DESIGN OF DIGITAL CONTROLLERS FOR UNCERTAIN CHAOTIC SYSTEMS USING FUZZY LOGIC

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ABSTRACT

A new and systematic method to design digital controllers for uncertain chaotic systems with structured uncertainties is presented in this paper. Takagi-Sugeno (TS) fuzzy model is used to model the chaotic dynamic system, while the uncertainties are decomposed such that the uncertain chaotic system can be rewritten as a set of local linear models with an additional disturbed input. Conventional control techniques are utilized to develop the continuous-time controllers first. Then, the digital controllers are obtained as the digital redesign of the continuous-time controllers using the state-matching approach. The performance of the proposed controller design is illustrated through numerical examples.

1. INTRODUCTION

Real world systems are uncertain in general. However, it is a common practice to describe a real system using nominal linear models; the primary reason to do this is the advantage of using the well-developed linear techniques for analysis and design of the controller. Unfortunately, under some condition like uncertainties, modeling errors, noise and other disturbances, the nominal linear approach might fail to produce satisfactory results. To overcome this problem, it is critical to design and implement controllers that take into consideration these aspects of the system's dynamics.

Digital controller in a state-matching sense was originally proposed by [8]. During the last two decades Shieh et al. proposed different digital redesign. A more suitable representation of a real world system is a continuous-time parametric-uncertain model with bounded disturbances and noise inputs. For this representation, most of the research efforts have been concentrated on the design of continuous-time controllers for continuous-time systems or discrete-time controllers for discrete-time systems. In this paper we consider the hybrid case; our objective is to design a discrete-time controller for a continuous-time uncertain chaotic system with structured uncertainties using fuzzy logic.

The idea of developing a discrete-time controller from a previously designed continuous-time one [12] had been applied to different types of systems, among them are PWM controllers [10], cascaded analog controllers [11], delayed systems [5], chaotic systems [4].

The application of digital redesign to chaotic systems was proposed on earlier works. However, the problem of applying digital redesign technique to control chaotic systems with uncertainties using fuzzy modeling has not been addressed. On this regard, the methodology proposed in this work is to decompose and incorporate the uncertainty in the system. Next, using Takagi-Sugeno (TS) fuzzy modeling, the uncertain chaotic system is expressed as set of linear models, then, for each linear model a controller is designed as the solution to a linear matrix inequality problem.

2. FUZZY MODELING

In this section we will discuss the fuzzy modeling of chaotic systems. The Takagi-Sugeno (TS) fuzzy model is used. In addition Chua's circuit is utilized to demonstrate the application of TS model to chaotic system modeling, We represent the chaotic system dynamics by set of local relations in the state space, The TS model represents every fuzzy rule by a linear model.

Consider a family of chaotic systems of the form

$$\dot{x}(t) = f(x) + g(x)u(t) \tag{1}$$

where $f:\Re^n \to \Re^n$ and $g:\Re^n \to \Re^{n\times m}$ are chaotic functions, $x(t) \in \Re^n$ is the state vector, and $u(t) \in \Re^m$ is the control input.

The TS system is described by IF-THEN statement[13]. Every rule represents a linear model of the system as follows

$$\begin{aligned} &Rule\ j : IF\ x_1(t)\ is\ M_1^j .....and\ x_n(t)\ is\ M_n^j \\ &THEN\ \dot{x}(t) = A_j x(t) + B_j u(t) \end{aligned} \tag{2}$$

where

$$j = 1, 2, ..., q,$$
$$x(t) = [x_1, x_2, ...x_n]^T,$$
$$u(t) = [u_1, u_2, ...u_m]^T.$$

The M_j is the fuzzy set and q is the number of rules. Given a pair of $(x(t), u(t))$, the fuzzy system is inferred as the following

$$\dot{x}(t) = A_j x(t) + B_j u(t) \tag{3}$$

where

$$A=\sum_{j=1}^{q}\mu_j(x)A_j, \qquad B=\sum_{j=1}^{q}\mu_j(x)B_j \text{ and}$$

$$\mu_j=\frac{\omega_j(x)}{\sum_{j=1}^{q}\omega_j(x)}\geq 0, \qquad \sum_{j=1}^{q}\omega_j(x)=1, \qquad \omega_j(x)=\prod_{j=1}^{q}M_j^i$$

The input system is given by

$$\dot{x}(t)=\frac{\sum_{j=1}^{q}\omega_j(x)\left(A_jx(t)+B_ju(t)\right)}{\sum_{j=1}^{q}\omega_j(t)}. \qquad (4)$$

Fuzzy modeling of Chua's chaotic circuits are developed as explained below. The electronic circuit of figure 1 is the realization of one of the most widely used benchmarks for chaotic dynamics [3]. There are different mathematical representations of Chua's circuit, one is suggested in equation (5).

$$\begin{aligned}\dot{v}_1(\tau)&=\frac{1}{C_1}\left[\frac{1}{R}\left(v_2(\tau)-v_1(\tau)\right)-f_{NL}(v_1(\tau))\right]\\ \dot{v}_2(\tau)&=\frac{1}{C_2}\left[\frac{1}{R}\left(v_1(\tau)-v_2(\tau)\right)+i_L(\tau)\right]\\ \dot{i}_L(\tau)&=-\frac{1}{L}v_2(\tau)\end{aligned} \qquad (5)$$

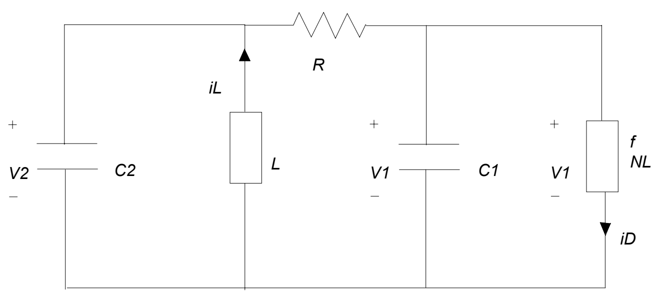

Figure 1. *Chua's circuit realization.*

Chua's circuit have two form: piecewise linear and nonlinear. These are considered below.

Piecewise-linear case:

In the piecewise-linear case, f_{NL} represents the nonlinear resistance of the circuit, which is represented by a piecewise-linear function and expressed as follows:

$$f_{NL}(v_1(\tau))=g_bv_1(\tau)+\frac{1}{2}\left(g_a-g_b\right)\left(\left|v_1(\tau)+E\right|-\left|v_1(\tau)-E\right|\right) \qquad (6)$$

where $g_a, g_b<0$. Or it can be expressed more conveniently as

$$f_{NL}(v_1(\tau))=\begin{cases} g_bv_1(\tau)+\left(g_a-g_b\right)E & v_1(\tau)\geq E\\ g_av_1(\tau) & -E<v_1(\tau)<E\\ g_bv_1(\tau)-\left(g_a-g_b\right)E & v_1(\tau)\leq -E\end{cases}$$

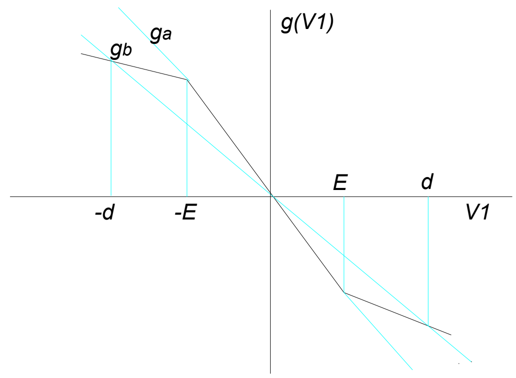

Figure 2. *Piecewise-linear representation of the Chua's resistor circuit realization.*

At this point we need to obtain the fuzzy model of Chua's chaotic system. Considering $f_{NL}(v_1(\tau))\in[-d,\, d],\ d>E>0,$ as shown in Figure 2, we obtain the following bounds

$$f_a(v_1(\tau))=g_av_1(\tau)$$

$$f_b(v_1(\tau))=\left(g_av_1+\frac{\left(g_a-g_b\right)E}{d}\right)(v_1(\tau))=G_{ab}(v_1(\tau)).$$

When $g_a=g_b$, the Chua's system becomes linear, otherwise we use the trapezoidal membership functions shown in Figure 3 to model the Chua's system.

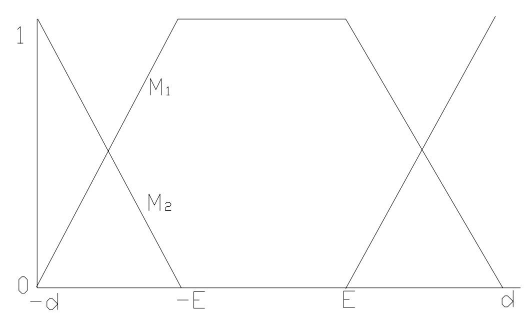

Figure 3. *Membership functions.*

Assigning state vector as $x=[vc_1, vc_2, i_L]$, Chua's circuit of (5) can be represented using the following model

$$\begin{aligned}&Rule1: IF\ v_1(t)\ is\ M_1(v_1)\ (near\ 0)\\ &\qquad THEN\ \dot{x}(t)=A_1x(t)+B_1u(t)\\ &Rule2: IF\ v_1(t)\ is\ M_2(v_1)\ (near\ \pm d)\\ &\qquad THEN\ \dot{x}(t)=A_2x(t)+B_2u(t)\end{aligned} \qquad (7)$$

where

$$A_1=\begin{pmatrix}-\frac{1}{C_1R}-\frac{g_a}{C_1} & \frac{1}{C_1R} & 0\\ \frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2}\\ 0 & -\frac{1}{L} & 0\end{pmatrix}, \quad A_2=\begin{pmatrix}-\frac{1}{C_1R}-\frac{G_{ab}}{C_1} & \frac{1}{C_1R} & 0\\ \frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2}\\ 0 & -\frac{1}{L} & 0\end{pmatrix}$$

and $B_1 = B_2 = I_{3\times3}$. substituting $\alpha = \dfrac{1}{C_1 R}$ *and* $\beta = \dfrac{1}{L}$ will result in

$$A_1 = \begin{pmatrix} -\alpha\left(1-\dfrac{g_a}{C_1}\right) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix},\quad A_2 = \begin{pmatrix} -\alpha\left(1-\dfrac{G_{ab}}{C_1}\right) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}. \tag{8}$$

Nonlinear case:

In the nonlinear case f_{NL} represents the nonlinear resistance as follows $g_{NL}(x_1(t)) = m_1 x_1(t) + m_2 x_1(t)^3$ for $m_1 = m_a R$ and $m_2 = m_c R$.

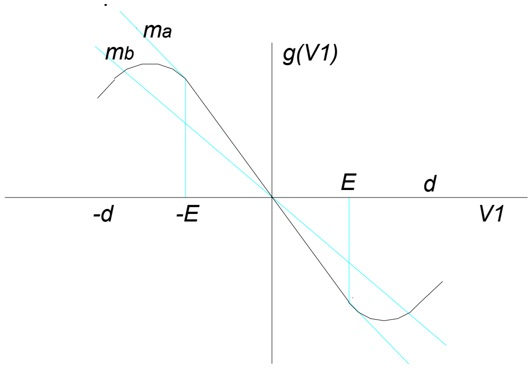

Figure 4. *Nonlinear representation of the Chua's resistor*

As in the previous case, consider $f_{NL}(v_1(\tau)) \in [-d,\, d],\ d > E > 0,$ we obtain the following bounds $g_1(v_1(\tau)) = m_a v_1(\tau),\ g_2(v_1(\tau)) = (m_1 + m_2 d^2) v_1(\tau) = m_{12} v_1(\tau)$. The membership function is given by [7]

$$M_1(v_1(\tau)) = 1 - \left(\frac{v_1(\tau)}{d}\right)^2,\quad M_2(v_1(\tau)) = 1 - M_1(v_1(\tau)) = \left(\frac{v_1(\tau)}{d}\right)^2.$$

Then the Chua's circuit can be represented using the following model

$$\begin{aligned} &Rule1: IF\ v_1(t)\ is\ M_1(v_1)\ (near\ 0) \\ &\qquad THEN\ \dot{x}(t) = A_1 x(t) + B_1 u(t) \\ &Rule2: IF\ v_1(t)\ is\ M_2(v_1)\ (near\ \pm d) \\ &\qquad THEN\ \dot{x}(t) = A_2 x(t) + B_2 u(t) \end{aligned} \tag{9}$$

where

$$A_1 = \begin{pmatrix} -\dfrac{1}{C_1 R} - \dfrac{m_a}{C_1} & \dfrac{1}{C_1 R} & 0 \\ \dfrac{1}{C_2 R} & -\dfrac{1}{C_2 R} & \dfrac{1}{C_2} \\ 0 & -\dfrac{1}{L} & 0 \end{pmatrix},\quad A_2 = \begin{pmatrix} -\dfrac{1}{C_1 R} - \dfrac{m_{12}}{C_1} & \dfrac{1}{C_1 R} & 0 \\ \dfrac{1}{C_2 R} & -\dfrac{1}{C_2 R} & \dfrac{1}{C_2} \\ 0 & -\dfrac{1}{L} & 0 \end{pmatrix},$$

and $B_1 = B_2 = I_{3\times3}$. Substituting $\alpha = \dfrac{1}{C_1 R}$ *and* $\beta = \dfrac{1}{L}$, will lead to

$$A_1 = \begin{pmatrix} -\alpha - \dfrac{m_a}{C_1} & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix},\quad A_2 = \begin{pmatrix} -\alpha - \dfrac{m_{12}}{C_1} & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}.$$

3. UNCERTAINTY DECOMPOSITION

In what follows, we present a method to incorporate the uncertainties of the chaotic system in the fuzzy linear model. In particular, we consider the case of parametric variations. This type of disturbance can be seen as a structured uncertainty of the system. This method consist of representing the uncertain chaotic system as a set of uncertain local linear models, then the uncertainty is decomposed and incorporated into the local models such that they can be rewritten as nominal linear models with an additional disturbance input.

Considering the following uncertain chaotic system of equation (1) and applying the fuzzy modeling discussed in section 2, the uncertain chaotic model of equation (9) can be represented by a set of local uncertain linear models of the form,

$$\dot{x}_c(t) = \tilde{A}_j\, x_c(t) + \tilde{B}_j\, u_c(t) = (A_{o,j} \pm \Delta A_j)\, x_c(t) + (B_{o,j} \pm \Delta B_j)\, u_c(t) \tag{10}$$

where $A_{o,j} \in \Re^{n\times n}$ and $B_{o,j} \in \Re^{n\times m}$ are the nominal system and nominal input matrices respectively, while $\Delta A_j \in \Re^{n\times n}$ and $\Delta B_j \in \Re^{n\times m}$ are unknown but bounded structured uncertainty matrices corresponding to the effects of the uncertain parameters.

The uncertainty matrices can be rewritten in terms of the uncertain elements Δ_{a_l} and Δ_{b_l} and the constant matrices $A_{j,l} \in \Re^{n\times n}$ and $B_{j,l} \in \Re^{n\times m}$ as

$$\Delta A_j = \sum_{l=1}^{k_a} \Delta_{a_l} A_{j,l} = M_{j,ac} \Delta_{j,a} N_{j,ar}, \tag{11}$$

$$\Delta B_j = \sum_{i=1}^{k_b} \Delta_{b_l} B_{j,l} = M_{j,bc} \Delta_{j,b} N_{j,br}. \tag{12}$$

From this representation and by letting $q_l \overset{\Delta}{=} rank(A_{j,l})$, $p_l \overset{\Delta}{=} rank(B_{j,l})$, the constant matrices $M_{j,ac}$, $M_{j,bc}$, $N_{j,ar}$ and $N_{j,br}$ are given by the equations

$$M_{j,ac} = [M_{ac,1},\ M_{ac,2},\ \ldots, M_{ac,k_a}]$$

$$M_{j,bc} = [M_{bc,1},\ M_{bc,2},\ \ldots, M_{bc,k_b}],$$

$$N_{j,ar} = \begin{bmatrix} N_{ar,1}^T & N_{ar,2}^T & \ldots & N_{ar,k_a}^T \end{bmatrix}^T$$

$$N_{j,br} = \begin{bmatrix} N_{br,1}^T & N_{br,2}^T & \ldots & N_{br,k_b}^T \end{bmatrix}^T$$

where $M_{ac,l} \in \Re^{n\times q_l}$ are the q_l nonzero column vectors of $A_{j,l}$, $M_{bc,l} \in \Re^{n\times p_l}$ are the p_l nonzero column vectors of $B_{j,l}$, while $N_{ar,l} = (M_{ac,l}^T M_{ac,l})^{-1} M_{ac,l}^T A_{j,l} \in \Re^{q_l\times n}$, $N_{br,l} = (M_{bc,l}^T M_{bc,l})^{-1} M_{bc,l}^T B_{j,l} \in \Re^{p_l\times m}$, $\Delta_{j,a} = block\ \ diag\ \ [\Delta_{a_1} I_{q_1},\ \Delta_{a_2} I_{q_2},\ \ldots, \Delta_{a_{k_a}} I_{q_{k_a}}]$ and

$\Delta_{j,b} = block \quad diag \; [\Delta_{b_1} I_{p_1},\; \Delta_{b_2} I_{q_2},\; \ldots, \Delta_{b_{k_b}} I_{p_{k_b}}]$. With I_{q_l} and I_{p_l}, being $q_l \times q_l$ and $p_l \times p_l$ identity matrices respectively.

Without loss of generality, we can assume that $\left|\Delta_{a_i}\right| \le 1$ and $\left|\Delta_{b_j}\right| \le 1$ for $i = 1, \; \ldots, \; k_a$; $j = 1, \; \ldots, \; k_b$. So that one can rewrite the uncertain linear system in equation (10) as a nominal linear system with a disturbance input as shown below

$$\begin{pmatrix} \dot{x}_c(t) \\ z_c(t) \\ y_c(t) \end{pmatrix} = \begin{pmatrix} A_{o,j} & B_{I,j} & B_{o,j} \\ C_{I,j} & 0 & D_{I,J} \\ C_{o,j} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_c(t) \\ \overline{w}_c(t) \\ u_c(t) \end{pmatrix} \tag{13}$$

where $B_{I,j} = \left[M_{j,ac}, M_{j,bc}\right]$, $C_{I,j} = \left[N^T_{j,ar} \quad 0\right]^T$, $D_{I,j} = \left[0 \quad N^T_{j,br}\right]^T$ and $C_{o,j} = I_n$.

From the uncertainty decomposition of (11) and (12) the fictitious disturbance input is given by

$$\overline{w}_c(t) = diag \;\; \left[\Delta_{j,a},\; \Delta_{j,b}\right] z_c(t).$$

A controller for the disturbed system (13) can be constructed using the general feedback structure. Where the disturbed local linear model (13) corresponds to the system under consideration and the objective is to find the feedback controller, in the form

$$\begin{pmatrix} \dot{\chi}_c(t) \\ u_c(t) \end{pmatrix} = \begin{pmatrix} A_{\chi,j} & B_{\chi,j} \\ C_{\chi,j} & D_{\chi,j} \end{pmatrix} \begin{pmatrix} \chi_c(t) \\ y_c(t) \end{pmatrix} \tag{14}$$

The closed-loop the system described in equations (13) and (14) should be internally stable. Moreover, the effects of the disturbed input $\overline{w}_c(t)$ on the desired output $z_c(t)$, measured in terms of the infinity norm of their transfer function $\left\|\hat{T}_{z_c\overline{w}_c}(s)\right\|_\infty$, should be less than a given bound $\gamma > 0$. Consequently, the closed-loop system becomes

$$\begin{pmatrix} \dot{x}_c(t) \\ z_c(t) \end{pmatrix} = \begin{pmatrix} A_{Cl} & B_{Cl} \\ C_{Cl} & D_{Cl} \end{pmatrix} \begin{pmatrix} x_c(t) \\ \overline{w}_c(t) \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} A_{o,j} + B_{o,j} D_{\chi,j} C_{o,j} & B_{o,j} C_{\chi,j} \\ B_{\chi,j} & A_{\chi,j} \end{bmatrix} & \begin{bmatrix} B_{I,j} \\ 0 \end{bmatrix} \\ \begin{bmatrix} C_{I,j} + D_{I,J} D_{\chi,j} C_{o,j} & D_{I,j} C_{\chi,j} \end{bmatrix} & 0 \end{pmatrix} \begin{pmatrix} x_c(t) \\ \chi_c(t) \\ \overline{w}_c(t) \end{pmatrix} \tag{15}$$

As proposed in reference [9], a static state feedback controller can be obtained by solving the following LMIs

$$Y > 0, \quad \begin{pmatrix} \left(A_{o,j} Y + B_{o,j} M\right)^T + \left(A_{o,j} Y + B_{o,j} M\right) & B_{I,j} & \left(C_{I,j} Y + D_{I,j} M\right)^T \\ B^T_{I,j} & -\gamma I & 0 \\ C_{I,j} Y + D_{I,j} M & 0 & -\gamma I \end{pmatrix} < 0. \tag{16}$$

A continuous-time state feedback controller for equation (1) can be constructed in the form

$$u_c(t) = -K_{c,j} x_c(t) \tag{17}$$

where the feedback gain $K_{c,j}$ is found from the solutions of (16) as $K_{c,j} = -M \; Y^{-1}$, around each operating point x_j of the nonlinear trajectory.

4. DIGITAL REDESIGN TECHNIQUES

Digital redesign can be defined as the process of converting a previously well-designed continuous-time controller to a discrete-time controller suitable for digital implementation. The states of the continuous-time and sampled-data closed-loop systems are matched at least at each sampling instant for the entire process. Consider a controllable and observable continuous-time linear system

$$\dot{x}_c(t) = A_{o,j}\, x_c(t) + B_{o,j}\, u_c(t), \qquad x_c(0) = x_0 \tag{18}$$

where $x_c(t) \in \Re^n$, $u_c(t) \in \Re^m$ and $(A_{o,j},\, B_{o,j})$ are the constant matrices of appropriate dimensions which form the linear model of a nominal chaotic system produced by the fuzzy modeling. The continuous-time controller $u_c(t)$ is the control law obtained in the previous section.

Applying equation (17) to equation (18) results in the continuous-time closed-loop system

$$\dot{x}_c(t) = (A_{o,j} - B_{o,j} K_{c,j}) x_c(t), \qquad x_c(0) = x_0 \tag{19}$$

Assume that we need to implement the continuous-time controller (17) using a digital system using a zero order hold device. The digitally implemented controller $u_d(t)$ is piecewise-constant, such that

$$u_d(t) = u_d(kT) = -K_{d,j} x_d(kT) \tag{20}$$

for $kT \le t < (k+1)T$ where $T > 0$ is the sample-hold period and $K_{d,j} \in R^{m\times n}$ is the digital feedback gain.

The discrete-time controlled closed-loop system becomes

$$\dot{x}_d(t) = A_{o,j} x_d(t) + B_{o,j}\left[-K_{d,j} x_d(kT)\right],\; x_d(0) = x_0. \tag{21}$$

Thus, the digital redesign problem is reduced to finding the digital gain $K_{d,j}$ in (20) from the continuous-time controller gain $K_{c,j}$ in (17), such that the closed-loop state $x_d(t)$ in (21) closely match the closed-loop state $x_c(t)$ in (19) at least for every sampling instant during the entire process.

Using prediction-based digital redesign method [6], we obtain the digital redesigned controllers

$$K_{d,j} = (I_m + K_{c,j} H_{0,j})^{-1} K_{c,j} G_{0,j} \tag{22}$$

where $G_{0,j} = e^{A_{0,j}T}$ and $H_{0,j} = (G_{0,j} - I_n) A^{-1}_{0,j} B_{0,j}$.

It is important to note that the matrices $(A_{0,j}, B_{0,j})$ are obtained from fuzzy modeling of the chaotic system and the matrices $(B_{I,j}, C_{I,j}, D_{i,j})$ are obtained from the uncertainty decomposition, while the discrete-time controller $u_d(kT)$ is calculated using the digitally redesigned gain $K_{d,j}$ in terms of the linear models obtained around each sampled value of the state as the operating point $x_j(kT)$.

5. NUMERICAL RESULTS

In this section, the proposed methodology will be used to find the digital tracker for two benchmark Chua's chaotic systems. The objective is to find a discrete-time controller that ensures tracking of the system in the presence of bounded parameter uncertainties.

Note that the parameters α and β of (7) and (9) are assumed to be uncertain but bound to a given interval. The controller $u_c(t)=\left[u_{c,1}(t),u_{c,2}(t),u_{c,3}(t)\right]^T$ for the uncertain Chua's circuit shown below

$$\begin{bmatrix}\dot{x}_{c1}(t)\\ \dot{x}_{c2}(t)\\ \dot{x}_{c3}(t)\end{bmatrix}=\begin{bmatrix}\tilde{\alpha}\left[x_{c2}(t)-x_{c1}(t)-g_{NL}(x_{c1}(t))\right]\\ x_{c1}(t)-x_{c2}(t)+x_{c3}(t)\\ -\tilde{\beta}x_{c2}(t)\end{bmatrix}+\tilde{B}\begin{bmatrix}u_{c,1}(t)\\ u_{c,2}(t)\\ u_{c,3}(t)\end{bmatrix} \quad (23)$$

is to be determined next.

The linear model of equation (23) can be rewritten using the fuzzy modeling as

$$\dot{x}_c(t)=\begin{cases}A_j^{*1}\left[\tilde{\alpha},\tilde{\beta}\right]\ x_c(t)+B_j^*\left[\tilde{B}_j\right]u_c(t), & (near\ 0)\\ A_j^{*2}\left[\tilde{\alpha},\tilde{\beta}\right]\ x_c(t)+B_j^*\left[\tilde{B}_j\right]u_c(t), & (near\ \pm d)\end{cases} \quad (24)$$

Here $\tilde{B}_j$ refers to the uncertainty of the input matrix of the local linear system, which will be treated as an extra uncertain parameter. Rewriting the uncertain parameters $(\tilde{\alpha},\tilde{\beta},\tilde{B}_j)$ as $\tilde{\alpha}=\alpha_0+\Delta\alpha_I$, $\tilde{\beta}=\beta_0+\Delta\beta_I$ and $\tilde{B}_j=B_0+\Delta B_I$ where $\alpha_0=\frac{19}{2}$, $\beta_0=\frac{100}{7}$, $B_0=I_{3\times3}$, $\alpha_I=\frac{19}{40}$, $\beta_I=\frac{5}{7}$, and $B_I=\frac{5}{100}$. the linear model of the uncertain Chua's circuit can be written as:

$$\dot{x}_c(t)=\begin{cases}\left(A_{0,j}^{*1}+\Delta A_{I,j}^{*1}\right)\ x_c(t)+\left(B_{0,j}^{*}+\Delta B_{I,j}^{*1}\right)u_c(t), & (near\ 0)\\ \left(A_{0,j}^{*2}+\Delta A_{I,j}^{*2}\right)\ x_c(t)+\left(B_{0,j}^{*}+\Delta B_{I,j}^{*1}\right)u_c(t), & (near\ \pm d)\end{cases} \quad (25)$$

with $A_{0,j}^{*1}=A_j^{*1}\left[\alpha_0,\beta_0\right]$, $A_{0,j}^{*2}=A_j^{*2}\left[\alpha_0,\beta_0\right]$ and $B_{0,j}^{*}=B_j^{*}\left[B_0\right]$. Note that both piecewise and nonlinear Chua's circuits has the same treatment since they differ only in the item $i_{1,1}$ of the system matrix A as shown in equations (7) and (9). Using the method presented in section 3, the uncertainties of equation (25) can be decomposed into the following matrices:

$$M_{ac,1}^*=M_{ac,2}^*=M_{ac,3}^*=\begin{cases}\left\{\begin{bmatrix}-\alpha_I\left(g_b-g_a\right)\\ 0\\ 0\end{bmatrix}\right\} & for\ piecewise\\ \left\{\begin{bmatrix}-\alpha_I\left(g_2-g_1\right)\\ 0\\ 0\end{bmatrix}\right\} & for\ nonlinear\end{cases}$$

$$N_{ar,1}^*=N_{ar,2}^*=N_{ar,3}^*=\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix},\quad N_{br,1}^*=N_{br,2}^*=N_{br,3}^*=1,$$

$$M_{ac,1}^*=M_{ac,2}^*=M_{ac,3}^*=\begin{cases}\left\{\begin{bmatrix}-\alpha_I\left(1+g_a/C_1\right)&\alpha_I\\0&0\\0&-\beta_I\end{bmatrix},\ (near\ 0)\quad \begin{bmatrix}-\alpha_I\left(1+G_{ab}/C_1\right)&\alpha_I\\0&0\\0&-\beta_I\end{bmatrix}\ (near\ \pm d)\right\} & for\ piecewise\\ \left\{\begin{bmatrix}-\alpha_I\left(1+m_a/C_1\right)&\alpha_I\\0&0\\0&-\beta_I\end{bmatrix},\ (near\ 0)\quad \begin{bmatrix}-\alpha_I\left(1+m_{12}/C_1\right)&\alpha_I\\0&0\\0&-\beta_I\end{bmatrix},\ (near\ \pm d)\right\} & for\ nonlinear\end{cases}$$

The control objective is to track a periodic orbit corresponding to a limit cycle known to encircle the double-scroll [2] given by the formula

$$\begin{aligned}x_{r1}(t)&=a\ \ \cos(\xi)\cos(wt)-b\ \ \sin(\xi)\sin(wt)+\\ &\quad c\ \ \cos(\xi)\cos(2wt)-d\ \ \sin(\xi)\sin(2wt)\\ x_{r2}(t)&=e\left[a\ \ \sin(\xi)\cos(wt)-b\ \ \cos(\xi)\sin(wt)\right]+\\ &\quad f\ \left[c\ \ \sin(\xi)\cos(2wt)-d\ \ \cos(\xi)\sin(2wt)\right]\\ \dot{x}_{r3}(t)&=-\beta\ \ x_{r2}(t)\end{aligned} \quad (26)$$

where $a=2.6$, $b=1.2$, $c=d=0.2$, $e=0.6$, $f=0.3$, $\xi=\pi/18$ and $w=1.77$.

The tracking error can be defined as

$$x_e(t)=x_c(t)-x_r(t) \quad (27)$$

Then, the continuous-time tracking controller is found to be

$$u_c(t)=-K_{c,j}x_e(t) \quad (28)$$

where the feedback gain is found as the solution to the linear matrix inequality (16) constructed in terms of tracking error variables.

Figures 5 through 8 show the trajectories under digitally redesigned controllers and continuous/digital control laws. These shown for both piecewise and nonlinear circuits. It is obvious from the figures below this methodology is capable of achieving good tracking performance even in the presence of parameter uncertainties. Furthermore, the discrete-time implementation obtained via state-matching digital redesign allows for a compatible performance between the continuous-time design and the digital implementation for significantly large sampled-hold period as shown in figure (6) and Figure (8).

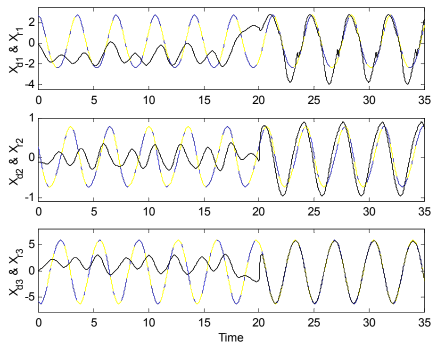

Figure 5. *Piecewise-linear Chua's circuit trajectories under digitally redesigned control.*

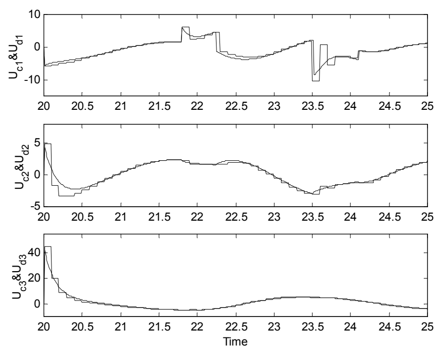

Figure 6. *Continuous and digitally redesigned control laws for piecewise-linear Chua's Circuit.*

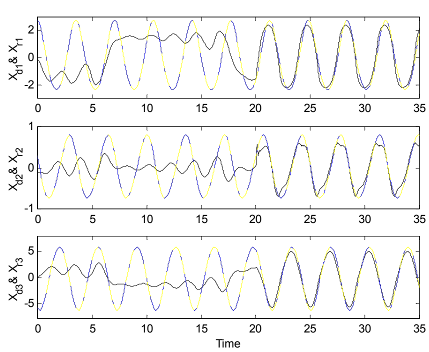

Figure 7. *Nonlinear Chua's circuit trajectories under digitally redesigned robust control.*

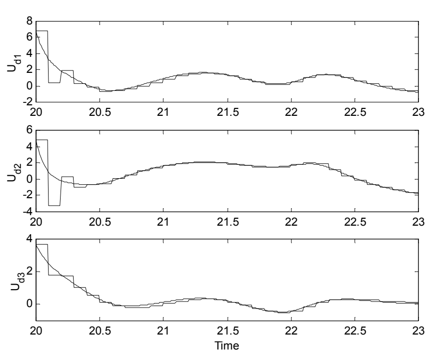

Figure 8. *Continuous and digitally redesigned robust control laws for nonlinear Chua's Circuit.*

6. CONCLUSIONS

In this paper a systematic methodology to design a discrete-time controller for a continuous-time uncertain chaotic system with structured uncertainties was presented. The proposed method consists of four basic steps: First, an alternative representation of the uncertain chaotic system is obtained applying the TS fuzzy modeling. Secondly, decomposing the uncertainties of the local model is carried out, such that the system can be rewritten as an uncertain linear system with an additional disturbed input. Then, a continuous-time controller, that makes the local model internally stable and satisfies a performance index given in terms of the infinite norm, is obtained as the solution of a linear matrix inequality problem. Finally, applying the recently proposed prediction-based digital redesign method, a discrete-time controller is found from the continuous-time design in the state-matching sense. The results of the proposed method demonstrated through the given numerical examples have shown good tracking performance. In the next stage of research the performance of the proposed method will be analyzed quantitatively including the system tracking error. Furthermore, the error resulting from different system parameters will be investigated.

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