

# INTERFERENCE EFFECT ON STRAIN-CONCENTRATION FACTOR OF CYLINDRICAL BARS WITH DOUBLE CIRCUMFERENTIAL U-NOTCHES UNDER STATIC TENSION

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## ABSTRACT

The Finite Element Method (FEM) is employed to study the interference effect on new strain concentration factor (SNCF), which has been defined under triaxial stress state. The employed specimens are cylindrical bars with double circumferentially U-notches under static tension. The new SNCF is constant in the elastic deformation and the range of this constant value increases with decreasing notch pitch  $l_0$ . It also increases with increasing notch radius. The elastic SNCF rapidly decreases from its value at  $l_0 = 0$  mm, i.e. single circumferential U-notch, and reaches minimum value at  $l_0 = 0.5$  mm with increasing notch pitch  $l_0$ . After that, it increases with increasing  $l_0$  and reaches a value nearly equal to that of a bar with single circumferential U-notch. This becomes prominent with decreasing notch radius. The new SNCF increases from its elastic value to a maximum value as the plastic deformation develops from the notch root. On further plastic deformation, the new SNCF decreases with plastic deformation. The maximum new SNCF shows the same variation with  $l_0$  as that of the elastic SNCF. The current results indicate that the maximum notch pitch where the interference effect occurs on the elastic and maximum new SNCF is  $l_0 \approx 5$  mm.

## 1 INTRODUCTION

Geometrical irregularities such as notches, grooves, holes, or defects are acting as local stress and strain raisers. Many numerical analyses and theoretical studies have been conducted to obtain the elastic stress-concentration factor (SSCF); the results have been published and used for engineering design [1-5]. Neuber's rule predicts that the plastic SNCF increases and the plastic SSCF decreases from their elastic values as plastic deformation develops from the notch root [6]. This prediction has been confirmed by many experimental or analytical studies [4, 6, 7-11]. These results indicate that the SNCF is more important than the SSCF [12]. This is because the plastic SNCF maintains a value much greater than unity, while the plastic SSCF decreases towards unity.

A new definition of elastic-plastic SNCF has been proposed for better understanding of strain concentration under static tension [12, 13]. This new SNCF is defined under the triaxial state of stress at the net section, while the conventional one is defined under uniaxial state of stress. This difference in stress state gives the new SNCF reasonable value [12, 13] and can remove the contradiction of the conventional definition. This is because the conventional SNCF becoming less than unity in spite of the concave distribution of axial strain. The effect of notch depth on the SNCF and SSCF under static tension has been studied by Tlilan et al. [13]. The results indicates that the new SNCF is more reasonable than the conventional SNCF and SSCF.

Some studies have been made on the interference effect on the elastic SSCF of the flat bars with double U- or semicircular notches under tension. The obtained relations between the elastic SSCF and the notch pitch have been published for engineering design [5]. These studies show that the SSCF subjected to the interference effect is less than the SSCF of a single notch. Few studies have been carried out on the interference effect on the elastic SSCF of cylindrical bars with double U- or semicircular notches under tension. Moreover, only two studies have been performed on the interference effect on strength such as yield point load and ultimate tensile strength and deformation properties of notched bars under tension [14, 15]. In these studies the interference effect has also been discussed on the elastic SSCF of cylindrical bars with double U-notches under tension.

Unfortunately, the interference effect on elastic and elastic-plastic new SNCF has not evaluated. In this paper, the cylindrical bar double circumferential U-notches is employed to study the interference effect on the elastic-plastic new SNCF.

## 2 STRAIN-CONCENTRATION FACTOR

A new SNCF has been defined for static axial loading as the ratio of the maximum axial strain at the notch root  $(\varepsilon_z)_{\max}$  to the new average axial or new nominal strain  $(\varepsilon_z)_{\text{av}}^{\text{new}}$  [12, 13].

$$K_{\varepsilon}^{\text{new}} = \frac{(\varepsilon_z)_{\max}}{(\varepsilon_z)_{\text{av}}^{\text{new}}} \quad (1)$$

It should be noted that this new SNCF has been defined under the triaxial stress state at the net section. This new SNCF is introduced by a new definition of the average axial strain. For circumferentially notched cylindrical bars  $(\varepsilon_z)_{\text{av}}^{\text{new}}$  is defined as follows [12]:

$$(\varepsilon_z)_{\text{av}}^{\text{new}} = \frac{1}{\pi r_n^2} \int_0^{r_n} \varepsilon_z(r) 2\pi r dr = 2 \int_0^1 \varepsilon_z(s) s ds \quad (2)$$

where  $s = r/r_n$ . In the elastic level of deformation,  $(\varepsilon_z)_{\text{av}}^{\text{new}}$  can be transformed into the following equation:

$$(\varepsilon_z)_{\text{av}}^{\text{new}} = \frac{1}{\pi r_n^2} \int_0^{r_n} \left[ \frac{\sigma_z}{E} - \frac{\nu}{E} (\sigma_{\theta} + \sigma_r) \right] 2\pi r dr = \frac{(\sigma_z)_{\text{av}}}{E} - \frac{\nu}{E \pi r_n^2} \int_0^{r_n} (\sigma_{\theta} + \sigma_r) 2\pi r dr = \frac{(\sigma_z)_{\text{av}}}{E} - \frac{2\nu}{E} \int_0^1 \{ \sigma_{\theta}(s) + \sigma_r(s) \} s ds$$

where  $E$  and  $\nu$  are the Young's modulus and Poisson's ratio, respectively. This equation indicates that  $(\varepsilon_z)_{av}^{new}$  is defined under the triaxial stress state at the net section. It should be noted that  $(\varepsilon_z)_{av}^{new}$  is defined under the triaxial stress state also in plastically deformed area at the net section. This is because the plastic component of the axial strain is directly related to the triaxial stress state, as is indicated by the theory of plasticity. The definition under the triaxial stress state gives reasonable results consistent with the concave distribution of the axial strain at any deformation level [12, 13].

The conventional SNCF under static tension has been defined as follows

$$K_\varepsilon^{con} = \frac{(\varepsilon_z)_{max}^{con}}{(\varepsilon_z)_{av}^{con}} \quad (5)$$

where  $(\varepsilon_z)_{av}^{con}$  is the conventional average. This conventional SNCF has been defined under uniaxial stress state at the net section. This is because  $(\varepsilon_z)_{av}^{con}$  has been defined under uniaxial stress state [10]. It should be noted that there is a small plastic deformation occurs around the notch root even in the range  $(\sigma_z)_{av} \leq \sigma_Y$ , when  $(\sigma_z)_{av}$  approaches  $\sigma_Y$ . Even in this range  $(\varepsilon_z)_{av}^{con}$  is given by

$$(\varepsilon_z)_{av}^{con} = \frac{(\sigma_z)_{av}}{E} \quad (6)$$

This equation indicates that the conventional definition has neglected the effect of tangential  $\sigma_\theta$  and radial  $\sigma_r$  stresses. On further development of plastic deformation, i.e. in the range  $(\sigma_z)_{av} > \sigma_Y$ ,  $(\varepsilon_z)_{av}^{con}$  is determined using the uniaxial true stress–total strain curve  $\sigma = f(\varepsilon)$ .

## 2 SPECIMEN GEOMETRIES AND FINITE ELEMENT MESH

The employed cylindrical bar with the double circumferential U-notches is shown in Fig. 1a. The net-section diameter  $d_o$  of 3.34 mm and the gross diameter  $D_o$  of 16.7 mm were selected to give the net-to-gross diameter ratio  $d_o/D_o$  of 0.2. Figure 1 shows that the specimen length is expressed as  $(2L_o+2l_o)$ , where  $L_o$  is the unnotched length, and  $2l_o$  is the notch pitch. The half notch pitch  $l_o = 0.0$  mm denotes the circumferential U-notch. The unnotched length is held constant, while the half notch pitch  $l_o$  is varied from 0.0 to 25 mm to examine the interference effect of the double circumferential U-notches. Two notch radii  $\rho_o$  of 0.5 and 2.0 mm are employed to vary the notch sharpness  $d_o/2\rho_o$ . Figure 1b shows a finite element mesh of one quarter of a notched specimen with the double circumferential U-notches. An eight-node axisymmetric ring element was chosen to model the notched specimens. The FEM calculations were performed under the axisymmetric deformation. The increments of the axial displacement were applied at the right end of the unnotched part of  $L_o$ . The magnitude of the increment was small enough to provide an elastic solution for the first few increments in each notched specimen.

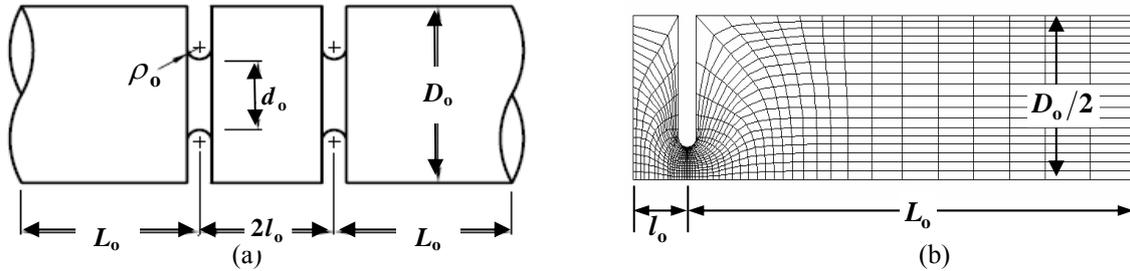


Figure 1. (a) Specimen geometries. (b) Finite element mesh.

## 3 SPECIMEN MATERIALS

The materials employed are an Austenitic stainless steel and a Ni-Cr-Mo steel. Young's modulus  $E$ , Poisson's ratio  $\nu$  and the yield stress  $\sigma_Y$  of these materials are given in Fig. 2. The true stress-plastic strain relation was obtained from tension tests. In order to express the stress-strain curve accurately the obtained relation was divided into a few ranges of plastic strain and in each range the following polynomial of 5<sup>th</sup> degree was fitted

$$\sigma = C_0 + C_1\varepsilon_p + C_2\varepsilon_p^2 + C_3\varepsilon_p^3 + C_4\varepsilon_p^4 + C_5\varepsilon_p^5$$

The values of these coefficients in the plastic strain ranges are given in Ref. 13. The true stress-plastic strain curves used in the calculations are shown in Fig.2.

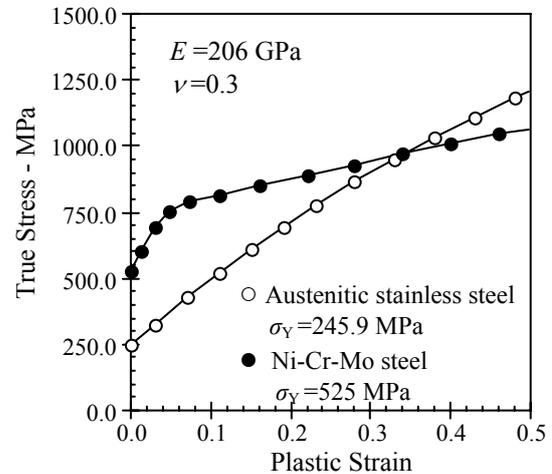


Figure 2. True stress – plastic strain curve.

#### 4 INTERFERENCE EFFECT ON THE VARIATION OF $K_{\epsilon}^{new}$ WITH DEFORMATION

The variations in the new SNCF  $K_{\epsilon}^{new}$  with  $2\ln(d_0/d)$  are given in Figures 3. The new SNCF is constant in elastic deformation and the range of  $2\ln(d_0/d)$  for this constant value increases with increasing notch radius. It also increases with increasing notch pitch up to  $l_0 \approx 0.5$  mm. On further increase in the notch pitch, this range decreases and becomes the same as that of the single circumferential U-notch. As the plastic deformation develops from the notch root, the new SNCF increases from its elastic value to its maximum and then decreases on further plastic deformation. Figure 4 shows the relation between the elastic  $K_{\epsilon}^{new}$  and half notch pitch  $l_0$ . The elastic  $K_{\epsilon}^{new}$  rapidly decreases from its value at  $l_0 = 0.0$  mm and reaches its minimum at  $l_0 \approx 0.5$  mm. On further increase in  $l_0$ , the elastic  $K_{\epsilon}^{new}$  gradually increases and finally reaches the value of the circumferential U-notch ( $l_0 = 0.0$  mm) at  $l_0 = 5$  mm and 2 mm for  $\rho_0 = 0.5$  and 2 mm, respectively. Beyond this value of  $l_0$  the elastic  $K_{\epsilon}^{new}$  is nearly constant up to  $l_0 = 25$  mm, the maximum half notch pitch in the FEM calculations. The elastic  $K_{\epsilon}^{new}$  in the range  $5 \text{ mm} \leq l_0 \leq 12.5 \text{ mm}$  are nearly equal to the elastic  $K_{\epsilon}^{new}$  of the circumferential U-notch. This indicates that the interference effect on the elastic  $K_{\epsilon}^{new}$  is extremely strong in a small range of  $l_0$  and nearly vanishes beyond  $l_0 \approx 5.0$  mm. It should be noted that the same results have been obtained for  $\rho_0 = 2$  mm. However, the results have not been introduced here because of the limitation of the paper length.

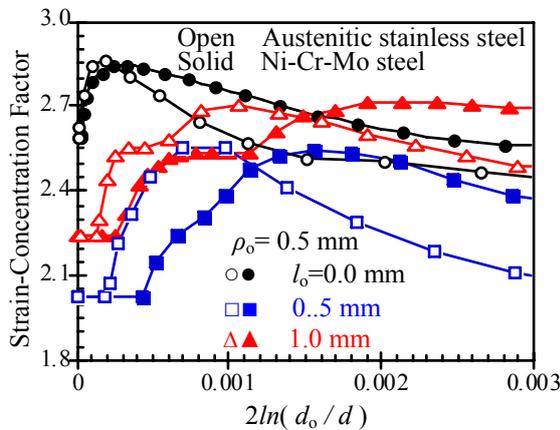


Figure 3. Variations in SNCF with deformation

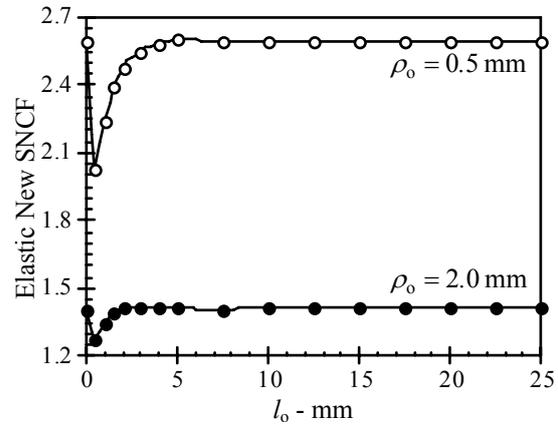


Figure 4. Interference effect on elastic SNCF

#### 5 CONCLUSIONS

The relation between the elastic new SNCF and half notch pitch  $l_0$  shows very rapid decrease from the value at  $l_0 = 0$  mm with increasing  $l_0$ . The elastic new SNCF becomes minimum at  $l_0 = 0.5$  mm. On further increase in  $l_0$  the elastic new SNCF gradually increases and reaches the maximum value at  $l_0 \approx 5$  mm. This maximum value is nearly equal to the elastic new SNCF at  $l_0 = 0$  mm. The elastic new SNCF is almost constant up to  $l_0 = 25$  mm for further notch pitch. This indicates that the maximum notch pitch where the interference effect occurs on this elastic new SNCF is  $l_0 = 5$  mm. The new SNCF rapidly increases from its elastic value to the maximum value with increasing  $2\ln(d_0/d)$  for  $l_0 = 0$  mm. On the other hand, It shows a gradual increase from its elastic value for  $l_0 = 0.5$  mm. Actually, the rate of increase in the new SNCF from its elastic value increases with increasing notch pitch in the range  $l_0 > 0.5$  mm.

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