Tolerance analysis example is given and explained in Section VI.

V. MODIFICATION OF POSES DISPLACEMENT MATRIX CONSIDERING TOLERANCE

Joint clearance tolerance would affect and produce a tolerance region for each coupler point to be positioned within. Research performed by [14] and [15] presented that the region of the moving pivot point in four-bar mechanism takes the shape of a rectangle with curved sides. If δ_x and δ_y applied on each coupler point pose, then a tolerance region of a box shape would predict the limits with reasonable accuracy. [16, 17and 18] concluded that the tolerance region for coupler point positions is an ellipse shape. Using reliability analysis in mechanism synthesis, [19] formed a reliable region S_R for RCCC Mechanism. Russell and Sodhi [20] considered point tolerance for RRSS mechanism by considering δ_x and δ_y for one pose only. This consideration would produce a tolerance region of a box shape if covariance is not calculated; this consideration still produces a tolerance region with reasonable accuracy. Similar consideration was performed by [21] in which a square tolerance region for one coupler point pose was suggested. Design sensitivity of an elliptical tolerance region versus square shape tolerance region was formulated by [22]. The tolerance region presented in this paper is considered as a square or a box shape. The work of [7], [20] and [21] for choosing the clearance value has been adopted. Therefore, (5) is defined to enumerate the maximum tolerance value formed for each coupler point pose 1 through 4. Fig. 3 shows the tolerance regions limited by $\pm\delta_x$ and $\pm\delta_y$ for coupler points **p**, **q** and **r**.

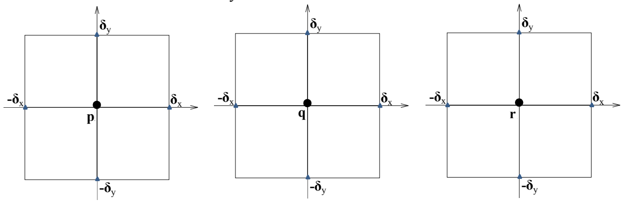

Fig. 3 Tolerance region

The tolerance calculated from (5) will be used in the poses matrix of the coupler points $[\mathbf{D}_j]$ and the displacement matrix $[\mathbf{D}_{1j}]$ as shown in (6) and (7). Several cases of tolerance limits (i.e. 0δ, $+\delta_x$, $-\delta_x$, $+\delta_y$, $-\delta_y$, $+\delta_x$ and $+\delta_y$, $-\delta_x$ and $-\delta_y$, $+\delta_x$ and $-\delta_y$, and $-\delta_x$ and $+\delta_y$) are investigated, moving pivots $\mathbf{a}_1$, $\mathbf{b}_1$ and Links L_1 and L_2 are synthesized for each case.

$$[\mathbf{D}_j] = \begin{bmatrix} p_{jx} \pm \delta_x & q_{jx} \pm \delta_x & r_{jx} \pm \delta_x \\ p_{jy} \pm \delta_y & q_{jy} \pm \delta_y & r_{jy} \pm \delta_y \\ 1 & 1 & 1 \end{bmatrix} \tag{6}$$

$$[\mathbf{D}_{1j}] = \begin{bmatrix} p_{jx} \pm \delta_x & q_{jx} \pm \delta_x & r_{jx} \pm \delta_x \\ p_{jy} \pm \delta_y & q_{jy} \pm \delta_y & r_{jy} \pm \delta_y \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{1x} & q_{1x} & r_{1x} \\ p_{1y} & q_{1y} & r_{1y} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \tag{7}$$

where j=2,3,4

VI. EXAMPLE

Dimensions used in this example are in SI units. Motion generation program can be used with prescribed values of $\mathbf{a}_0$=(0, 0), $\mathbf{b}_0$=(0.5080, 0), and initial guesses of $\mathbf{a}_1$=(0.2540, 0.3048), L_1=0.3810, $\mathbf{b}_1$=(0.6096, 0.3048), and L_2=0.5080. Worst case tolerance model (δ) is calculated in Table 1 which later will be used in (6) and (7) to generate the area described in Section V. Table 2 shows the prescribed rigid body poses for planar four bar mechanism. Joint number and clearance are based on Fig. 2. Coupler poses can fall anywhere within that region. Therefore, nine tolerance cases has been discussed and investigated as shown in Table 3. Rigid-body poses 1 through 4 correspond to link $\mathbf{a}_0$-$\mathbf{a}_1$ rotation angles of θ_1= 45°, 70°, 120°, and 150° respectively. Therefore the displacement angles $(\delta\theta)_{1j}$ for link $\mathbf{a}_0$-$\mathbf{a}_1$ are 25°,75° and 105° respectively.

TABLE1
WORST CASE TOLERANCES ANALYSIS

Joint	Clerance Fit		Nominal Size	LL	UL	Centered Dimension	$\lvert\delta\rvert$
1	RC5	Hole	0.0127	0	2.540E-05	0.012713	1.270E-05
		Pin		-4.826E-05	-3.048E-05	0.012661	3.937E-05
2	RC6	Hole	0.0127	0	4.064E-05	0.012720	2.032E-05
		Pin		-5.588E-05	-3.048E-05	0.012657	4.318E-05
3	RC6	Hole	0.0127	0	4.064E-05	0.012720	2.032E-05
		Pin		-5.588E-05	-3.048E-05	0.012657	4.318E-05
4	RC5	Hole	0.0127	0	2.540E-05	0.012713	1.270E-05
		Pin		-4.826E-05	-3.048E-05	0.012661	3.937E-05
						$\sum\lvert\delta_i\rvert$	0.00023

TABLE 2
PRESCRIBED RIGID-BODY POSES FOR PLANAR FOUR-BAR MECHANISM

	p	q	r
Pose 1	0.1085, 0.4339	0.1479, 0.7330	0.3494, 0.5668
Pose 2	-0.0515, 0.5528	0.0056, 0.8490	0.1969, 0.6712
Pose 3	-0.4418, 0.5096	-0.3981, 0.8081	-0.1990, 0.6391
Pose 4	-0.6201, 0.3188	-0.6086, 0.6202	-0.3926, 0.4735

Synthesized mechanisms which consist of the achieved moving pivot points $\mathbf{a}_1$ and $\mathbf{b}_1$ and link lengths L_1 and L_2 shown in Table 3 and prescribed values of $\mathbf{a}_0$, $\mathbf{b}_0$, were constructed. All rigid-body poses achieved by the constructed mechanisms were investigated. They were found to be comparable with the prescribed poses and were within the calculated worst case tolerance range. One example (highlighted case shown in Table 3) is presented from this investigation, others can be done similarly. This case produces the longest link lengths L_1 and L_2. Table 4 includes the rigid-body poses calculated after incorporating the parameters of the synthesized mechanism (a_{1x}, a_{1y}, L_1, b_{1x}, b_{1y}, L_2) for the highlighted case. The displacement angles $(\delta\theta)_{1j}$ for link $\mathbf{a}_0$-$\mathbf{a}_1$ are 24.9810°, 75.0002° and 105.0232° respectively.

TABLE 3
CALCULATED COORDINATES OF THE MOVING PIVOT VARIABLES $\mathbf{a}_1$ AND $\mathbf{b}_1$ AND SCALAR LINK LENGTHS L_1 AND L_2 FOR NINE COMBINATION CASES OF WORST CASE TOLERANCE δ

	0δ	$+\delta_x$	$-\delta_x$	$+\delta_y$	$-\delta_y$	$+\delta_x$, $+\delta_y$	$-\delta_x$, $-\delta_y$	δ_x , $-\delta_y$	$-\delta_x$, δ_y
a_{1x}	0.3233	0.3230	0.3235	0.3233	0.3233	0.3231	0.3235	0.3230	0.3235
a_{1y}	0.3233	0.3232	0.3233	0.3230	0.3236	0.3229	0.3236	0.3236	0.3230
L_1	0.4572	0.4572	0.4572	0.4571	0.4573	0.4571	0.4573	0.4573	0.4571
b_{1x}	0.8466	0.8464	0.8469	0.8467	0.8466	0.8464	0.8469	0.8464	0.8469
b_{1y}	0.5068	0.5067	0.5070	0.5064	0.5072	0.5063	0.5074	0.5071	0.5066
L_2	0.6095	0.6094	0.6097	0.6094	0.6097	0.6093	0.6098	0.6096	0.6095

TABLE 4
Rigid-body poses achieved by synthesized planar four-bar mechanism for chosen $\mathbf{a_1}$, $\mathbf{b_1}$ from Table 3

	p	**q**	**r**
Pose 1	0.1085, 0.4339	0.1479, 0.7330	0.3494, 0.5668
Pose 2	-0.0517, 0.5529	0.0054, 0.8491	0.1967, 0.6713
Pose 3	-0.4423, 0.5096	-0.3986, 0.8080	-0.1995, 0.6391
Pose 4	-0.6207, 0.3185	-0.6093, 0.6199	-0.3932, 0.4733

The ranges of the achieved pivot variables for the given tolerance region are represented by the perimeter of the solid line in the plots of Fig. 4. The perimeter represents the value of these pivot variables for which, the rigid-body position tolerances will be within the prescribed limit. For the given tolerance, a least square best fit can be obtained for each of the variables. These best fit curves are represented in Fig. 4 using dashed-line format. Since only nine cases were analyzed here, the shape of the best fit curve is a nine-sided polynomial. But, a close examination of the data clearly indicates that for the entire square tolerance region (Fig. 3) the best fit curve will be a circle. The radius of this best fit curve represents the values of the pivot variable for which the given tolerances will always be met.

Using ADAMS module in Solidworks, the achieved coupler poses in Table 4 were measured for the four-bar motion generator (Fig. 6). Figure 5 includes the scalar differences ($\left|point_{prescibed} - point_{achieved}\right|$) between the prescribed (Table 2) and achieved (Table 4) coupler poses of the synthesized motion generator.

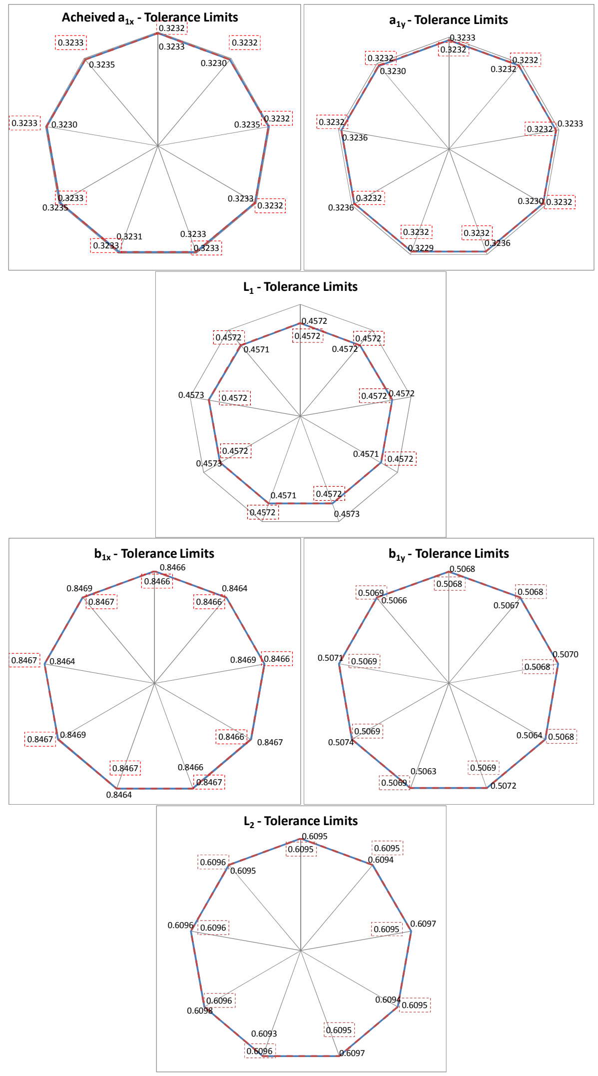

Fig. 4. Plots of synthesized moving pivot points with tolerance limits, dotted line denotes the best fit curve for the achieved moving pivots pose

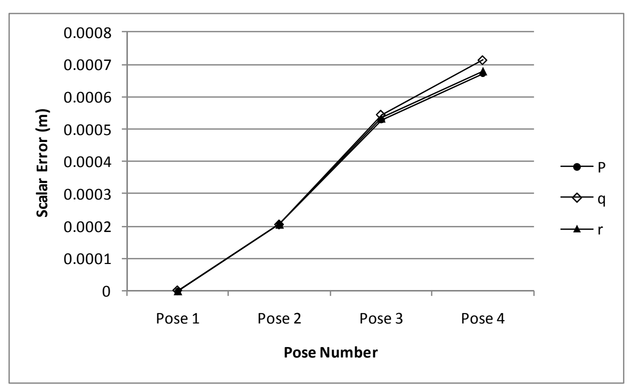

Fig. 5. Plots of synthesized moving pivot points with tolerance limits, dotted line denotes the best fit curve for the achieved moving pivots pose

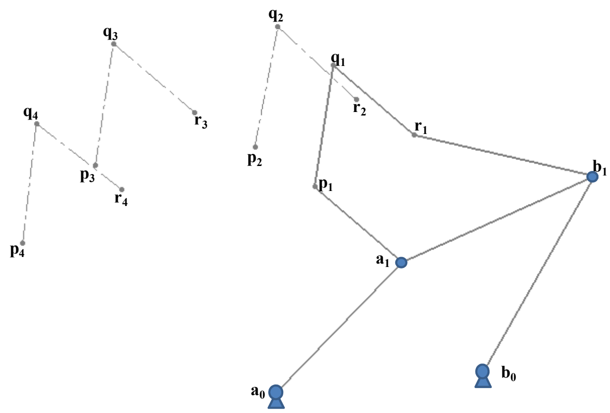

Fig. 6: Plots of synthesized motion generator

VII. DISCUSSION

When the pivots $\mathbf{a}_1$, $\mathbf{b}_1$, and $\mathbf{b}_0$ are collinear, the four-bar mechanism reaches a "lock-up" or binding position. Modeling the prescribed rigid-body poses and concept mechanisms via CAD software could enable one to specify initial guesses for the unknown mechanism more judiciously than by arbitrary guessing. The mathematical analysis software MathCAD was used to codify and solve the formulated motion with tolerance program. For future work, the tolerance modeling technique adopted by [26] will be considered.

VIII. CONCLUSIONS

Four-bar motion generation is used to synthesize a mechanism which passes through or approximates prescribed rigid-body positions. This work discussed the motion generation of four-bar mechanism with poses worst case tolerance which is due to joint clearance during assembly stage. ANSI standard for clearance fit tolerances was incorporated in the rigid-body poses displacement matrix. The synthesized mechanism approximate the prescribed rigid-body positions within the calculated gear train tolerances.

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