Enhanced SBS Instability Growth Rate of Extraordinary Electromagnetic Waves in Strongly Coupled, Magnetized Plasma

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Abstract— Stimulated Brillouin Backscattering (SBBS) of a large amplitude, extraordinary electromagnetic wave travelling across an ambient dc-magnetic field in a strongly coupled plasma is investigated. Using a magnetohydrodynamic model, a system of coupled equations that describes the problem is derived and then solved for a strongly coupled regime. The SBBS maximum growth rate in a magnetized plasma is obtained numerically and compared to that for non-magnetized plasma known in literature. Result shows an enhancement in the instability growth rate in the presence of the magnitic field.

1. INTRODUCTION

Parametric instabilities occur when the incident electromagnetic wave resonantly decays into a scattered electromagnetic wave and an electrostatic plasma mode, which, in laser-produced plasma, could lead to possible absorption, as well as scattering of the wave [1–7]. Stimulated Brillouin scattering (SBS) of electromagnetic waves in plasma is the parametric instability, where an incident light wave with frequency ω_0 and wavenumber $\vec{k}_0$ decays resonantly into a scattered light wave with frequency ω_1 and wavenumber $\vec{k}_1$ and an ion acoustic wave with $(\omega, \vec{k})$. The matching conditions for the frequencies and wave numbers are given by the following equations;

$$\omega_0 = \omega_1 + \omega \quad \text{and} \quad \vec{k}_0 = \vec{k}_1 + \vec{k}, \tag{1}$$

where the equation of the wave number matching condition for stimulated Brillouin bachscattering (SBBS) reads $k_1 = k_0 + k$.

In inertial confinement fusion (ICF), since the driving energy for the implosion is provided by the incident light wave, the occurance of backscattered light could greatly reduce the laser energy absorption efficiency. The full understanding and controll of SBBS instability growth is still of a fundamental concern in more understanding of ICF. This makes SBBS recieve a great deal of attention both theoretically [8–11] and experimentally [12–14]. Megagauss magnetic fields, that have been known to exist for a long time in laser-produced plasmas [15–19], have significant effects on the dynamics of the plasma; The dispersion relations of electromagnetic waves inside the plasma are greatly modified by the presence of such fields, hence magnetized plasma. In previous studies [7, 9], Bawa'aneh et al. have dealt analytically with the SBBS and filamentation problem considering weak field coupling in magnetized plasma. In the present work, SBBS instability in strongly coupled, magnetized plasma is considered. The problem is formulated in Section 2 and a system of coupled equations that describes SBBS in magnetized palsma is derived. The dispersion relation of SBBS in strongly coupled, magnetized plasma is obtained, compared with previous result in the literature for the non-magnetized case and solved for SBBS groth rate in Section 3, where the effect of the dc-magnetic field is obtained numerically. Finally, the results are discussed in Section 4.

2. THE MODEL

To study the coupling of a large amplitude laser beam, propagating in a plasma across a magnetic field, into a scattered electromagnetic wave and an ion acoustic wave we begin with Ampere's and Faraday's laws, given respectively by $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$ and $\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E}$, and the definition of the electric current density in plasma, given by $\vec{J} = q_j n_j \vec{v}_j$, where j stands for electrons and ions, q_j, n_j and $\vec{v}_j$ are the charge, density and velocity for the jth species of the plasma, respectively. This yields the following equation governing the electric field in the plasma;

$$\frac{\partial^2 \vec{E}}{\partial t^2} + c^2 \vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\frac{q_j}{\epsilon_0}\frac{\partial \left(n_j \vec{v}_j\right)}{\partial t} \tag{2}$$

In this equation, the electric field is coupled with the charge, density and velocity as seen on the right hand side, which follow from the force equation and the continuity equation, given for the jth specieis, respectively, by

$$\frac{\partial n_{\mathrm{j}}}{\partial t} + \vec{\nabla} \cdot (n_{\mathrm{j}} \vec{v}_{\mathrm{j}}) = 0 \tag{3}$$

and

$$m_j \left(\frac{\partial \vec{v}_j}{\partial t} + \left(\vec{v}_j \cdot \vec{\nabla} \right) \vec{v}_j \right) = q_j \vec{E} + q_j n_j \vec{v}_j \times \vec{B} - \frac{\vec{\nabla} p_j}{n_j} + m_j \nu \vec{v}_j, \tag{4}$$

where ν is the effective electron-ion collisional frequency. The closed system of equations, namely Equations (2)–(4), can be linearized following the standard procedure known in literature [see for example Refs. [6, 7]] considering immobile, neutralizing background ions in the fast time scale of the electron motion and the coupling of the incident laser beam with the plasma to come through the electrons [6]. Considering magnetized plasma (with a static dc-magnetic field be given along the z-axis), both longitudinal and transverse components of the electron perturbation velocity take high and low frequency components $[\vec{v}_{1e} = (v^h_{ex} + v^l_{ex})\hat{x} + (v^h_{ey} + v^l_{ey})\hat{y}]$, where the number 1 in the subscript denotes a perturbed quantity and the superscripts l and h denote low and high frequency, respectively. The ion perturbation velocity is $v_{ix}\hat{x} + v_{iy}\hat{y}$, where v_{ix}, v_{iy} are the low frequency ion velocity components in the x and y-directions, respectively. Also, the electric field is given by $\vec{E}_1 = E_x\hat{i} + E_y\hat{j}$, where both E_x and E_y have low and high frequency components. The presence of such high and low components for the electron and ion perturbation velocities and the electric field are well explained in Ref. [7].

We consider now the geometry where electrostatic fluctuations are along the x-axis, the incident laser beam is polarized along the y-direction, where both the pump field and the direction of propagation are perpendicular to the static dc-magnetic field. At modest pump power, the nonlinear behaviour would be negligible and linear theory would be good enough to describe the system. The linearized form of Equations (2) and (4) give the x and y-components for the low and high frequency electric field, and for the elctron low and highy frequency velocity components, respectively, and the linearized form of the force equation, namely Equation (4), gives the x and y-components for the ion low frequency velocity [see Ref. [7]]. A very lengthy process of vector algebra done by simplifying the electron and ion velocities to the first order that leads to first order iterational solution of the equations describing the low and high frequency perturbation in the electric field and the low frequency density perturbation that is used in literature [4–7], ignoring collisions, considering $Zn_{0i} = n_{0e} \to n_0$, where the number 0 in the subscript denotes an equilibrium quantity, the plasma approximation $Zn_{1i} \approx n_{1e} \to n_1$, a linearly polarized pump of the form $\vec{E}_0(x,t) = \hat{j}\ E_0 \cos(k_0 x - \omega_0 t) = \frac{1}{2}\left(E_0\ e^{i(k_0 x - \omega_0 t)} + c.c.\right)\ \hat{j}$, where $cc.$ denotes complex conjugate, also considered the dispersion relations of the X-waves and ion cyclotron waves that occure in magnetized plasma, and finally considering resonant terms only, the equation that governs the high frequency electric field perturbations of the backscattered wave yields

$$\left[\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} + \omega^2_{\mathrm{pe}} \frac{\omega_1^2 - \omega^2_{\mathrm{pe}}}{\omega_1^2 - \omega^2_{UH}} \right] E_1 = \frac{-iev_{0e}\omega_1}{\epsilon_0} n_1^* \,, \tag{5}$$

and that governing the low frequency ion density perturbation yields

$$\left[\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2} - \Omega_i^2 \right] n_1 = \frac{iZen_0 k^2 v_{0e}}{m_i} \frac{E_1^*}{\omega_1} \frac{\omega_1^2 - \omega^2_{\mathrm{pe}}}{\omega_1^2 - \omega^2_{UH}} \left(1 + \frac{\Omega_i^2}{c_s^2 k^2} \right) \,, \tag{6}$$

where $c_s = \sqrt{(\gamma_e k_B T_e + \gamma_i k_B T_i)/m_i}$ is the ion acoustic speed, $\omega^2_{UH} = \omega^2_{\mathrm{pe}} + \Omega_e^2$ the upper hybrid frequency and γ_j the specific heat ratio for the jth species. Equations (5) and (6) form a closed system of equations. As seen in the system of equations, eleminating the magnetic field, the equations will reduce to the well known system of coupled equations for SBS in nonmagnetized plasmas known in literature [6].

3. SBS GROWTH RATE IN STRONGLY COUPLED PLASMA

In Section 2, we have derived the system of equations that describes SBBS instability growth rate in weakly magnetized plasma, namely Equations (5) and (6). Upon using the fourier transformation in space and time, making use of the frequency and wave number matching conditions of Equation (1), and solving the resulting two linear equations for the electric field and density simultaneously, our system of equations yields the following dispersion relation

$$\left(\omega_1^2 - c^2k_1^2 - \omega_{pe}^2 \frac{\omega_1^2 - \omega_{\rm pe}^2}{\omega_1^2 - \omega_{UH}^2}\right)\left(\omega^2 - c_s^2k^2 - \Omega_i^2\right) = \frac{1}{4}\omega_{pi}^2 k^2 \upsilon_{os}^2 \frac{\omega_1^2 - \omega_{\rm pe}^2}{\omega_1^2 - \omega_{UH}^2}\left(1 + \frac{\Omega_i^2}{c_s^2k^2}\right) \tag{7}$$

This dispersion relation reduces, when the magnetic field is ignored, to that formula well known in literature for nonmagnetized plasma [see Refs. [5, 6] for example], which has been analyzed analytically for weak and strong plasma coupling. Equation (7) has been analyzed for weakly coupled plasma [9], where the instabiliy growth rate is much smaller than the ion acoustic frequency. However, in this work we analyze Equation (7) analytically for strongly coupled plasma.

Following [6], we consider the strong coupling condition $|\omega| \gg kc_s$. The first term on the left hand side of Equation (7) reduces to $-2\omega_0\omega - (4\omega_0^2c_s^2/c^2)(1 + (\Omega_e^2\omega_{pe}^2)/(\omega_0^2 - \Omega_i^2))$, and the second term on the left hand side reduces to $\omega^2 + \Omega_i^2$. Also, the maximum growth rate occures when the electromagnetic wave is resonant, with k taking the following values; $k \approx 2k_0 - (2\omega_0c_s/c^2)(1 + \alpha)$, where $\alpha = \left(\Omega_e\omega_{\rm pe}/(\omega_0^2 - \Omega_e^2)\right)^2$. Back substitution in Equation (7), considering $\beta = (\omega_1^2 - \omega_{\rm pe}^2)/(\omega_1^2 - \omega_{UH}^2)$, yields the following cubic equation;

$$\omega^3 - D_2\omega^2 - D_1\omega + D_0 = 0\ , \tag{8}$$

where

$$D_2 = k_0c_s\frac{\Omega_e}{\omega_0}(1+\alpha)\left(1 + \frac{\omega_0}{k_0c}(1+\alpha)\right) \tag{9}$$

$$D_1 = \Omega_i^2 \tag{10}$$

$$D_0 = D_2D_1 + \frac{k_0^2\upsilon_{os}^2}{2}\frac{\omega_{pi}^2}{\omega_0}\beta\left(1 + \frac{\Omega_i^2}{c_s^2k^2}\right) + \frac{1}{2}\omega_{pi}^2\beta\left(\frac{c_s\upsilon_{os}}{c^2}\right)^2(1+\alpha)(\omega_0 - 4ck_0)\left(1 + \frac{\Omega_i^2}{c_s^2k^2}\right), \tag{11}$$

and α and β are given by the expressions in the paragraph just above Equation (8). In Equation (8), ignoring the magnetic field will yield the frequency for the strongly coupled, nonmagnetized plasma that is well known in the literature [6] with one difference that we use $k \approx 2k_0 - (2\omega_0c_s/c^2)(1+\alpha)$, while Ref. [6] uses $k \approx 2k_0$.

The numerical solution of the cubic Equation (8) shows one real root and two complex conjugates, as expected. Figure 1 shows the imaginary part of the unstable root of Equation (8). It represents the growth rate (normalized to γ_0, the imaginary part of ω for non magnetized plasma [see Ref. [6]]) versus the magnetic field given in terms of the electron cyclotron frequency Ω_e normalized to incident laser frequency ω_0. for the different plasma densities $n/n_{cr} = 0.1, 0.4, 0.8$, respectively,

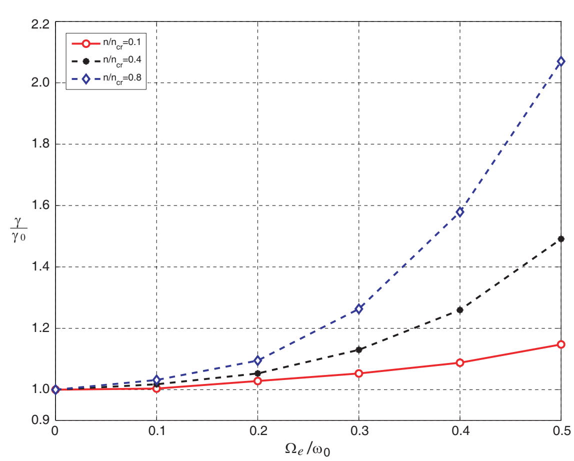

Figure 1: Normalized maximum growth rate versus normalized electron cyclotron frequency for different densities. $I_0 = 10^{15}\,\mathrm{W/cm^2}$, $\lambda_0 = 1.06\,\mu\mathrm{m}$, $T_e = 3\,\mathrm{keV}$ and $T_i = 0.3T_e$.

where the solid line corresponds to $n = 0.1n_{cr}$. In obtaining the figure, the following parameters, typical for ICF, are used for the incident laser intensity and wavelength; $I_0 = 10^{15}\,\mathrm{W/cm}^2$, $\lambda_0 = 1.06$ microns, and the following for the electron and ion temperatures; $T_e = 3\,\mathrm{keV}$ and $T_i/T_e = 0.3$, respectively. The figure shows enhancement of the instability by the magnetic field. This effect is stronger for plasma with higher density.

4. CONCLUSION

Stimulated Brillouin Backscattering (SBBS) of a large amplitude, extraordinary electromagnetic wave travelling across an ambient dc-magnetic field in a strongly coupled plasma is investigated. In this study, we consider a static magnetic field perpendicular to the direction of the electrostatic density fluctuation and to the direction of polarization of the incident laser field. Using a magneto-hydrodynamic model, a system of coupled equations that describes the problem is derived [7]. This system of equations is then solved for a strongly coupled regime to obtain a modified expression for the maximum growth rate in magnetized plasmas, where it is found to be the solution of a cubic equation. In the absence of the magnetic feld, both the system of coupled equations and the cubic equation for the maximum growth rate, reduce to the corresponding expressions known in the literature for SBBS in non-magnetized plasmas [6].

The numerical solution of the cubic equation shows a real solution and two complex conjugates. For the unstable mode, the static magnetic field is found to enhance the instability growth rate. The growth rate doubles at a value of Ω_e/ω_0 just below 0.5 for $n = 0.8n_{cr}$.

REFERENCES

1. Perkins, F. W. and J. Flick, "Parametric instabilities in inhomogeneous plasmas," *Phys. Fluids*, Vol. 14, 2012–2019, 1971.
2. Forslund, D. W., J. M. Kindel, and E. L. Lindman, "Parametric excitation of electromagnetic waves," *Phys. Rev. Lett.*, Vol. 29, 249–252, 1972.
3. Rosenbluth, M. N., "Parametric instabilities in inhomogeneous media," *Phys. Rev. Lett.*, Vol. 29, 565–567, 1972.
4. Chen, F. F., *Introduction to Plasma Physics*, Plenum, New York, 1985.
5. Banmgärtel, K. and K. Sauer, *Topics on Nonlinear Wave — Plasma Interaction*, Birkhäuser, Berlin, Germany, 1987.
6. Kruer, W. L., *The Physics of Laser Plasma Interactions*, Westview, Boulder, CO, 2003.
7. Bawa'aneh, M. S., G. Assayed, and S. Al-Awfi, "Filamentation instability of electromanetic radiation in magnetized plasma," *IEEE Transactions on Plasma Science*, Vol. 38, No. 5, 1066–1072, 2010.
8. Tikhonchuk, V. T., J. Fuchs, C. Labaune, S. Depierreux, S. Hller, J. Myatt, and H. A. Baldis, "Stimulated brillouin and raman scattering from a randomized laser beam in large inhomogeneous collisional plasmas. II. Model description and comparison with experiment," *Phys. Plasmas*, Vol. 8, 1636–1350, 2001.
9. Bawa'aneh, M. S., "Stimulated brillouin backscattering in magnetized plasma," *Contrib. Plasma Phys.*, Vol. 43, 447–455, 2003.
10. Maximov, A. V., J. Myatt, W. Seka, R. W. Short, and R. S. Craxton, "Modeling of stimulated Brillouin scattering near the critical-density surface in the plasmas of direct-drive inertial confinement fusion targets," *Phys. Plasmas*, Vol. 11, No. 6, 2994–3000, Jun. 2004.
11. Bawa'aneh, M. S. and T. J. M. Boyd, "Enhanced levels of stimulated brillouin reflectivity from non-maxwellian plasmas," *Journal of Plasma Physics*, Vol. 73, 159–167, 2007.
12. Fuchs, J., C. Labaune, S. Depierreux, H. A. Baldis, and A. Michard, "Modification of spatial and temporal gains of stimulated brillouin and raman scattering by polarization smoothing," *Phys. Rev. Lett.*, Vol. 84, 3089–3092, 2000.
13. Malka, V., J. Faure, S. Hüller, V. T. Tikhonchuk, S. Weber, and F. Amiranoff, "Enhanced spatiotemporal laser-beam smoothing in gas-jet plasmas," *Phys. Rev. Lett.*, Vol. 90, 075002–075006, 2003.
14. Maximov, A. V., I. G. Ourdev, D. Pesme, W. Rozmus, V. T. Tikhonchuk, and C. E. Capjack, "Plasma induced smoothing of a spatially incoherent laser beam and reduction of backward stimulated Brillouin scattering," *Phys. Plasmas*, Vol. 8, 1319–1329, 2001.
15. Stamper, J. A. and B. H. Ripin, "Faraday-rotation measurements of megagauss magnetic fields in laser-produced plasmas," *Phys. Rev. Lett.*, Vol. 34, 138–141, 1975.

16. Boyd, T. J. M., G. J. Humphreys-Jones, and D. Cooke, "Structure of magnetic fields generated in laser produced plasmas," *Phys. Lett. A*, Vol. 88, 140–143, 1982.
17. Pukov, A. and J. Meyer-ter-Vehn, "Laser hole boring into overdense plasma and relativistic electron currents for fast ignition of ICF targets," *Phys. Rev. Lett.*, Vol. 79, No. 14, 2686–2689, Oct. 1997.
18. Boyd, T. J. M., A. Tatarinov, M. S. Bawa'aneh, and A. Dyson, "Magnetic field generation and penetration into dense, laser-produced plasma," *Proceedings of the 1996 International Conference on Plasma Physics*, edited by H. Sugai and T. Hayashi, Vol. 2, 1714–1718, Japan Society of Plasma Science and Nuclear Fusion Research, Nagoya, 1997.
19. Bawa'aneh, M. S., "Magnetohydrodynamics of megagauss magnetic fields generated in plasma," *IEEE Transactions on Plasma Science*, Vol. 38, No. 8, 1808–1814, 2010.