Useful Bounds for the BEP of Partially Coherent Faded PSK Signals

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***Abstract*— Useful upper and lower bounds are presented for the average bit-error probability (BEP) of uncoded partially coherent BPSK and QPSK systems. For the first time, the bounds are derived taking into consideration wireless fading impairment. The bounds are shown to be tight and easy to evaluate for phase error noise parameters likely to be encountered in practice and for SNR that are needed in moderate fading environment. A precise relationship between the lower bounds and the average degradation caused by the carrier phase error are given. Also, the analysis of the upper bound establishes expression for the irreducible errors due to imperfect phase recovery for coherent QPSK system. The accuracy of both bounds is illustrated by comparing the bound results to those obtained by more complex exact series analysis.**

I. Introduction

Exact evaluation of the error rate and the resulting degradation of partially coherent PSK systems involves the summation of complex series [1], [2]. However, for preliminary design purposes, bound solutions that allow for quick and easy to compute estimates of both the BEP and the average signal-to-noise ratio (SNR) degradation caused by the imperfect phase recovery are of primary concern. Throughout our literature search, no upper or lower bounds on the BEP of partially coherent PSK signals under fading environments were found. In this paper, such bound expressions are proposed and being derived for both BPSK and QPSK systems. Although most practical faded systems utilized some kind of channel coding, we consider the analysis of uncoded systems since our attempt here is to reach bound solutions that permit easy estimates of the performance. Also, we do expect that the coding gain (usually in practical order of < 1 dB) will equally attribute the exact as well as the bound solutions.

The problem of evaluating the error rate performance of coherent PSK systems in the presence of additive white Gaussian noise (AWGN) and carrier phase error is extensively studied in the literature [1], [3]-[10]. Since exact evaluation is usually complex, the BEP of partially coherent PSK systems was obtained by numerically integrating the conditional BEP expression for a fixed phase error over the phase error statistic [3], [4]. Later on, many authors have devoted this problem by using infinite series approaches [1], [5], giving approximate solutions [6], [8], and [9], or deriving upper and lower bounds [1], [7], and [10].

The previous studies did not take channel fading into account, a phenomenon important in wireless communications. And such extension is first targeted by [11], but provided only limited details. Most recently, the authors in [12] used Maclaurin series to obtain accurate approximation for the average BEP of partially coherent BPSK and QPSK for several channel fading models. But in their analysis linear fading channel behavior over the entire region of the instantaneous SNR was assumed, as well as, linearly increased tracking-loop SNR ρ_c with respect to the thermal noise SNR γ_b. However, for analytical purposes, it is desirable to have a good approximation, or tight bounds with less restrictive assumptions on the system parameters making them valid to arbitrary values of ρ_c and γ_b.

II. Derivation of the Bounds

In this section, we will extend the work in [10] by considering channel fading impairment in the analysis to derive lower bounds on the error probabilities of partially coherent PSK systems. Then we use a well-known bound on the error function to derive for the upper bounds.

A. Lower Bounds

1) BPSK Case: The BEP of BPSK in the presence of AWGN, and for a given flat channel fading magnitude α and carrier phase error ϵ can be written as [1]

$$P_2(e|\alpha,\epsilon) = \frac{1}{2}\text{erfc}\left(\sqrt{\gamma_b}\alpha\cos\epsilon\right), \tag{1}$$

where $\gamma_b = E_b/N_0$ is the average SNR per bit. Here, we assume a slowly Nakagami-m faded channel such that α would remain constant over the data symbol duration T. Also, the introduced phase error ϵ varies slowly compared to the fading process so that the phase-locked loop (PLL) is assumed to attain its steady state in every phase tracking interval. Mathematically, the Nakagami-m fading is characterized by the the probability density function (pdf) [2]

$$p(\alpha) = \frac{2m^m}{\Omega^m\Gamma(m)}\,\alpha^{2m-1}e^{-\frac{m}{\Omega}\,\alpha^2}, \quad \alpha \geq 0 \tag{2}$$

where $\Omega = E[\alpha^2]$ is the envelope average power, $\Gamma(\cdot)$ in the gamma function, and $m \geq 0.5$ is the fading severity parameter. The Nakagami-m distribution covers many fading distributions. For $m = 0.5$ it becomes one-sided Gaussian