A Unified Semi-Analytical Technique to Evaluate the Distributions of Products of Non-Identically Distributed Weibull Random Variables

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Abstract: In this paper we derived the probability density function of the product of independent non-identically distributed Weibull RVs. The derivation is based on the use of the Mellin transformation along with the Residue theorem to evaluate the attained complex integral. The resultant integrands have only elementary functions, such as exponentials and algebraic power functions and can, therefore, be explicitly evaluated to any degree of accuracy using any symbolic mathematics based software. The distribution of the product of independent and identically distributed (i.i.d) Weibull RVs are also evaluated. The proposed method avoids the computational burden of residuals of a product of functions. Hence our results are expected to be of significant practical use to evaluate the error rate performance of any M-ary orthogonal faded systems in indoor and outdoor wireless environments.

Keywords– Weibull distribution; channel modeling and characterization; wireless communications.

I. INTRODUCTION

The two-parameter Weibull distribution is widely used in reliability and life data analysis due to its flexibility. An important aspect of the Weibull distribution is how the values of the shape parameter, α, and the scale parameter, β, affect such distribution characteristics as the shape of the pdf curve, the reliability and the data failure rate. In particular, the Weibull model usually fits some empirical fading data for both indoor and outdoor mobile multipath propagation environments [1, 2]. Also, Weibull distribution includes the well-known Rayleigh and exponential distributions as special cases. Moreover, it has been recommended by the IEEE Vehicular Technology Society to compensate any limitations of the Rayleigh model in theoretical studies [3]. However, just recently, Weibull fading over wireless communication channels begun to receive rehabilitated interest in the literatures.

The product of independent random variables takes place in many applications of wireless communications to model the channel or to characterize the receiver combining technique. In terms of channel modeling, the cascaded channel of a multihop wireless relaying system using the amplify-and-forward protocol with fixed amplification factor can be modeled as the product of the RVs that describe the channel gains [4]. Also, in a MIMO keyhole system, the electromagnetic wave propagates through several keyholes can be modeled as the product of the RVs that describe the individual keyhole channels [5]. On the other hand, the well-known product combining technique, which is employed in fast frequency hopping (FFH), is mathematically characterized by the product of the fading amplitudes [6, 7]. More generally, the received signal in cascaded wireless systems is a function of the product of the channel fading gains [8-11]. Hence, it is of great importance to develop an efficient analytical technique to evaluate the distribution of product of RVs.

Previous works on the distribution of the product of independent RVs include the following. In [6], the PDF of the product of independent Rayleigh RVs is derived using the Mellin transform to evaluate the error rate performance M-FSK system under FFH. In [8], [9] and [10], The Moments-generating function method along with the Meijer's G-function were used to evaluate the distributions of the product of several independent RVs. In [11], the level crossing rate of the product of independent N-Rayleigh envelopes was derived in terms N-folds complex integral. The fact that the product of Fox's H-functions is also an H-function was used to derive the distributions of product of independent generalized Nakagami-m variables in [12]. Moreover, the authors in [13] derived an infinite series solution for the PDF of product of two correlated K_G RVs. Recently, the authors in [14] derived closed-form approximation for the distributions (PDF and CDF) of independent fading variables including generalized Gamma, Nakagami-m and Gaussian distributions. The previous work is based on the Mellin transform to derive the distributions of the product of the specified RVs. The same Mellin transform method was used in [15] to calculate the PDF of product of independent Rayleigh and Nakagami-m variables and the results were given in terms of infinite series expansion.

All the pervious works expressed the distribution of product of RVs using infinite series, special functions or approximate solution methods. Moreover, all of them consider the case of the product of identically distributed RVs, which is not of practical interest in real wave propagation scenario. In this paper, we generalized the analysis to consider the case of non-identically distributed products of Weibull RVs. To the best of our knowledge, such analysis has not been tackled in the literature before.