A New Four-Dimensional Chaotic Attractor

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***Abstract*—A new four-dimensional chaotic system is introduced in this paper, it is mainly consist of four multiplier terms. The fundamental characteristics of the new system are investigated by means of equilibrium points, their stabilities, and power spectrum. Furthermore, an optimal controller using Riccati equation is established to run trajectories to the origin. The dynamic of the new system is simulated using Matlab and Simulink.**

Keywords—Chaos attractor; Lyapunov exponents; equilibrium points; Riccati equation; and stability.

I. INTRODUCTION

Chaotic systems are nonlinear systems which are sensitive to initial conditions and exhibits rich dynamic behavior [1]. Furthermore, a chaotic attractor is defined as a chaotic set toward which a dynamic system tends to evolve [2]. Chaos systems have wide range of applications in many Engineering and non-Engineering fields, some of its applications are found in intelligent control [3], power systems [4], secure communication [5], biology [6], and mathematics [7].

Furthermore, chaotic attractors were discovered by Lorenz while he was studying atmospheric convection, then, he introduced the first three-dimensional chaotic system in 1963 [8]. Afterwards, Rossler continued this work of dissipative dynamical system and proposed a new chaotic system in 1976 [9]. More work has been conducted since then, for example Chen introduced a new three-dimensional attractor in 1999 which is not topologically equivalent to Lorenz system [10].

Proposing of new chaotic attractors with new structures and dynamics is very useful to field of chaos theory and its application [3, 4, 5]. In this paper, a new four-dimensional chaotic system is introduced; it is mainly consist of four multiplier terms and four simple terms. The contribution of this work is that it propose a novel system which has different structure and topology of existing four-dimensional systems [11, 12, 13] The fundamental characteristics of the new system are investigated by means of equilibrium points, their stabilities, and power spectrum. Further investigations of the system are performed in next section. However, the system is completely new and does not belong to a known family of known systems.

The rest of paper is organized as follows. In section II, the new four-dimensional chaotic system is presented. In section III, the numerical analysis and simulation of system dynamics are shown. In section IV, an optimal controller design based on Riccati equation is derived. Finally, conclusions are presented in section V.

II. THE NEW 4D CHAOTIC SYSTEM

First, The new chaotic system has the following set of four dynamic equations:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 - x_1x_3 \\ \dot{x}_3 &= x_1x_2 - x_2x_4 \\ \dot{x}_4 &= x_2x_3 - x_3 \end{aligned} \tag{1}$$

where $x = [x_1, x_2, x_3, x_4]^T$ is the state vector of the system. The significance of the system in (1) is that it present a new 4D chaotic system with a unique dynamic. The equilibrium points are found by setting equation (1) to zero. This produces two equilibrium points: (0, 0, 0, 0) and (1, 1, b, 1). In order to study their stabilities the Jacobian of (1) is found as follows:

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ b-x_3 & 0 & -x_1 & 0 \\ x_2 & x_1-x_4 & 0 & -x_2 \\ 0 & x_3 & x_2-1 & 0 \end{bmatrix} \tag{2}$$

The linearized system at first equilibrium point (0, 0, 0, 0) is given by the following Jacobian matrix:

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \tag{3}$$

The characteristics equation for (0, 0, 0, 0) is $\lambda^2(\lambda^2 + a\lambda - ab)$ and the roots are given as double poles at origin and the other poles at $\frac{-a \mp \sqrt{a^2+4ab}}{2}$ which makes the point is unstable point.

The linearized system at second equilibrium points (1, 1, b, 1) is given by the following Jacobian matrix:

$$J=\begin{bmatrix} -a & a & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & b & 0 & 0 \end{bmatrix} \tag{4}$$

Moreover, The characteristics equation for the point is $\lambda^4 + a\lambda^3 + (a-b)\lambda - ab$ and applying Routh-Hurwitz criterion yield that this point is $V(t) = V(0)e^{-at}$ stable only for $a > 0$ and $ab < 0$.

The new system is proved to be dissipative system for all positive a since the divergence of flow of the system is:

$$\frac{1}{V}\frac{dV}{dt} = divV = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} + \frac{\partial \dot{x}_4}{\partial x_4} = -a \tag{5}$$

And $V(t) = V(0)e^{-at}$ with rate of contraction $\frac{dV}{dt} = V(0)e^{-a}$. Equation (5) prove the existence of a bounded and attracting chaotic set that form the attractor.

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III. Prepare Your Paper Before Styling

The new chaotic system has been tested for a wide range of parameters values and proved to yield chaotic behavior for many selections of parameters a and b. By choosing $a = 23, b = 9$ the chaotic system (1) is dissipative and the two equilibrium points are unstable. The portrait for two dimensions and three dimensions, for initial conditions (1, 1, 1, 1), are shown below. First, the phase-plane for x2-x1 is shown in figure 1, and the phase-plane for x4-x3 is shown in figure 2.

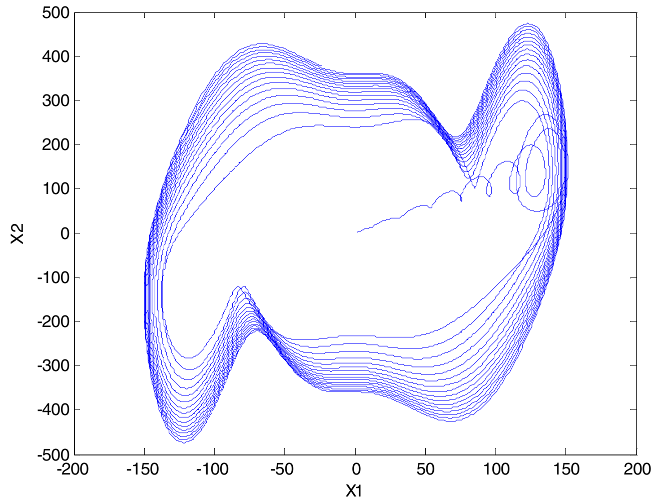

Fig. 1. A new chaotic attractor x2-x1 phase plane.

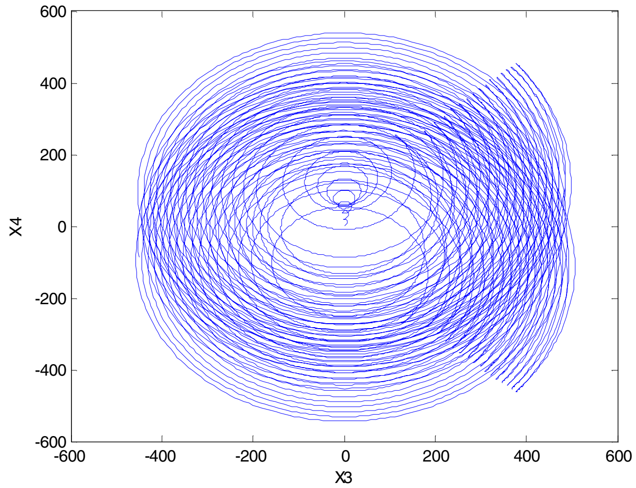

Fig. 2. A new chaotic attractor x4-x3 phase plane.

Then, the 3D portraits are shown in figures 3, 4, and 5, for different arrangements of the axes.

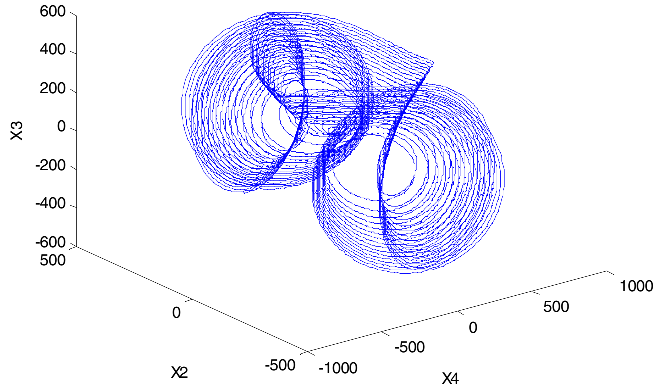

Fig. 3. Three-dimensional x3-x2-x4 portrait

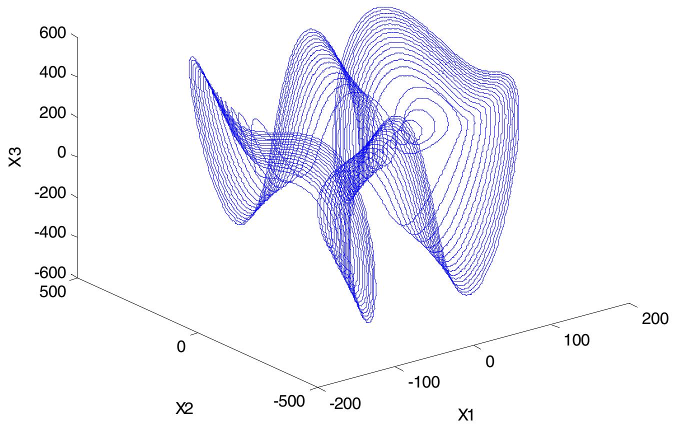

Fig. 4. Three-dimensional x3-x2-x1 portrait.

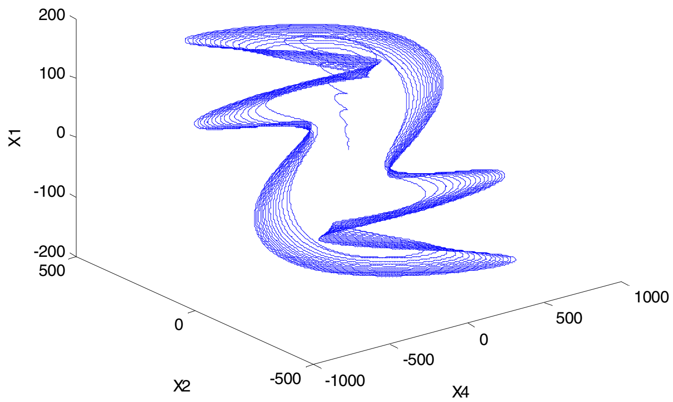

Fig. 5. Three-dimensional x1-x2-x4 portrait.

The power spectrum of signal describes how the variance of the data is distributed over the frequency domain [11]. Figure 6 shows the power spectrum of the signal x1(t) of the system (1). In this system the bandwidth is roughly between about 0–50 Hz,

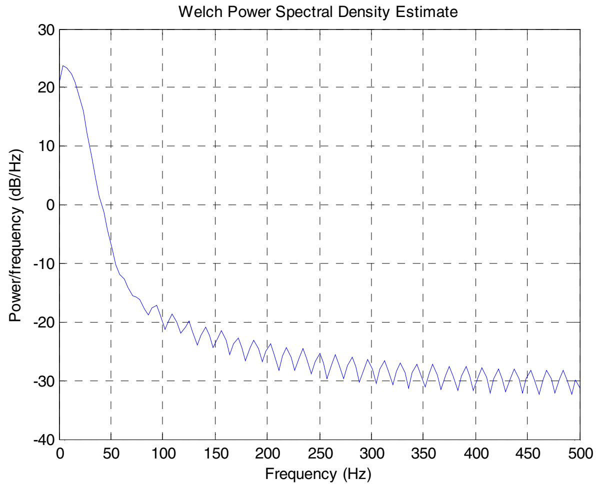

Fig. 6. Power spectral density.

IV. OPTIMAL CONTROL DESIGN

Linear Quadratic Regulator (LQR) is a popular case of optimal control where a measure of the quadratic continuous time cost function

$$J = \frac{1}{2}\int_0^{\infty}\left[X^T(t)Q(t)X(t) + u^T(t)R(t)u(t)\right]dt \qquad (6)$$

is minimized. Subject to linear dynamic constraints as given in the linearzed Jacobian system discussed in section II. Both Q and R matrices are positive definite matrices to ensure the cost measure remain positive. Furthermore, the negative feedback controller, shown in Fig 7, is in the form

$$u(t) = -Kx(t) \qquad (7)$$

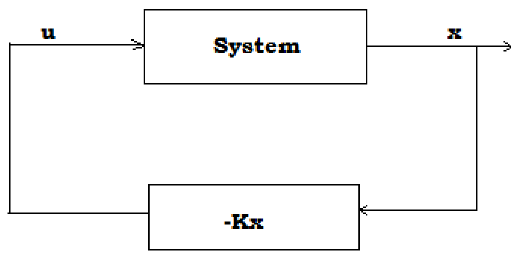

Fig. 7. System block diagram with optimal controller.

It has been shown in optimal control theory that the feedback controller K is given by $K = R^{-1}B^T S$ where S is the solution of the well known Algebraic Riccati equation

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \qquad (8)$$

Where S is symmetrical solution matrix [10, 11]. This controller is calculated around every point of the trajectory. Figures 8 and 9 show the four trajectories with the controller is applied after 3 second to drive the trajectories to the zero equilibrium.

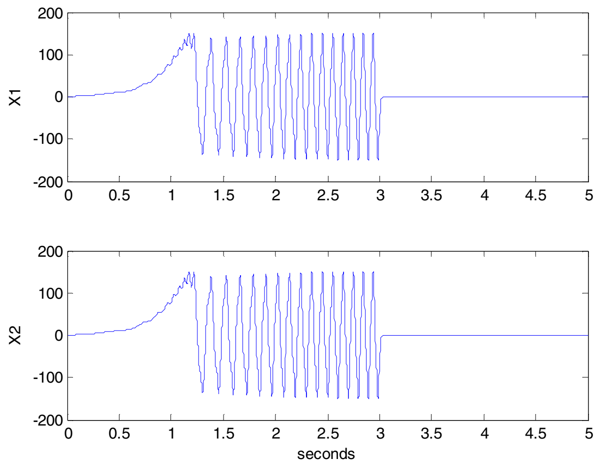

Fig. 8. x1 and x2 trajectories with controller after 3 sec.

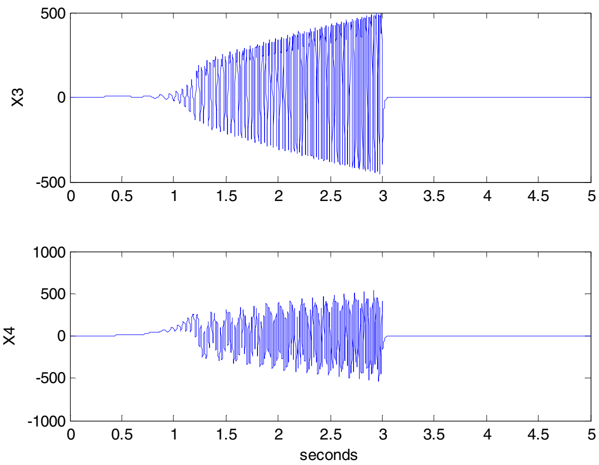

Fig. 9. x3 and x4 trajectories with controller after 3 sec.

V. CONCLUSIONS

A new chaotic system is introduced in this paper, it has four multiplier terms and four simple terms. The system produces two equilibrium points at (0, 0, 0, 0) and (1, 1, b, 1). The new chaotic system has been tested for a wide range of parameters values and proved to yield chaotic behavior for many selections of parameters a and b. The new system is proved to be dissipative system for all positive a and for all values of b. Therefore, choosing $a = 23, b = 9$ makes the chaotic system (1) dissipative and the two equilibrium points unstable. Optimal controller, based on Riccati equation, is designed and controlled system trajectories to the zero equilibrium after applying the controller.

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