Performance Analysis of QPSK System with Nakagami Fading Using Efficient Numerical Approach

Mahmoud A. Smadi
Department of Electrical Engineering
The Hashemite University
Zarqa, Jordan 13115

ABSTRACT

An efficient numerical method is developed to analyze the bit error rate of PSK Nakagami-*m* fading systems with imperfect carrier recovery. We evaluate the detection loss suffered due to the carrier recovery for different rms phase error values when coherent QPSK system are used in wireless channels. Our results are useful in the design of practical systems and will enable designers to determine the phase precision of PSK systems in wireless environments.

I. Introduction

It is well known that the Nakagami-m distribution [1] usually fits some empirical fading data that cannot be modeled by either Rayleigh or Rician distributions. Basically, this type of fading gives the best fit to land and indoor-mobile multipath propagation [2], [3] and scintillating ionospheric radio links [4]. On the other hand, the phase error recovery problem and its negative effect on the performance of digital coherent systems has been studied previously. The most common model to characterize the phase error is the Tikhonov model. Such model was shown to be much suitable when a first order phase-locked loop (PLL) (or even a second order PLL) is used to synchronize the carrier phase from a pilot carrier in the presence of thermal Gaussian noise [5]. Hence, the Tikhonov model will be considered in this paper. Note that we will refer to such systems corrupted by phase error recovery as non-ideal systems.

Dating back to the early 1960s, many research studies have analyzed the bit error rate (BER) of non-ideal binary and quaternary phase-shift keying (BPSK and QPSK) in additive white Gaussian noise (AWGN) channel [5] -[6]. Since exact evaluation is usually complex, the BER of non-ideal PSK systems was obtained by numerically integrating the conditional BER expression for a fixed phase error over the phase error statistic [5], [7]. Later on, many authors have devoted this problem by using infinite series approaches [8], [9], giving approximate solutions [10], [11], and [12], or deriving upper and lower bounds [8], [13], and [6].

The previous studies did not take channel fading into account, a phenomenon important in wireless communications.

Such extension was first targeted by [14], but provided only limited details. The authors in [15] used Maclaurin series to obtain accurate approximation for the average BER of non-ideal BPSK and QPSK for several channel fading models. But in their analysis, linear fading channel behavior over the entire region of the instantaneous SNR was assumed, as well as, linearly increased tracking-loop SNR ρ with respect to the thermal noise SNR γ_b . However, for analytical purposes, it is desirable to have a good approximation, or tight bounds with less restrictive assumptions on the system parameters making them valid to arbitrary values of ρ and γ . Most recently, we evaluate the error rate performance for non-ideal BPSK and QPSK systems but under Rayleigh fading channel [16]. However, as we mentioned before, many wireless environments are not accurately described by Rayleigh fading, whereas, Nakagami-m model gives better description to these environments.

In this paper, the BER of non-ideal QPSK system over Nakagami-*m* fading channel is derived based on the numerical approach discussed in [16]. The approach makes use of the fact that many highly efficient numerical analysis software packages exist nowadays. The paper is organized as follows: the receiver model structure is given in section II. The error rate performance for QPSK when the carrier phase error is subject to Tikhonov distribution is derived in section III. Moreover, numerical results with discussion were given in section IV. Finally, our conclusion remarks are in section V.

II. RECEIVER MODEL

Consider a general coherent M-ary PSK system. Hence, the l-th received complex envelope can be written as

$$r_l(t) = g(t)s_l(t) + n(t), \quad l = 1, 2, \dots, M$$
 (1)

where

$$s_l(t) = u(t)e^{j\theta_l},\tag{2}$$

is the transmitted symbol, in which θ_l is the modulation phase of the l-th symbol. A rectangular signaling pulse u(t) is assumed over the symbol interval T. That is

$$u(t) = \sqrt{2P_s}, \quad 0 \le t \le T \tag{3}$$