

# Error Rate Analysis of DCPSK over Generalized Fading Channels

Mahmoud A. Smadi

Department of Electrical Engineering  
The University of Texas at Arlington  
Arlington, Texas 76019  
Email: smadi@ieee.org

Vasant K. Prabhu

Department of Electrical Engineering  
The University of Texas at Arlington  
Arlington, Texas 76019  
Email: prabhu@uta.edu

**Abstract**—Binary and quaternary -differential coherent phase-shift keying (DBPSK, DQPSK) are widely used modulations in digital wireless systems. However, a simple expression for the probability density function (pdf) of the decision statistic, and hence the error rate performance, of such systems under post-detection equal gain combining (EGC) and generalized fading channels is not currently available in the literature. This will be the subject of our paper. Our analytic study is based on a convergent infinite series for the sum of independent random variables that has been derived using Fourier series approximation method. Generalized fading channels including Rayleigh, Ricean, and Nakagami- $m$  are investigated. The channels are assumed to be independent, frequency-nonselective, and slowly varying. The accuracy of our results is verified through simulation.

## I. INTRODUCTION

In general, because of thermal noise and time-variant characteristics of the fading channel, phase tracking devices cannot perfectly recover the phase of the received signal. This will indeed degrade the performance of the coherent reception [1]. Hence, differential coherent phase-shift keying (DCPSK), in which there is no need to estimate the carrier phase, is an interesting alternative to ideal coherent phase-shift keying (CPSK). The price to pay here is the degradation in the error rate performance. It is well known that for the simple case without channel fading and diversity, DBPSK is approximately 2.3 dB poorer in performance than BPSK at low SNR and less than 1 dB at high SNR. On the other hand, these gain penalties are about 4 dB and 2.3 dB in the DQPSK case, respectively [2]. To evaluate the error rate performance of DCPSK systems and compare it to that of CPSK systems in the presence of channel fading and EGC diversity reception is one aim of this study.

Since diversity reception techniques are effective methods for compensating the abusive effect of channel fading, we consider the effect of diversity reception when signals are subject to multipath fading. Although, maximal ratio combining (MRC) is shown to be the optimum diversity reception scheme, it is not practical for differentially coherent detection since, for this kind of detection, we no longer need to co-phase and weight the diversity channels. Instead, postdetection EGC has a simple structure and it is more appropriate for this kind of detection [3].

We have a lot of previous contributions in this field. Proakis

[2] has derived a closed-form expressions for the bit error probability (BEP) of DCPSK over  $L$  independent and identically distributed (i.i.d.) Rayleigh fading channels. In their paper [4], Weng and Leung introduced a closed-form solution for the BEP of DBPSK over  $L$  i.i.d. Nakagami- $m$  fading channels. As an extension to the work done in [4], Patenaude *et al.* [5] evaluated the error probability performance of DBPSK for  $L$  independent but not necessarily identically distributed Nakagami- $m$  fading channels. On the other hand, the error probability analysis of DBPSK over correlated fading channels was studied in [6], [7], and [8]. In [6] Zhang derived a general BEP solution of DBPSK over arbitrary correlated Nakagami- $m$  channels. Mallik *et al.* [7] used the same approach as in [6] to evaluate the BEP of DBPSK signals over correlated Ricean channels. More generally, the BEP of DCPSK over correlated Ricean channels was studied in [8]. The aforementioned analyses are based on the characteristic function (CF) method. This method requires  $L - 1$  order derivatives, which becomes difficult to evaluate for large diversity order  $L$ , and cannot often be used to compute error probabilities. Using different approaches, Tjhung [9] *et al.* and Tanda [10] used the series expansion of the first order Marcum  $Q$ -function to evaluate the BEP of DQPSK system with Gray coding over Ricean and Nakagami- $m$  fading channels, respectively. Furthermore, Tellambura [11] expressed the BEP of DQPSK over Ricean and Nakagami- $m$  fading distributions in a single finite integral form based on an alternating definition of the first order Marcum  $Q$ -function. However, all the analyses above are for single channel reception. Recently, Simon and Alouini [12] have given a unified error rate analysis for DCPSK over many types of independent fading channels. Their analysis is based on an alternating definition of the generalized Marcum  $Q$ -function and the results are expressed in term of one-fold finite-range integral, which need to be evaluated numerically.

In this paper, we add to the above results by providing simple closed-form expressions for the pdf and the BEP of DCPSK systems over generalized fading channels in term of a convergent infinite series of elementary functions. This series has been derived in [13] for the pdf of a sum of independent random variables (RV's). The derivation of this series is based on the assumption that for any unbounded RV  $X$ , one can choose  $T_S$  large enough such that  $\Pr [X > T_S]$  is a very small