Proceedings of the ASME 2007 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2007
September 4-7, 2007, Las Vegas, USA

DETC2007-35781

EXPERIMENTAL OBSERVATIONS OF NONLINEAR NONPLANAR OSCILLATIONS OF A CANTILEVER BEAM

Haider N. Arafat*

Senior Engineer
Dynamic Loads Group
Cessna Aircraft Company
Wichita, Kansas 67215, USA
Email: hnarafat@cessna.textron.com

Ali H. Nayfeh and Bashar K. Hammad

University Distinguished Professor
Graduate Research Assistant
Department of Engineering Science and Mechanics, MC 0219
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061, USA
Email: anayfeh@vt.edu, bkhammad@vt.edu

ABSTRACT

The dynamics of a thin cantilever beam undergoing combined torsion and bending vibrations are examined experimentally. The beam's fundamental natural frequencies in the two orthogonal bending motions and in torsion are $f_{v1} = 5.719$ Hz, $f_{w1} = 189.730$ Hz, and $f_{\phi 1} = 138.938$ Hz, respectively. A base-excitation shaker imparts a harmonic load that acts parallel to the width of the beam. First, the response of the beam is examined when the excitation frequency is equal to the fundamental torsion natural frequency (i.e., $f = 138.9$ Hz). For low levels of excitation, the motion consists mainly of hardly noticeable twisting vibrations. For high levels of excitation, the energy of the first torsion mode excites the first out-of-plane bending mode. In this case, the beam responses exhibit modulated vibrations containing both high-frequency and low-frequency components. Second, the beam is excited at the frequency $f = 132.0$ Hz, which is in the neighborhood the difference of these two natural frequencies. For large excitation levels, the beam vibrates with large-amplitude out-of-plane bending motions that exhibit chaotically intermittent behaviors.

Keywords: *Beams, high-to-low energy transfer, chaos, intermittency, modal interaction.*

BACKGROUND AND MOTIVATION

A common belief in the engineering community is that high-frequency excitations of systems are presumed to create minute amplitude vibrations, and hence it makes sense to ignore their effects altogether. Nevertheless, in recent years, several theoretical and experimental studies have exposed this misconception, especially in lightly-damped highly-flexible structures. With the right conditions, inherently existent nonlinear mechanisms are capable of transforming the high-frequency energy in such systems into significantly large-amplitude dangerous motions.

Large-amplitude low-frequency vibrations have been demonstrated in structures with cubic nonlinearity (e.g., taut strings and flat beams and plates) that are excited at a frequency near: (i) three times a natural frequency (subharmonic resonance of order-$\frac{1}{3}$), (ii) the sum or difference of two natural frequencies (external or parametric combination resonance), (iii) a natural frequency that is in a three-to-one internal resonance with a lower natural frequency, and/or (vi) a natural frequency that is in a combination internal resonance with two or more natural frequencies. In all of these cases, the excitation frequency is much larger than the natural frequency of the lowest activated mode. Many experimental and analytical studies on this subject are discussed in Nayfeh and Mook [1] and Nayfeh [2].

Moreover, high-frequency excitations have been found to excite low-frequency modes through internal energy transfer from the excited high-frequency mode, despite the fact that the natural frequencies of the modes involved do not conform to a well-defined internal resonance relationship, such as a three-to-one, two-to-one, one-to-one, or any combinations or subcombination internal resonance relationship. In fact, in all of the cases where this energy transfer has been observed, the natural frequencies where so widely spaced that their ratios were on the order of 20:1 and higher [2,3].

Concentrating our attention to cantilever beams that undergo combined bending-torsion motions, a few theoretical and exper-

*Address all correspondence to this author.