THE OCCURRENCE PROBABILITY AND RETURN PERIOD OF EXTREME HYDROLOGICAL DROUGHTS

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ABSTRACT

In arid and semi-arid regions the traditional probabilistic definition of drought events may not show flexibility to capture extreme droughts where the management of water resources systems is a crucial issue. This paper defines the drought event as a run of consecutive deficits of extreme magnitude regardless where the run starts as long as it satisfies the continuity. Theoretical models to estimate the occurrence probability and return period of extreme hydrological droughts are presented assuming deficits magnitude is Beta distributed. Historical annual streamflow data for the Poudre River is used to characterize hydrological droughts in the Poudre River basin, Colorado. Estimates of the occurrence probability and return period of extreme droughts obtained using the theoretical models developed here were found to well agree with empirical results obtained by analyzing simulated flows of the Poudre River. In the Poudre River basin, the analysis shows that hydrological droughts of moderate magnitude ($\gamma = 0.5$) and duration of 2 – 3 years are the most recurrent events having return period of nearly 6 years, while droughts of extreme magnitude ($\gamma = 1$) and duration of 3 – 5 years are the most recurrent events having return period of nearly 15 years.

INTRODUCTION

Managing water resources systems in arid and semi-arid regions during dry conditions is considered a crucial issue since water supply systems may fall short to fulfill water requirements for urban, agricultural, industrial and environmental needs. This water shortage may extend over several years and cover large areas leading to unfavorable economic and social consequences. Generally the term drought is used in literature to describe the complex natural phenomenon of moisture lack during time period over a region (Sen, 1998).

Drought indices (e.g. the Palmer Drought Severity Index, the Palmer Hydrological Drought Index, and the Drought Monitor) have been used intensively in literature to study droughts (Heim, 2002). Such indices are useful to monitor drought severity in a given year and describe its spatial evolution; however common weaknesses of current drought indices have been reported (Byun et al., 1999; Blenkinsop and Fowler, 2007).

Furthermore, drought indices can not explain how often extreme historical droughts may occur (Rossi and Cancelliere, 2003; Salas et al., 2005). Alternatively, to better estimate useful drought statistics like the probability of drought occurrence or the average interarrival time between recurrent events, it is essential to define drought events considering probabilistic framework. For example, Yevjevich (1967) defined the drought event as an uninterrupted sequence of observations below a predefined threshold given that the dry sequence is preceded and followed by at least one observation above that threshold. The observations below the predefined threshold are called deficits (Salas et al., 2005). On the other hand, Fernandez and Salas (1999) considered the definition of runs as in Schwager (1983) to define the event of drought as a run of consecutive time-steps of observations below a predefined threshold regardless where the run starts as long as it satisfies the continuity. The later definition is probably more useful to capture episodes of consecutive deficits that have extreme magnitude and impeded in a run that starts and ends by deficits of small magnitude. Figure (1) demonstrates the differences between the two probabilistic definitions of drought events. In this paper, to characterize droughts the second definition is adopted.

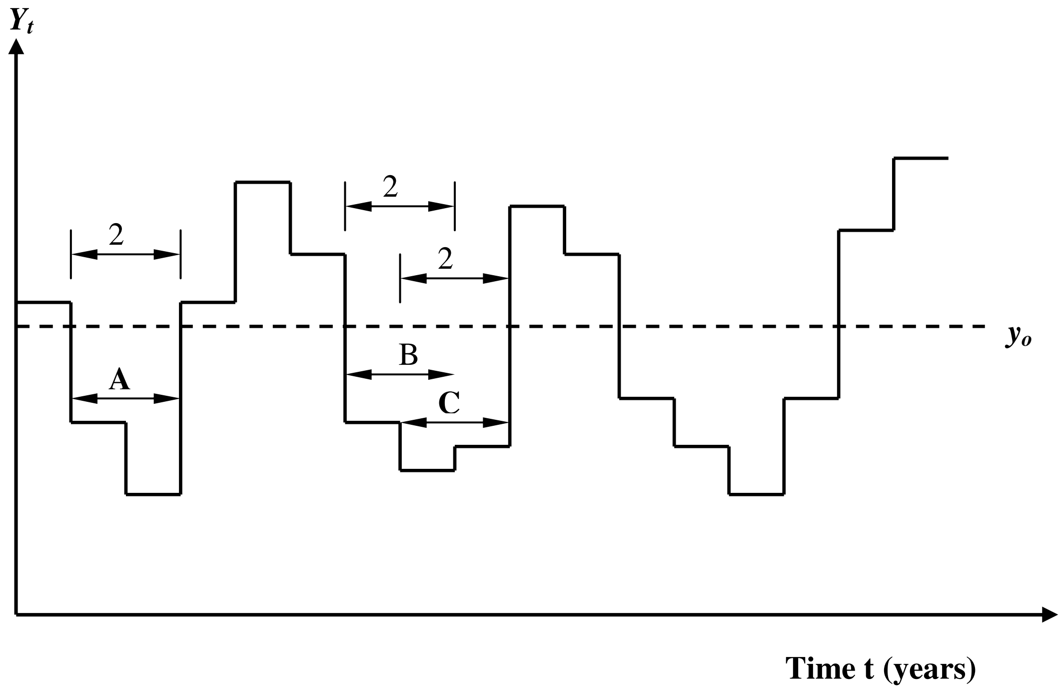

Figure 1. Typical two years drought events resulted after truncating the series Y_t at the level y_o. The event A is defined according to the definition of runs as in Yevjevich (1967), while events B and C are defined according to the definition of runs as in Fernandez and Salas (1999)

Considering the duration of drought, Fernandez and Salas (1999) derived the distribution of the first occurrence and recurrence time, and resented expressions to evaluate the return period and the risk of single time-step drought events arise from either independent or simple Markov processes. Likewise, Chung and Salas (2000) derived expressions for the distribution of the first occurrence and recurrence time, and the risk of single time-step drought events arise from process that exhibits longer time dependence structure, i.e. DARMA(1,1). The goal of this paper is to present theoretical models to evaluate the occurrence probability and return period of extreme droughts when drought components of duration and magnitude are considered jointly assuming that drought events are defined according to the definition of runs as in Schwager (1983).

The drought magnitude is evaluated by summing the single deficits over the length of the drought (Salas et. al., 2005). However since successive drought deficits are auto-correlated then deriving an exact distribution of the drought magnitude, i.e. sum of successive deficits, given the distribution of the single deficit is rather complex procedure unless the single deficits are normally distributed (Shiau and Shen, 2001). Instead the single deficits are usually assumed independent and identically distributed and in that case the distribution of the deficits sum conditioned to a fixed drought length can be found under certain conditions (Salas at al, 2005). Usually, distributions like the Gamma or the Exponential are employed to represent the distribution of drought magnitude (e.g. Shiau and Shen, 2001; Gonzalez and Valdes, 2003; Biondi et al., 2005; Salas et al., 2005). However since the single deficits or their conditional sum are bounded by lower and upper values, then the upper tail unbounded distributions like the Gamma or the Exponential may not well represent the distribution of the single deficits or their sum. It is suggested in this paper that the single deficits and their conditional sum be fitted as Beta distributed in the essence that the single deficits or the sum of deficits are bounded by the 0 and maximum value as will be seen later. The proposed models to evaluate the occurrence probability and return period will be used to characterize droughts in the Poudre river basin, Colorado, the United States.

The occurrence probability of a run of deficits

In any year t, if the streamflow observation Y is less than a specified truncation level y_o, then that is a deficit year. Let X_t be the sequence of deficits and surpluses resulted after truncating the series Y_t at the level y_o such that X_t takes 0 when the year t is a deficit year otherwise takes 1,

$$X_t = \begin{cases} 0 & when\ Y_t < y_o \\ 1 & otherwise \end{cases} \tag{1}$$

According to the alternative definition of runs as in Schwager (1983), the drought event $A_{l,t}$ occurs when a run of l successive deficits occurs starting at the time t. The occurrence probability of the event $A_{l,t}$ is:

$$P[A_{l,t}] = P[X_t = 0, X_{t+1} = 0, ..., X_{t+l-1} = 0] \tag{2}$$

If the sequence of deficits and surpluses X_t is assumed simple Markov process, then $P[A_{l,t}]$ can be expressed in terms of the unconditional and conditional state probabilities as:

$$P[A_{l,t}] = P[X_t = 0]\left(P[X_{t+1} = 0 \mid X_t = 0\right)^{l-1} \tag{3}$$

If X_t is stationary, the occurrence probability $P[A_l]$ that the event A_l occurs is:

$$P[A_l] = p_0 \left(p_{00}\right)^{l-1} \tag{4}$$

The unconditional and conditional deficit state probabilities p_0 and p_{00} can be estimated from the 0,1 process X_t as $p_0 = P[X_t = 0]$ and $p_{00} = P[X_{t+1} = 0 \mid X_t = 0]$. Equation (4) shows the exponential decay of the occurrence probability as the run length increases. This trend is similar to the findings of other studies (e.g. Loaiciga and Leipnik, 1996; Shiau and Shen, 2001; Salas et al, 2005).

The distribution of deficits

In a given year, when the streamflow Y_t is below the truncation level y_o, then the amount of the deficit d_t is:

$$d_t = y_o - Y_t \tag{5}$$

Equation (5) indicates that the random variable d_t takes the minimum value of 0 when Y_t equals y_o and the maximum value of y_o when Y_t equals 0. Given a run of l successive deficits, the magnitude D of the drought is obtained by accumulating the single deficits over the length l,

$$D = \sum_{t=1}^{l} d_t \tag{6}$$

The drought event is considered an extreme event in terms of the magnitude when D exceeds a specified value D_o, where D_o is defined as:

$$D_o = \gamma\, y_o \tag{7}$$

where γ is constant.

Since the single deficit d_t and the deficits sum D are bounded by lower and upper values then the unbounded upper tailed distributions such as the Gamma or the Exponential may not well represent the distribution of the single deficits and consequently the sum D. Referring to equation (5) and Figure (2) the single deficit random variable d_t is bounded by the lower value a = 0 and the upper value $b = y_o$, therefore it will be convenient if the single deficit d_t is assumed Beta distributed with the density function:

$$f_d(s) = \frac{1}{B(\alpha,\beta)} \frac{(s-a)^{\alpha-1}(b-s)^{\beta-1}}{(b-a)^{\alpha+\beta-1}} \quad , \qquad a \le d \le b \tag{8}$$

Where a, b, α and β are the distribution parameters.

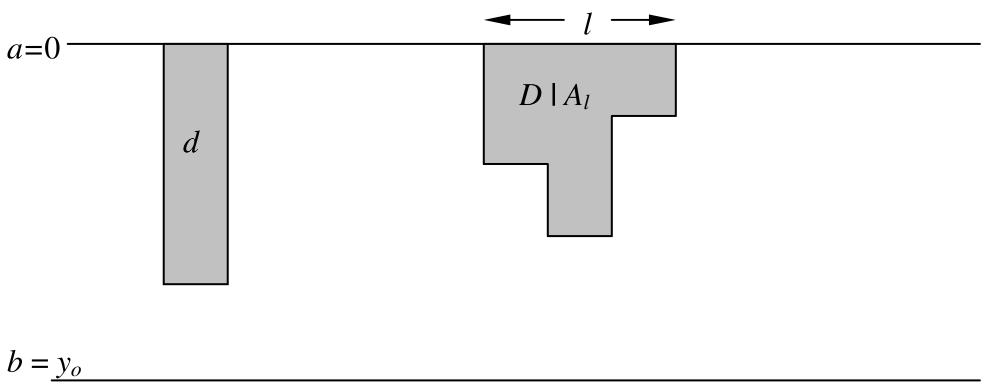

Figure 2. The single deficit random variable d_t and the conditional sum of deficits $D \mid A_l$. The d_t is bounded by a = 0 and $b = y_o$, while $D \mid A_l$ is bounded by a = 0 and $lb = ly_o$

If the single deficits are Beta distributed, then deriving an expression to represent the distribution of the deficits sum $D \mid A_l$ conditioned to the occurrence of the event A_l considering the auto-correlated single deficits is too complicated procedure. However the random variable $D \mid A_l$ is also bounded by a lower value of a = 0 and an upper value of lb, where $b = y_o$, therefore the assumption that $D \mid A_l$ is Beta distributed with parameters (α_l, β_l, a, lb) appears plausible. The probability density function of the Beta distributed variable $D \mid A_l$ is:

$$f_{D|A_l}(s) = \frac{1}{B(\alpha_l,\beta_l)} \frac{(s-a)^{\alpha_l-1}(lb-s)^{\beta_l-1}}{(lb-a)^{\alpha_l+\beta_l-1}} \quad , \qquad a \le D \le lb \tag{9}$$

Eventually, the probability distribution of the conditional deficit that exceeds the threshold D_o, $P[(D > D_o) \mid A_l]$, is:

$$P[(D > D_o) \mid A_l] = \int_{D_o}^{lb} \frac{1}{B(\alpha_l,\beta_l)} \frac{(s-a)^{\alpha_l-1}(lb-s)^{\beta_l-1}}{(lb-a)^{\alpha_l+\beta_l-1}} ds \tag{10}$$

The parameters α_l and β_l of the distribution can be estimated using the method of moments given the mean ($\mu_{D|A_l}$) and the variance ($\sigma^2_{D|A_l}$) of the deficits sum D. The moments $\mu_{D|A_l}$ and $\sigma^2_{D|A_l}$ can be determined either empirically (e.g. Shiau and Shen, 2001) or theoretically given the mean (μ_d) and the variance (σ^2_d) of the single deficits that can be determined from the historical single deficits. Assuming the process of the single deficits d_t is Auto-Regressive of order 1, AR(1), then the moments $\mu_{D|A_l}$ and $\sigma^2_{D|A_l}$ of the deficits sum D corresponding to the drought event of the fixed length l are (Sen, 1980):

$$\mu_{D|A_l} = l\,\mu_d \tag{11}$$

and

$$\sigma^2_{D|A_l} = \sigma^2_d \left\{ l + 2\rho \frac{l(1-\rho) - (1-\rho^l)}{(1-\rho)^2} \right\} \tag{12}$$

where ρ is the lag1 serial correlation between the single deficits. If the deficit D, $D \in [0, lb]$, is adjusted to $\tilde{D}$ such that $\tilde{D} \in [0,1]$, with mean and variance of $m_{\tilde{D}|A_l}$ and $s^2_{\tilde{D}|A_l}$ respectively, then the moment estimators of the parameters α_l and β_l are:

$$\hat{\alpha}_l = \frac{m^2_{\tilde{D}|A_l}(1 - m_{\tilde{D}|A_l})}{s^2_{\tilde{D}|A_l}} - m_{\tilde{D}|A_l} \tag{13}$$

$$\hat{\beta}_l = \frac{(1 - m_{\tilde{D}|A_l})\hat{\alpha}_l}{m_{\tilde{D}|A_l}} \tag{14}$$

For any drought length l, the mean and the variance of $\tilde{D}$ are related to mean and variance of D as $m_{\tilde{D}|A_l} = \dfrac{\mu_{D|A_l}}{lb}$ and $s^2_{\tilde{D}|A_l} = \dfrac{\sigma^2_{D|A_l}}{(lb)^2}$.

The occurrence probability and return period of multiyear droughts

The single site drought events are usually distinguished by a random duration that is associated to a random magnitude D, therefore to analyze droughts both random quantities should be considered (Salas et al., 2005). The drought event $\{(D > D_o) \cap A_l\}$ is defined as the event made of a run of length l and has total deficit D that exceeds the value D_o. Given the event $\{(D > D_o) \cap A_l\}$, the occurrence probability $P[(D > D_o) \cap A_l]$ can be evaluated using the law of the conditional probability as:

$$P[(D > D_o) \cap A_l] = P\,[A_l] \times P[(D > D_o) \mid A_l] \tag{15}$$

The probability $P[A_l]$ is given by equation (4) and the probability $P[(D > D_o) | A_l]$ is the probability distribution of D conditioned to the occurrence of A_l that is given by equation (10). The occurrence probability of the event $\{(D > D_o) \cap A_l\}$ is:

$$P[(D > D_o) \cap A_l] = P[A_l] \int_{D_o}^{lb} \frac{1}{B(\alpha_l, \beta_l)} \frac{(s-a)^{\alpha_l - 1} (lb - s)^{\beta_l - 1}}{(lb - a)^{\alpha_l + \beta_l - 1}} ds \qquad (16)$$

The return period of any specified drought event is defined as the expected value of the interarrival time *W* between recurrent drought events of the same kind (Fernandez and Salas, 1999). To analytically evaluate the return period, the drought event $\{(D > D_o) \cap A_l\}$ is considered as a renewal process and consequently the concept of the renewal theory (e.g. Loaiciga et al., 1992) can be used. The renewal theory states simply that the expected number $E[N_{\{D > D_o \cap A_l\}, n}]$ of drought events of the kind $\{(D > D_o) \cap A_l\}$ in a given time period *n* divided by the period *n* converges to $1/T$ assuming that *n* is sufficiently long, where *T* is the expected value of the renewal time *W* (Loaiciga et al., 1992). The number $N_{\{D > D_o \cap A_l\}, n}$ of drought events of the kind $\{(D > D_o) \cap A_l\}$ that may arise from a sequence of length *n* is:

$$N_{\{(D > D_o) \cap A_l\}, n} = \sum_{t=1}^{n-l+1} \mathrm{I}_t \qquad (17)$$

where I_t is an indicator function that is defined as:

$$\mathrm{I}_t = \begin{cases} 1 & when \ \{(D > D_o) \cap A_l\} \ occurs \ starting \ at \ the \ time \ t \\ 0 & otherwise \end{cases} \qquad (18)$$

The value *n*-*l*+1 in equation (17) is the maximum number of drought events of the kind $\{(D > D_o) \cap A_l\}$ that may occur in a sequence of length *n* when drought events are defined according to the alternative definition of runs (Schwager, 1983). The expected number $E[N_{\{D > D_o \cap A_l\}, n}]$ is obtained by taking the expectation of both sides of equation (18):

$$E[N_{\{(D > D_o) \cap A_l\}, n}] = (n - l + 1)\, E[\mathrm{I}_t] \qquad (19)$$

The expected value $E[\mathrm{I}_t]$ is:

$$E[\mathrm{I}_t] = \sum_{i \in \{0,1\}} i \ P[\mathrm{I}_t = i] = P[(D > D_o) \cap A_l] \qquad (20)$$

If n is sufficiently long such that $n \approx n\text{-}l+1$ then according to the renewal theory the return period T of recurrent drought events of the kind $\{(D > D_o) \cap A_l\}$ is:

$$T = E(W) = \frac{n}{E[N_{\{(D>D_o)\cap A_l\},\,\mathrm{n}}]} = \frac{1}{P[(D > D_o) \cap A_l]} \tag{21}$$

The return period T in equation (21) defined as $1/P$ appears plausible in the light that successive drought events emerge completely independent of each other (Sen, 1980; Fernandez and Salas, 1999).

Application and parameter estimation

Annual streamflow series of the Poudre River at the mouth of Canyon gauging station is used to characterize hydrological droughts in Colorado. The series consists of 119 annual streamflow observations for the period 1884 – 2002. Estimates of the streamflow statistics of the mean, standard deviation, skewness, and the lag-1 autocorrelation coefficient are 299011 acre-ft, 106512 acre-ft, 0.98, and 0.153 respectively. Figure (3) shows the annual streamflow series truncated at the level of the long-term mean. The sequence of deficits and surpluses resulted after truncating the streamflow series was converted to 0,1 sequence with the value 0 assigned when deficit state was observed and 1 otherwise. Assuming that the 0,1 process of deficits and surpluses (X_t) is simple Markov stationary process the estimated unconditional state probabilities p_0 and p_1 are 0.563 and 0.437 respectively, while the estimated conditional state probabilities are:

$$\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = \begin{bmatrix} 0.605 & 0.395 \\ 0.509 & 0.491 \end{bmatrix}$$

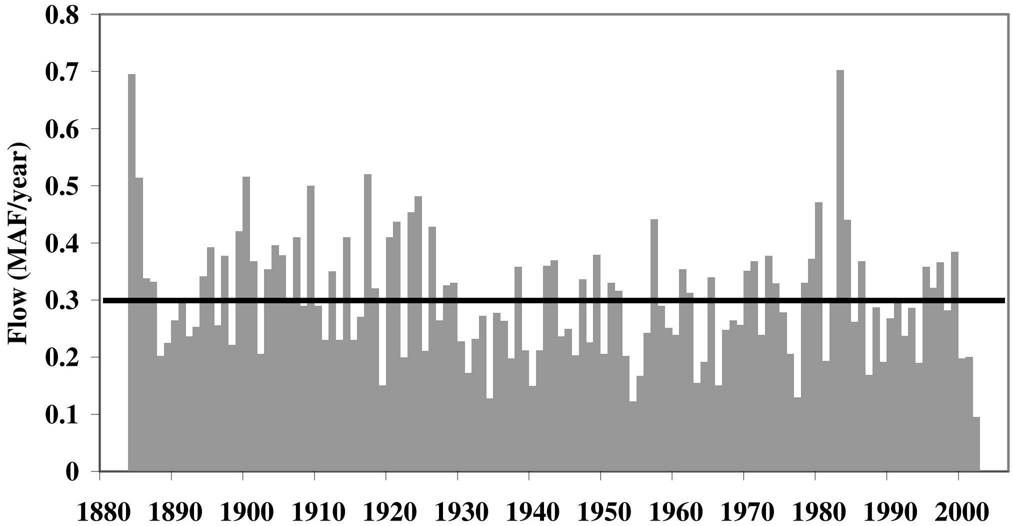

Figure 3. The annual streamflow series of the Poudre River at the mouth of canyon gauging station. The line represents the long-term mean of the streamflow for the period 1884 - 2002

For the Poudre River, the estimated single deficits statistics of the mean (μ_d), the standard deviation (σ_d), and the correlation between deficits (ρ) are 74257 acre-ft, 47865 acre-ft, and 0.082 respectively. The mean $\mu_{D|A_l}$ and the variance $\sigma^2_{D|A_l}$ of the deficit sum D, $D \in [0, lb]$ where b = 299011 acre-ft, were estimated using equations (11) and (12) respectively. Since the single year deficit d_t and deficits sum $D \mid A_l$ conditioned on the occurrence of run of deficits of length l are assumed Beta distributed, the parameters α_l and β_l versus the drought length l were determined using equations (13) and (14) respectively after calculating the adjusted mean $m_{\tilde{D}|A_l}$ and variance $s^2_{\tilde{D}|A_l}$.

The analysis of the short historical streamflow records usually results in relatively few extreme drought events and therefore clear picture about the natural variability of extreme droughts may not be utilized. Stochastic simulation models are used to synthesize longer streamflow series that can be analyzed to capture possible scenarios of extreme droughts that may occur in the future (Salas et al., 2005). An Auto-Regressive model of order 1, AR(1), has been fitted to the annual streamflow data of the Poudre River. The same model has been used in literature to generate synthetic annual streamflow series for the Poudre River that sufficiently reproduces the basic flow statistics of the historical streamflow data (e.g. Tarboton, 1994; Salas et al., 2005). Eventually a series of synthetic streamflow of 50000 years long was generated for the purpose of drought analysis.

RESULTS AND DISCUSSION

Considering the drought event $\{(D > D_o) \cap A_l\}$, the analytical procedures developed in this paper were employed to characterize extreme hydrological droughts in the Poudre River basin, Colorado. In specific the occurrence probabilities and return periods of recurrent drought events of length 1 – 10 years have been estimated using the proposed procedures and verified empirically by analyzing the synthesized streamflow series. Figure (4) shows the occurrence probability of the drought event $\{(D > D_o) \cap A_l\}$ evolves in the Poudre River basin versus the length of the drought at different critical deficit levels, i.e. at γ 0, 0.5, and 1.0 respectively. The figure shows the well agreement between theoretical estimates of the occurrence probability using equation (16) and results obtained from the empirical analysis of the truncated simulated streamflow series. Figure (4) also shows the uncertainty in the empirical estimates of the occurrence probability when short historical streamflow records are used only. In that case the estimated occurrence probability departs from its expectations especially for droughts of extreme duration (4 years or more) or extreme magnitude ($\gamma > 0$).

Considering the drought length l only (γ = 0), the chance that a drought occurs decreases as the length l increases as shown by Figure (4). That result is logical and expected because droughts is shifted more toward extremeness as the duration increases. Figure (4) also shows that when the magnitude of the drought is considered (γ > 0) and for a given duration (l = 1 – 6 years) then as the magnitude of the drought increases (γ increases) droughts become less frequent to occur due to the high unusual drought magnitude, however for l > 6 years the less frequent occurrence of droughts is only due to the extreme drought duration. Moreover, for a given deficit level D_o (γ is constant), Figure (4) depicts that in the Poudre River basin droughts of moderate duration (2 – 3 years when γ = 0.5 and 3 – 4 years when γ = 1) are the more often occurring events while droughts of duration shorter are less frequent to occur due to the high unusual drought magnitude, and droughts of duration longer than the moderate duration the chance of occurrence is less due to the extreme drought length.

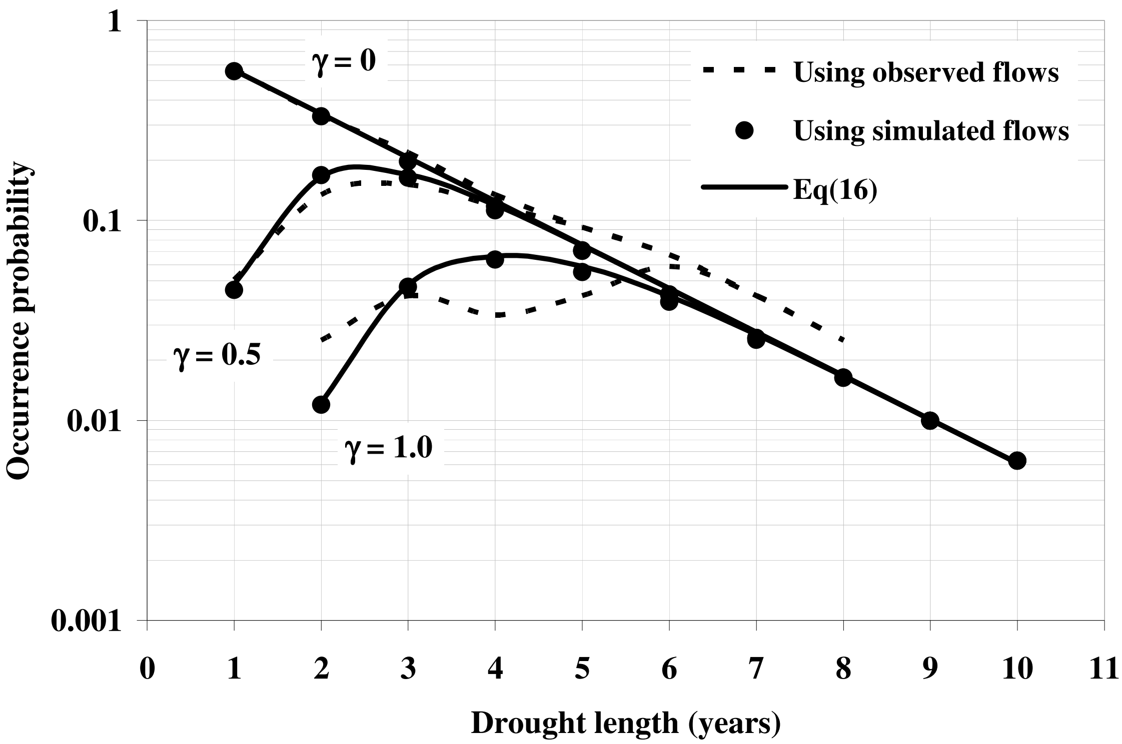

Figure 4. The occurrence probability of the multiyear drought event $\{(D > D_o) \cap A_l\}$ arises in the Poudre River basin

The return period T of the multiyear drought event $\{(D > D_o) \cap A_l\}$ estimated as the expected value of the interarrival time $E(W)$ between recurrent events was determined using equation (21) at different deficit levels (γ = 0, 0.5, and 1). Figure (5) shows the return period of recurrent droughts in the Poudre River basin versus the drought length. Generally, the estimates of the return period obtained analytically using equation (21) agree with the results obtained from the empirical analysis of the simulated streamflow. The results shown in Figure (5) indicate that the return period T of recurrent droughts in the Poudre River increases as the drought magnitude increases (γ increases) for drought duration of 6 years or less, and generally increases as the drought length l increases. Moreover, as Figure (5) shows the most frequent droughts, i.e. droughts of low return period, are of moderate duration (2 – 3 years when γ = 0.5 and 3 – 4 years when γ = 1). On average, such droughts occur every 6 years when γ = 0.5 and 15 years when γ = 1. The unexpected variations in the estimates of the average occurrence time of extreme hydrological droughts using only the historical streamflow records is clear in Figure (5). This uncertainty is associated to the limited number of drought events resulted after truncating the relatively short historical streamflow records for the purpose drought characterization. In fact droughts of longer durations or higher magnitude may not be observed at all in streamflow records of around 100 years like the case of the Poudre River. Generally the pattern of change in the return period estimates shown in Figure (5) agrees with the results of similar studies (e.g. Salas et al., 2005) although using different probabilistic definitions of the drought event.

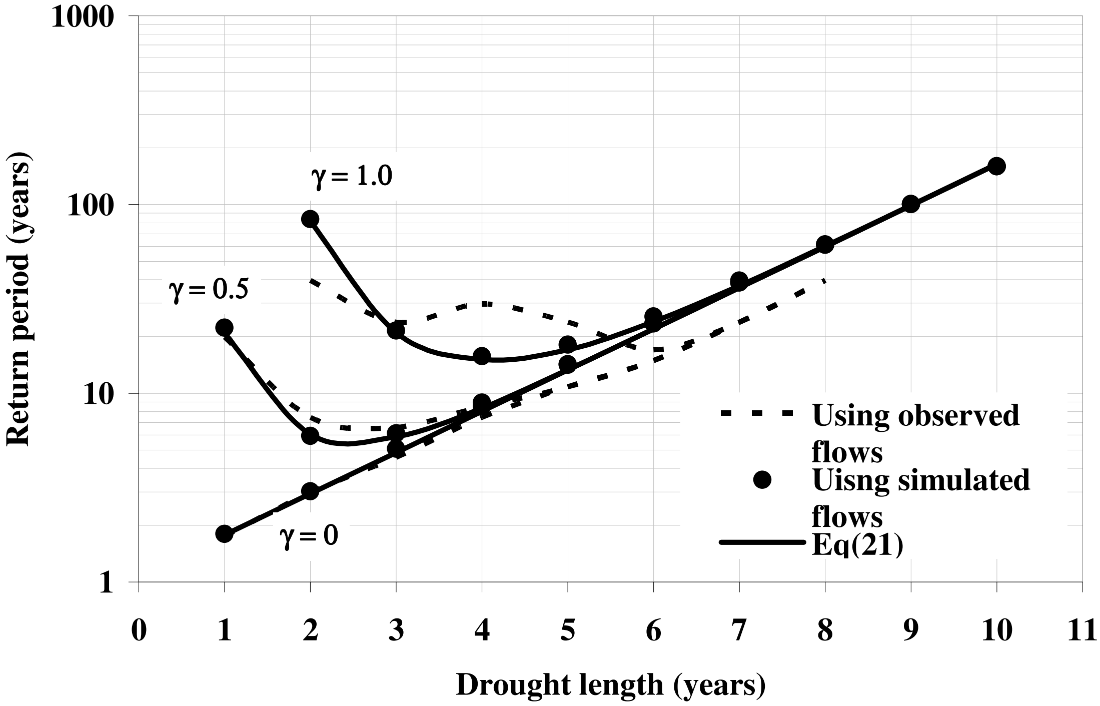

Figure 5. The return period of the multiyear drought event {$(D > D_o) \cap A_l$} arises in the Poudre River basin

The theoretical procedures developed here to estimate the occurrence probability and return period of droughts were built assuming drought deficits are Beta distributed. To assess the improvement of fitting the deficits as Beta distributed, drought return period was estimated using equation (21) assuming the distribution of deficits is Beta for one case while Gamma for the other. Figure (6) depicts the return period of the multiyear drought event $\{(D > D_o) \cap A_l\}$ occurring in the Poudre River assuming deficits are Beta distributed versus the Gamma deficits. At deficit level γ of 0.5 and 1.0, Figure (6) indicates that return period estimated assuming deficits are Beta distributed appears to quite well match the results obtained from analyzing the simulated flows, whereas results estimated assuming deficits are Gamma distributed depart to some extent from the empirical results in particular when droughts of moderate or short duration having large deficit (γ = 1) are considered. In fact when γ = 1, equation (21) with Gamma distributed deficits estimates the return period of the 1-year drought event $\{(D > y_o) \cap A_{l=1}\}$ of around 1300 years, however practically that event can not occur because drought deficit can not exceed the value y_o as can be seen from Figure (2).

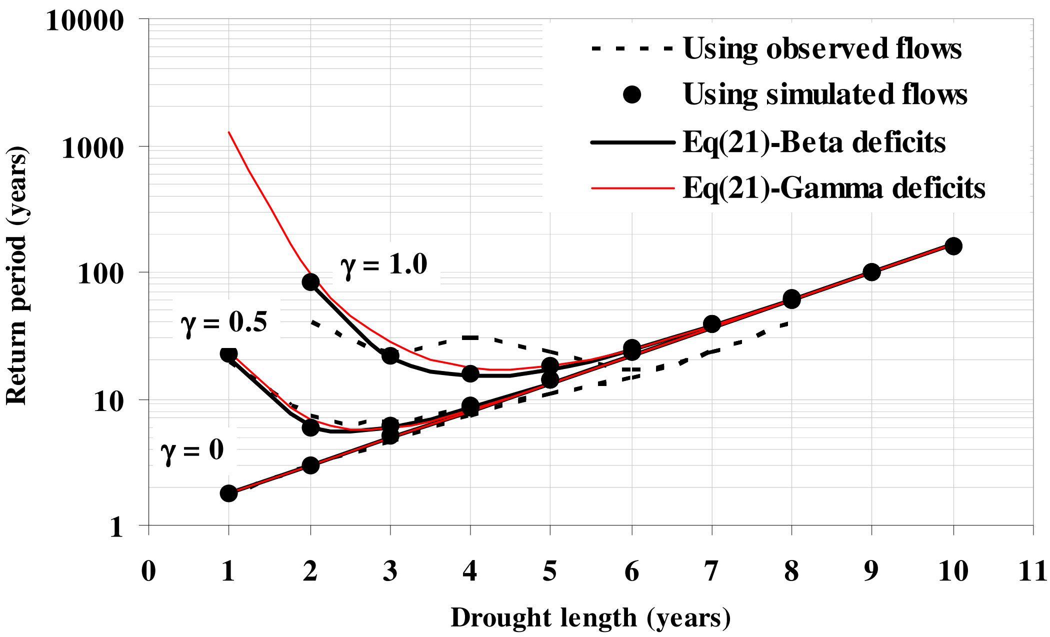

Figure 6. The return period of the multiyear drought event $\{(D > D_o) \cap A_l\}$ arises in the Poudre River basin

CONCLUSIONS

The multiyear drought of the length l is defined as a run of l consecutive deficits regardless where the run starts as long as the run satisfies the continuity. This definition appears more flexible in terms of the capability to capture drought events of extreme magnitude regardless of being proceeded and followed by at least one surplus that is a requirement of other definitions. Analytical procedures to estimate the occurrence probability and return period of multiyear droughts were developed and applied to characterize extreme hydrologic droughts in the Poudre River basin, Colorado. Assuming drought deficit are Beta distributed, the estimated occurrence probability and return periods using the procedures developed were compared with results obtained empirically from the analysis of the simulated annual streamflow of Poudre River. The analytical procedures developed here provided more reliable estimates of drought statistics when compared with the uncertain and the less informative results obtained from the analysis of the short observed streamflow series. Fitting drought deficits as Beta distributed improves the characterization of droughts of extreme magnitude and short or moderate duration when compared with the Gamma distributed deficits. In the Poudre River basin, the analysis shows that droughts of moderate magnitude (γ = 0.5) and duration of 2 – 3 years are the most frequent events having average recurrence time of nearly 6 years, while droughts of extreme magnitude (γ = 1) and duration of 3 – 5 years are the most frequent events having average recurrence time of nearly 15 years.

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