Proceedings of IMECE2007
2007 ASME International Mechanical Engineering Congress and Exposition
November 11-15, 2007, Seattle, Washington, USA

IMECE2007-42186

A DISCRETIZATION APPROACH TO MODELING CAPACITIVE MEMS FILTERS

Bashar K. Hammad[1], **Ali H. Nayfeh**[1], **Eihab Abdel-Rahman**[2]
[1]Dept. of Engineering Science and Mechanics, MC 0219, Virginia Tech,
Blacksburg, Virginia 24061, USA
[2]Dept. of Systems Design Engineering, University of Waterloo,
Waterloo, ON N2L 3G1, Canada
Email: anayfeh@vt.edu

ABSTRACT

We present a reduced-order model and closed-form expressions describing the response of a micromechanical filter made up of two clamped-clamped microbeam capacitive resonators coupled by a weak microbeam. The model accounts for geometrical and electrical nonlinearities as well as the coupling between them. It is obtained by discretizing the distributed-parameter system using the Galerkin procedure. The basis functions are the linear undamped global mode shapes of the unactuated filter. Closed-form expressions for these mode shapes and the coressponding natural frequencies are obtained by formulating a boundary-value problem (BVP) that is composed of five equations and twenty boundary conditions. This problem is transformed into solving a system of twenty linear homogeneous algebraic equations for twenty constants and the natural frequencies.

We predict the deflection and the voltage at which the static pull-in occurs by solving another boundary-value problem (BVP). We also solve an eigenvalue problem (EVP) to determine the two natural frequencies delineating the bandwidth of the actuated filter. Using the method of multiple scales, we determine four first-order nonlinear ODEs describing the amplitudes and phases of the modes. We found a good agreement between the results obtained using our model and the published experimental results. We found that the filter can be tuned to operate linearly for a wide range of input signal strengths by choosing a DC voltage that makes the effective nonlinearities vanish.

NOMENCLATURE

ℓ	Length of the primary beams
c	Ratio of the length of the coupling beam to ℓ
l_c	Position of attaching coupling beam to the primary beams
h_{p}	Thickness of the structure
A_{p}	Area of the cross-sections of the primary beams
I_{p}	Moment of inertia of the cross-sections of primary beams
A_{c}	Area of the cross-section of the coupling beam
I_{c}	Moment of inertia of the cross-section of coupling beam
N_{p_k}	Applied tensile axial forces in the primary beams
N_{c}	Applied tensile axial forces in the coupling beam
b_{p}	Width of the primary beams
b_{c}	Width of the coupling beam
E	Young's modulus
ρ	Material density
d	Gap width
ϵ_{o}	Dielectric constant of the capacitor gap medium

1 Introduction

A majority of the current transceiver systems used in radio frequency (RF) and intermediate frequency (IF) applications utilize a number of discrete resonant components, such as quartz-crystal, ceramic, and surface acoustic wave filters. These vibration-based components are superior to their transistor-based counterparts. But these filters require ultra-fine machining and occupy a large area of the system compared to other components. More importantly, these devices are not CMOS-compatible, consequently being off-chip components, they have to be interfaced