An Efficient Numerical Technique for Error Rate Analysis of Wireless Systems

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The bit error probability (BEP) of binary and quaternary PSK (BPSK and QPSK) signals over Rayleigh fading channel with imperfect carrier phase recovery is derived. The channel is assumed to be slow and frequency-nonselective in which the received signal is constant over at least two symbol intervals. Numerical values for the average BEP are obtained by integrating the conditional BEP expression over the carrier phase error distribution. The resultant integrands have only elementary functions, such as exponentials and algebraic power functions and can, therefore, be explicitly evaluated to any degree of accuracy.

***Index Terms*— Carrier phase errors, Raleigh fading and PSK systems.**

I. Introduction

THE MOBILE propagation channel deviates from the classical additive white Gaussian noise (AWGN) model due to the multipath propagation. This multipath propagation affects the transmitted signal by introducing random fluctuations to both the signal amplitude and phase, causing what we call channel multipath fading. The Rayleigh distribution is usually used to characterize the amplitude randomness of the received signal based on the assumption that there is no line-of-sight path between the transmitter and the receiver sides. On the other hand, the introduced carrier phase is uniformly distributed over the interval $(0, 2\pi]$.

In order to detect the signal coherently, the channel has to be slow and frequency-nonselective, while the carrier phase should be synchronized and completely known at the receiver side. Generally, the carrier phase is derived either from a separate unmodulated pilot carrier or from the modulated signal itself. In both cases, the receiver uses a phase-looked loop (PLL) to acquire and track the carrier phase. Since the thermal noise always exists, the PLL will never estimate the carrier phase perfectly; and hence; degradation on the system performance appears. Two distributions of the random carrier phase error ε have been studied well in the literature: the Gaussian model and the Tikhonov model. The Gaussian model is suggested by the central limit theorem assuming that the phase error is a narrow-band filtered version of the thermal noise and the random phase modulation. The second model; Tikhonov model; was shown to be much suitable when a first order PLL is used to synchronize the carrier phase from a pilot carrier in the presence of thermal Gaussian noise [1]. The Tikhonov model will be considered in the analysis.

Unfortunately, in the presence of carrier phase error with Tikhonov distribution, no closed form solution exists for the demodulator decision variable probability density function (PDF) [2]. Many authors attempt to evaluate the BEP of PSK signals in the presence of carrier phase error recovery. Prabhu [3] presented an efficient infinite series for computing the BEP of BPSK and QPSK signals. His approach is based on expanding the error function complementary in Fourier series which is shown to be strictly alternating and fast converging series. In [4] Kaplan and Ram have derived a Chernoff upper bound for BEP of non-faded BPSK system and [5] extended the bounds for faded wireless environment. An optimum value for a parameter λ is to be computed for each signal-to-noise ratio (SNR) γ_b and each loop SNR γ_l. Approximate results for BPSK BEP in the phase error environment were derived by Kam [6], *et, al* for non faded channel and this approximation gives good results only for high γ_l with either a comparable low γ_b or high γ_b (i.e. for $\gamma_l >> 1$ and $\gamma_b < \gamma_l / 2$ or $\gamma_b >> \gamma_l / 2$). However, none of the author above included the channel fading gain into the analysis. Recently, Najib and Prabhu have presented in [7] a technique based on Gram-Charlier series expansion for the evaluation of the BEP of the equal gain combining (EGC) with imperfectly coherent Rayleigh PSK signals (our case is a special case of EGC with the number of diversity channels is equal to 1). This approach is a little bit complicated and needs high precision computing either at small number of diversity channels or at high γ_b. Finally, the authors in [8] gave approximate solution to characterize the performance of partially coherent QPSK system.

In this paper, the BEP of BPSK and QPSK signals over Rayleigh fading channel with imperfect carrier phase recovery is derived. The results are obtained by numerically integrating the conditional BEP expression over the carrier phase error pdf. And since nowadays many highly efficient numerical analysis software packages exist; our results are readily obtainable with any degree of accuracy without using any approximations or bounds.