Linear Vibrations of MEMS Filters

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ABSTRACT

We present an analytical solution for the linear vibration problem of micromechanical filters and obtain closed-form expressions for the mode shapes. The filters are made up of two clamped-clamped beam resonators connected via a coupling beam. The model described in this work treats the filter as a distributed-parameter system. We solve a boundary-value problem (BVP) composed of five equations and twenty boundary conditions for the undamped, unforced natural frequencies and mode shapes of the filter. By solving the characteristic equation we develop in this work, we can obtain an infinite number of natural frequencies and their corresponding mode shapes. This is extremely useful when used to further analyze the filter response, such as obtaining the spatial integral of mode shapes and their derivatives necessary to perform Galerkin discretization of the distributed-parameter formulation of the filter.

1 Introduction

A majority of the current transceiver systems used in radio frequency (RF) and intermediate frequency (IF) applications utilizes a number of discrete resonant components, such as quartz-crystal, ceramic, and surface acoustic wave (SAW) filters. These vibration-based components are superior to their transistor-based counterparts. But these filters require ultra-fine machining and occupy large area of the system compared to other components. More importantly, these devices are not CMOS-compatible, consequently being off-chip components, they have to be interfaced with integrated electronics at the board level, which presents a bottleneck in the miniaturization of modern communication systems. However, the recent evolution of MEMS technology and the successful implementation of high-Q micromechanical resonators offer an opportunity to replace the discrete frequency-selective components in transceivers and create highly selective filters more amenable to miniaturization and on-chip integration with other IC components [?, ?].

The most common approach in the literature to model micromechanical filters adopts an underlying mass-spring-damper (lumped-parameter) approximation to represent each resonator with an equivalent circuit [?]. Linear (small) vibration theory is adopted and a small electric signal is assumed in order to linearize the inherently nonlinear electrostatic forces and suppress the geometrical nonlinearity in the resonators. In these models, the center frequencies and the pull-in voltages of the filters are investigated by considering a single resonator identical to the resonators comprising the filter. But a new approach based on a discretization of the filter distributed-parameter formulation using a Galerkin procedure was developed in [?]–[?].