The Role of Interface Element for Strip Plate on Elastic Media

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ABSTRACT: The influence of interface conditions in static problems of soil-structure interaction is demonstrated using a simple thin layer element. Plates of infinite length resting on different supporting medium and subjected to various loadings have been analyzed to reflect such influence. The far domain of the elastic half-plane is modeled by three-nodded infinite element, and the contact between the plate and the supporting medium has been simulated by incorporating thin-layer element.

Comparisons are made with closed-form solutions, and with other models of the supporting medium, from such comparisons the versatility of the proposed thin-layer element and infinite is clearly achieved.

KEY WORDS: Elastic Media, Infinite Element, Interface Element, Soil-Structure Interaction, Thin-Layer Element.

INTRODUCTION

There are numerous problems that involve soil structure interaction, e.g. soil-footing, and soil-culvert systems.

In the context of the finite element procedure, special interface or joint elements are used in order to account for the relative motions, and associated deformation modes.

One of the commonly used interface elements in soil-structure interaction is based on the joint element proposed by (Goodman, 1968). The element formulation is derived on the basis of relative nodal displacements of the solid elements surrounding the interface element, the thickness of the element is assumed to be zero.

Ghaboussi and Wilson, (1973) proposed a formulation which is derived by considering relative displacements as the independent degree of freedom for the joint element.

Hermann, (1978) presented an algorithm for joint element similar to the element of (Goodman, 1968) with certain improvements through introduction of constraint conditions. He also considered various modes of deformations such as sliding and debonding. Wilson, (1977) extended the joint element developed by (Ghaboussi and Wilson, 1973) to two-dimensional and three-dimensional interface elements. He advocated that numerical ill-conditioning of the assembled elastic equations may be avoided by using relative displacements as an independent parameter for the joint element.

Thin-layer element is developed and incorporated in soil structure interaction problems by many investigators, the first efforts carried out in this field were those by (Desai and Zaman, 1984). The element formulation is like any other solid element, but special constitutive model for stress-strain relationships was adopted, Desia element was formulated by considering its behavior either to be linear elastic, or non-linear elastic, and elastic–plastic. The proposed elements have been introduced in displacements, mixed and hybrid finite element method (Desai and Sergant, 1984; Desai and Lightner, 1982), the concept is also incorporated in dynamic finite element analysis (Zaman and Desai, 1984).

Zaman, (1985) used the proposed Desia element to compute dynamic response of embedded pile in medium-dense sand interaction, and he found that for realistic prediction of structural response, it is desirable to model relative motions at interfaces.

In the last few decades, many investigators (Kneifati, 1985; Hooper 1974; Hooper 1983), have solved many soil-structure interaction problems by using different methods. Regarding the behavior of the supporting soil masses, numerous ways of modelling soil strata have been proposed.

Hooper (1974); Hooper (1983), applied finite element method to analyze a circular raft in adhesive contact with thick elastic layer, the computations indicated that interfacial adhesion can significantly reduce the differential raft settlements. His investigation is extended to include strip raft with either smooth or adhesive contact with the elastic layer.

Kneifati, (1985) studied the structural response of an infinitely long plate subjected to different loading cases, and founded on different models (Winkler, Pasternak and Kerr), he found that the Kerr model gives more accurate results than the Winkler and Pasternak models.

The main objective of this paper is to formulate a thin layer element to simulate the bonded and smooth contacts between strip plates and the supporting medium.

The influence of interface conditions is also investigated, the far region of the elastic half-plane is modeled by an infinite element with 1/R decay function (Lynn and Hadid, 1981).

THIN LAYER INTERFACE ELEMENT

In order to take into account the relative slip that occurs at the interface between the structural plate and the supporting medium, due to materials discontinuities and under load transformation, for different plate flexural stiffnesses thin layer element is proposed and implemented. The proposed thin layer element has the following features, and several assumptions are conducted with its behavior:

a) The element has thin finite thickness with $t_1/B = 0.01$, where B is the length of the contact elements, and t_1 is the thickness of the interface element.
a) It was assumed that no separation between the plate and the supporting media will be occurred.
b) The element has very high rigidity to compression in the direction of the applied load.
c) The behavior of the interface element is assumed to be linear elastic conducted with the behavior of the adjacent materials, so that all the system behaves linearly.

Constitutive Modeling

For linear elastic isotropic material, the constitutive matrix in case of plane strain condition is given [14]:

$$[C]=\begin{bmatrix} C_1 & C_2 & 0 \\ C_2 & C_1 & 0 \\ 0 & 0 & G_1 \end{bmatrix} \tag{1}$$

Where:

$$C_1=\frac{E_1(1-\nu_1)}{(1+\nu_1)(1-2\nu_1)}\ ;\ C_2=\frac{E_1\nu_1}{(1+\nu_1)(1-2\nu_1)}$$

and E_1 is the elastic modulus of the interface, ν_1 is Poisson's ratio, and G_1 is the shear modulus of the interface.

Normal practice is to assume that the interfaces are either smooth or bonded, and all the closed-form solution of the plate contact problems assume the interface to be frictionless, through this study two interface conditions are considered.

Bonded Interface:

In this case the relative movement between the two different materials is prevented, this is similar to that considered by (Hooper, 1974; Hooper, 1983) in the analysis of plate in adhesive contact with the supporting soil media. Two approaches are used for this case:

i) First Approach:

On setting ν_1 (i.e. Poisson's ratio) equal to zero. Eq. (1) then becomes:

$$[C]=\begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_1 & 0 \\ 0 & 0 & G_1 \end{bmatrix} \tag{2}$$

E_1 is assigned to a large value of order (10^8 – 10^{10}) units, and G_1 is also assigned to a large value in the limits (10^9 – 10^{11}) to prevent the relative movements.

ii) Second Approach:

This takes the interface properties E_1, ν_1 similar to the corresponding properties of the adjacent supporting media, and G_1 is still assigned to large value.

Smooth Interface

In considering the behavior of the interface within the linear elastic ranges, and to match the frictionless contact assumption of the theoretical solutions of plate contact problems, this case is incorporated, and again two approaches have been used to set the element constitutive matrix in similar manner to that of bonded interfaces but in this case the value of shear modulus G_1 is assigned to a very small values.

One of the major improvements provided by the thin-layer element lies in its capability to provide consistent and satisfactory computations of stresses in the interfaces themselves, whereas with the zero thickness element, it has been often found difficult to obtain consistent and stable stresses in the adjoining elements (Desia and Zaman, 1984).

Plane Strain Interface Element

The development of the six and eight nodded interface element stiffness matrix characteristic follows essentially the same procedure as the solid elements, that is the stiffness matrix for the plane strain elastic problems is written as (Zienkiewicz, 1977):

$$[K]=\int_A [B]^T[C][B]\,dA \tag{3}$$

Where [B] is the strain-displacement matrix, [C] is the constitutive matrix, specified for each interface condition, and dA is the elemental area.

Testing the Interface Element

To ensure the accuracy and the applicability of the formulated interface element under loading transfer, the element has been tested in a simple manner under plane strain formulation, the test include the two interface conditions (bonded, and smooth) for the six and eight-nodded elements. Fig. (1) shows a single interface element with eight nodes, the element has a length equal to unity, and t_1/B ratio is equal to 0.01. A surface shear traction, T_x, equal to 10 units is applied at the upper boundary of the element. In order to achieve the bonded condition, the linear material properties are assumed as follows:

$E_1 = 10^9$ units, $\nu_1 = 0.0$, and $G_1 = 10^9$ units.

The computed results for the shear stress τ_{xy} are shown in Table (1), it is clearly seen that they are very close to the applied shear stress. The computed horizontal relative displacement u_r at the nodes are in agreement with the exact solution given by:

$$\gamma_{xy} = \frac{u_r}{t_1} = \frac{\tau_{xy}}{G_1} \tag{4}$$

then

$$u_r = \frac{\tau_{xy} * t_1}{G_1} \tag{5}$$

where:

γ_{xy} is the shearing strain, u_r is the relative horizontal displacement, and t_1 is the thickness of the interface element, the output displacements are also shown in Table (1). The procedure mentioned herein is also adopted for the six-nodded elements as shown in Fig. (2), and the results are also given in Table (1). It is found that the six- and eight-nodded elements give very close results. On this basis, only the six-nodded element is used to simulate the contact between the plates and the supporting medium through this paper.

NUMERICAL ANALYSIS AND DISCUSSION

1) Strip Foundation on Homogeneous Elastic Half-Plane:

Half–plane is frequently considered in strip footing and dam analysis. To model the far field of the half-plane, infinite elements in plane strain condition are incorporated in the analysis, the element formulation and application is given in (Lynn, and Hadid, 1981). The smooth interface element is introduced to simulate the frictionless contact requirement of the analytical solution. The flexural behavior of the strip relative to the half-plane stiffness is defined here as:

$$K_s = \frac{E(1-\nu_s^2)}{E_s(1-\nu^2)}(t/2b)^3 \tag{6}$$

where b is the half-width of the strip plate.

Fig. (3) shows the structural response of uniformly-loaded flexurally stiff strip foundation ($k_s > 1$) in frictionless contract with elastic, isotropic, and homogeneous half-plane. The results are given in non-dimensional from for the deflection and bending moment, and they are close to those obtained by (Hooper, 1983).

The results of line-loaded flexurally stiff strip foundation are shown in Fig. (4).

2) Strip Raft in Frictionless Contact with Homogeneous Elastic Layer:

The inteface element in palne strain condition is adopted to simulate the frictionless contact between the raft and the elastix layer. Fig. (5) shows the finite element mesh used in the analysis of this problem; it consists of 255 nodes and 73 elements. The rigid boundary base is located at a depth equal to the width of strip, whereas roller lateral boundary is substituted at a distance of 5*b from the raft center.

Fig. (6) illustrates the computed results for uniformly loaded strip foundation ($K_s = 1.0$, $\nu = \nu_s = 0.2$). The results are expressed in a non-dimensional form, and they are in agreement with the results obtained by Hooper.

Fig. (7-a) shows the variations of differential settlements between the center and edge, for different values of K_s. It is clear that the differential settlement decreases with increasing K_s, and it vanishes for K_s =10, the analytical solutions are also plotted in the figure. Fig. (7-b) illustrates the variations of central bending moments for smooth interface, as the relative raft rigidity is changed. The central bending moment increases with decreasing K_s (flexible raft).

Fig. (8) gives the influence of relative raft stiffness Ks with the interface conditions on central deflection of the loaded strip. It is seen that the interface condition is more dominant at flexible rafts, for raft of K_s = 0.01 the central raft deflection for smooth contact is 13.4 % higher than for bonded contact. When K_s is increased to 1, the strip raft behaves as rigid or stiff, in this case the central deflection for smooth case is 4.9 % higher than the corresponding value of bonded interface.

3) Infinitely Long Plate Resting on Different Models:

To demonstrate the versatility of the proposed smooth interface element, comparisons are presented with other developed models to represent the supporting media. (Kneifati, 1985) studied the foundation model consisting of two spring layers interconnected by a shearing layer to generalize Kerr model. Winkler and Pasternak models were also considered. Fig. (9) shows the results of uniformly-loaded strip plate resting on different models.

The present work given in terms of smooth interface is in agreement with Kerr model. Fig. (10) gives the influence of interface conditions on the deflection and stress distribution of the loaded plate. Fig. (11) illustrates the comparative results of edge-loaded infinitely long plate. Fig. (12) gives the results of an infinitely long plate subjected to edge moment, the moment is simulated by two horizontal couple forces applied at the plate edges.

Conclusion

Examples of geotechnical interest are presented to demonstrate the applicability of the proposed interface model. The numerical results obtained compared favourably with available closed-form solutions and other models of the supporting soil medium. Based on the results, the following conclusions are achieved:

1. The contact between dissimilar materials is fulfilled by the formulated interface element especially in smooth interface condition.
2. The existence of interface element between dissimilar materials affects the displacements and the stress distributions.
3. The influence of interface condition is more dominant for flexible plates than the stiff or rigid plates.
4. The present work of smooth interface gives results, accurate and in agreement with Kerr model, whereas it differs from the Winkler and Pasternak models.

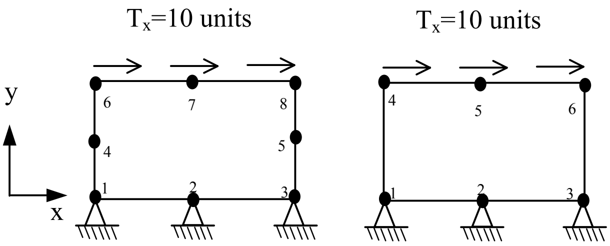

Table 1. Results for interface elements tests

	Node	Computed Shear Stress (τ_{xy})	Computed relative displacement 10^{-10}	Exact relative displacement in Eq. (5) 10^{-10}
8-noded interface element	1	9.9999	0.0	0.0
	2	9.999	0.0	0.0
	3	10.00001	0.0	0.0
	4	9.9999	0.50061	0.5
	5	10.00001	0.49976	0.5
	6	9.9999	1.00223	1.00
	7	9.9999	1.00033	1.00
	8	10.00001	0.99882	1.00
6-noded interface element	1	10.003	0.0	0.0
	2	10.003	0.0	0.0
	3	10.003	0.0	0.0
	4	9.997	1.00012	1.00
	5	9.997	1.05538	1.00
	6	9.997	1.00012	1.00

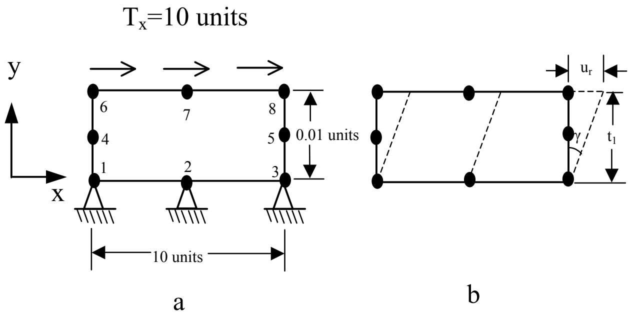

Fig. 1. Linear elastic test for eight-nodded interface element.
(a) Element Geometry
(b) Horizontal displacement behavior

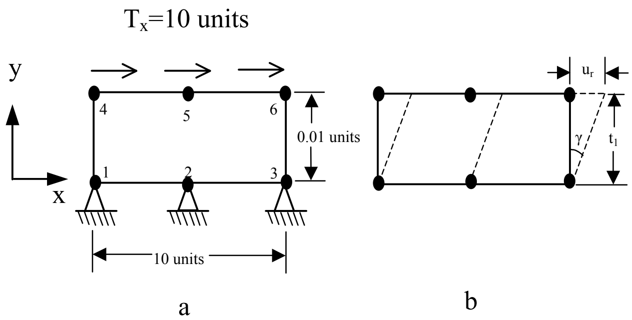

Fig. 2. Linear elastic test for six-nodded interface element.
(a) Element geometry
(b) Horizontal displacement behavior

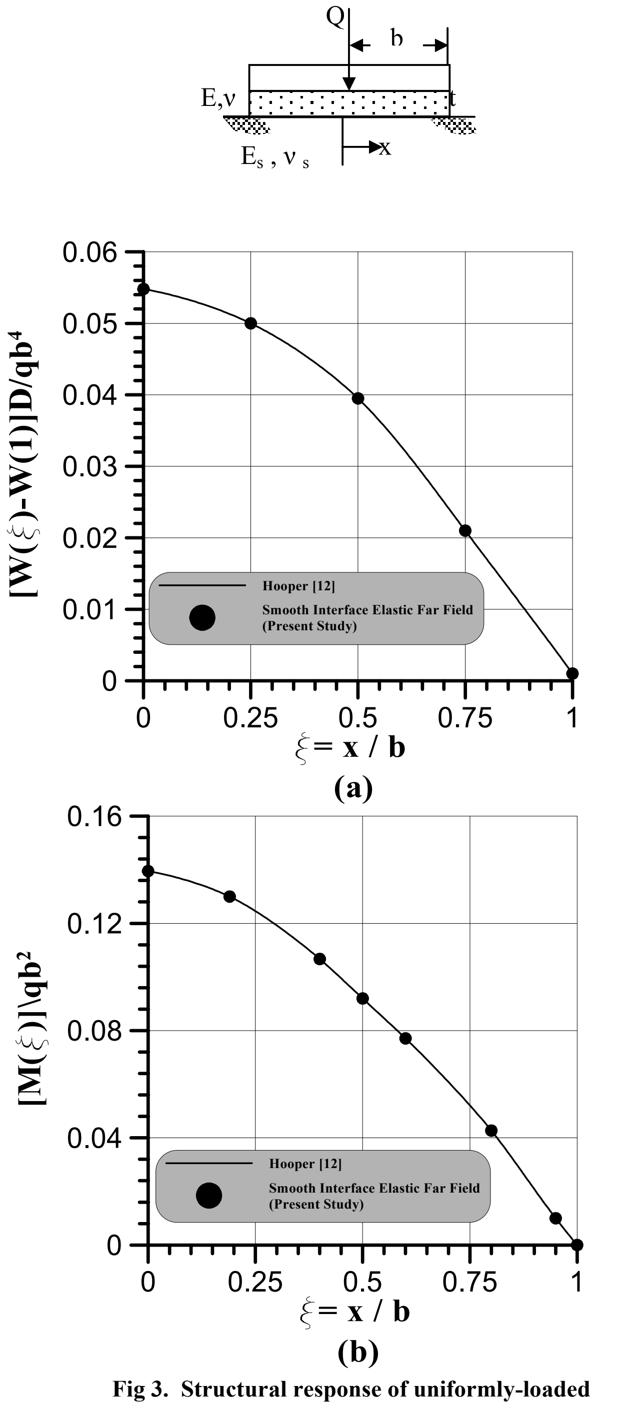

Fig 3. Structural response of uniformly-loaded flexurally stiff strip foundation (ks>1) in frictionless contact with homogeneous isotropic elastic half-plane. (a) : Deflected shape (b) : Bending moment

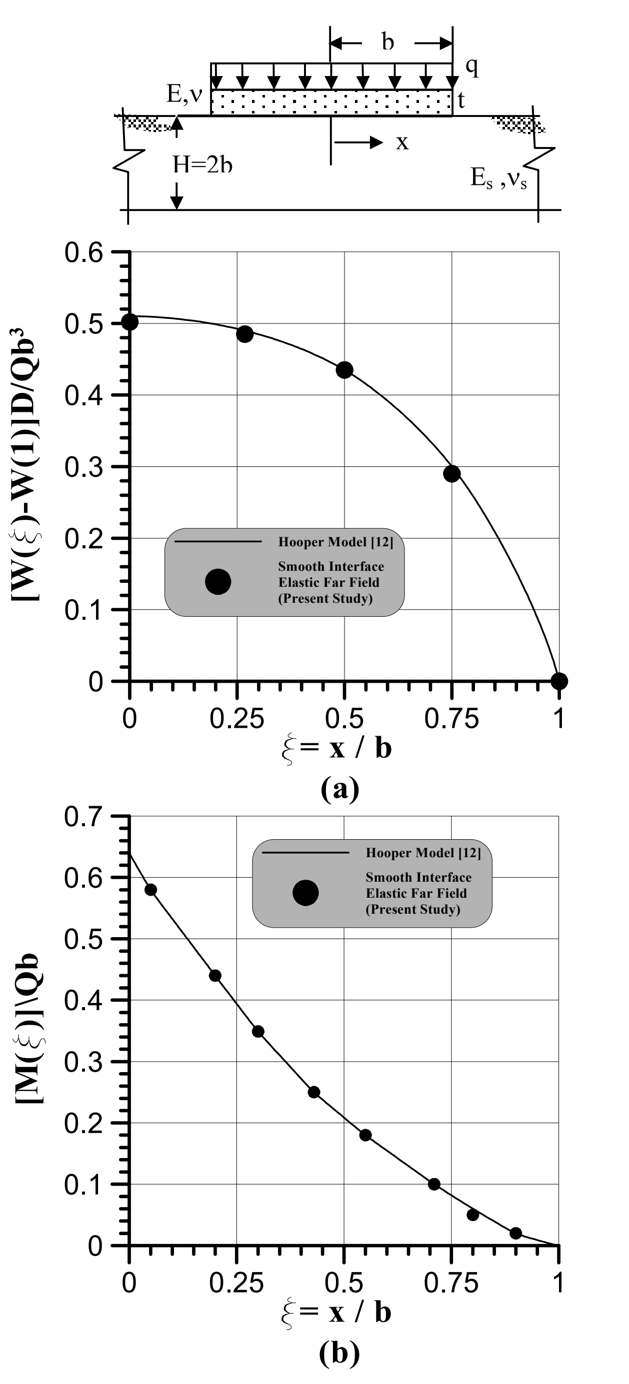

Fig 4. Computed results of line-loaded flexurally stiff strip foundation (Ks > 1) in frictionless contact with homogeneous isotropic elastic half-plane. (a): Deflected shape; (b): Bending moment

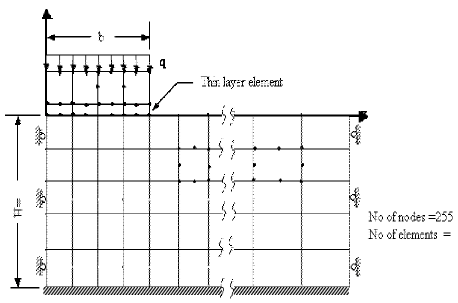

Fig. (5): Finite element discretization of strip plane on an elastic layer underlain by horizontal rigid base (

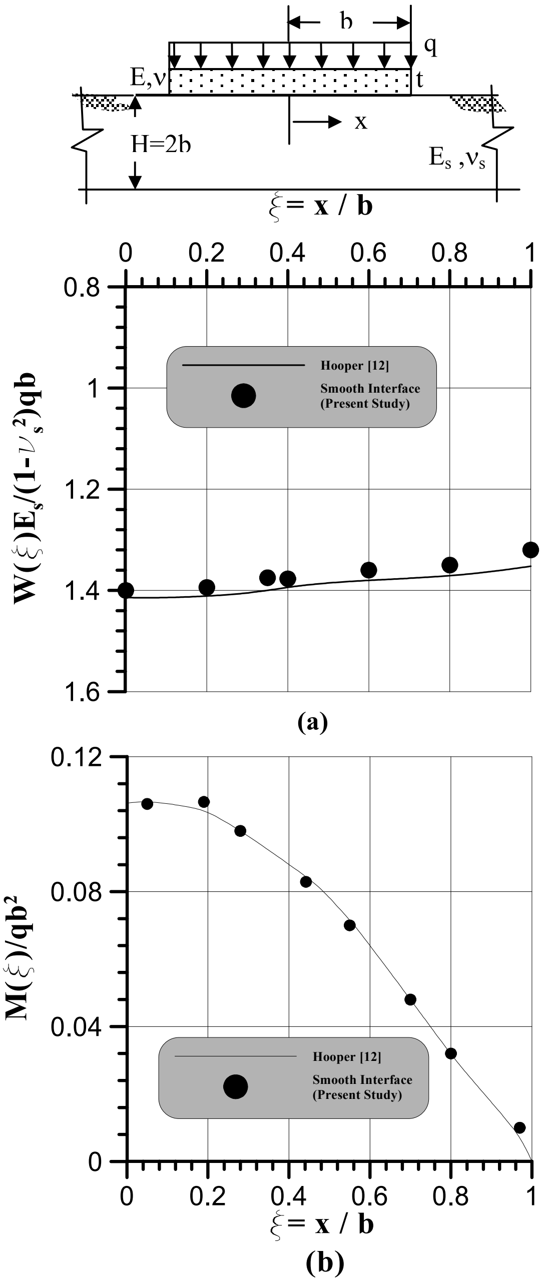

Fig. 6. Computed results for uniformly-loaded strip foundation in frictionless contact with homogeneous elastic layer underlain by horizontal rigidbase (Ks = 1.0, $\nu = \nu s = 0.2$).
(a): Settlement; (b): Bending moment

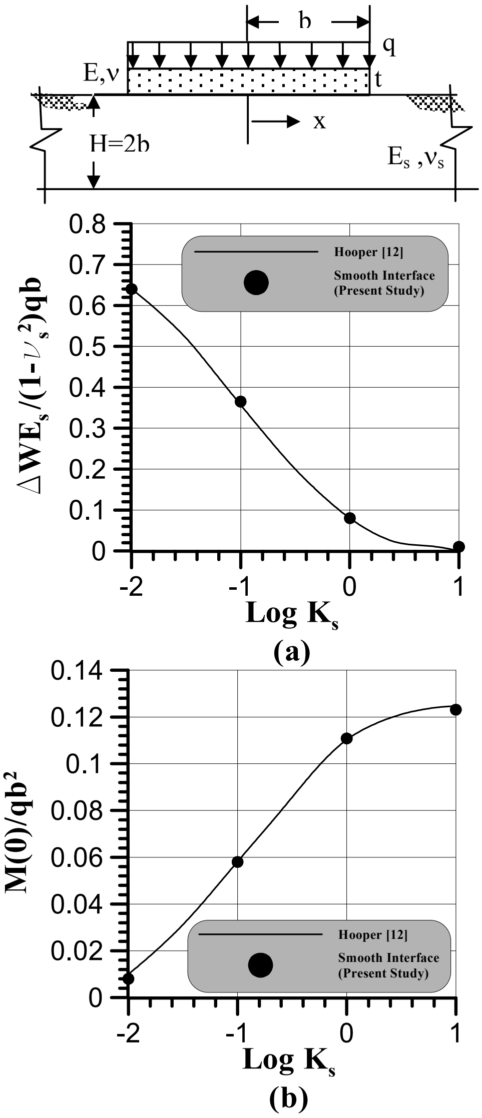

Fig. 7. Comparative results of uniformly-loaded strip foundation in frictionless contact with homogeneous elastic layer underlain by horizontal rigid base (different Ks, $\nu = \nu_s = 0.2$).
(a): Differential settlement between the center and edge
(b): Central bending moment

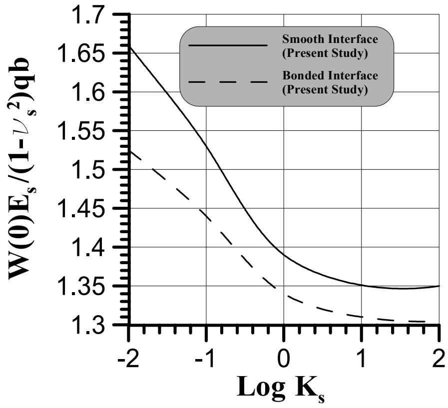

Fig. 8. Influence of relative raft stiffness (K_s) with the interface conditions on the central deflection of uniformly-loaded strip raft underlain by horizontal rigid base ($\nu = 0.2$).

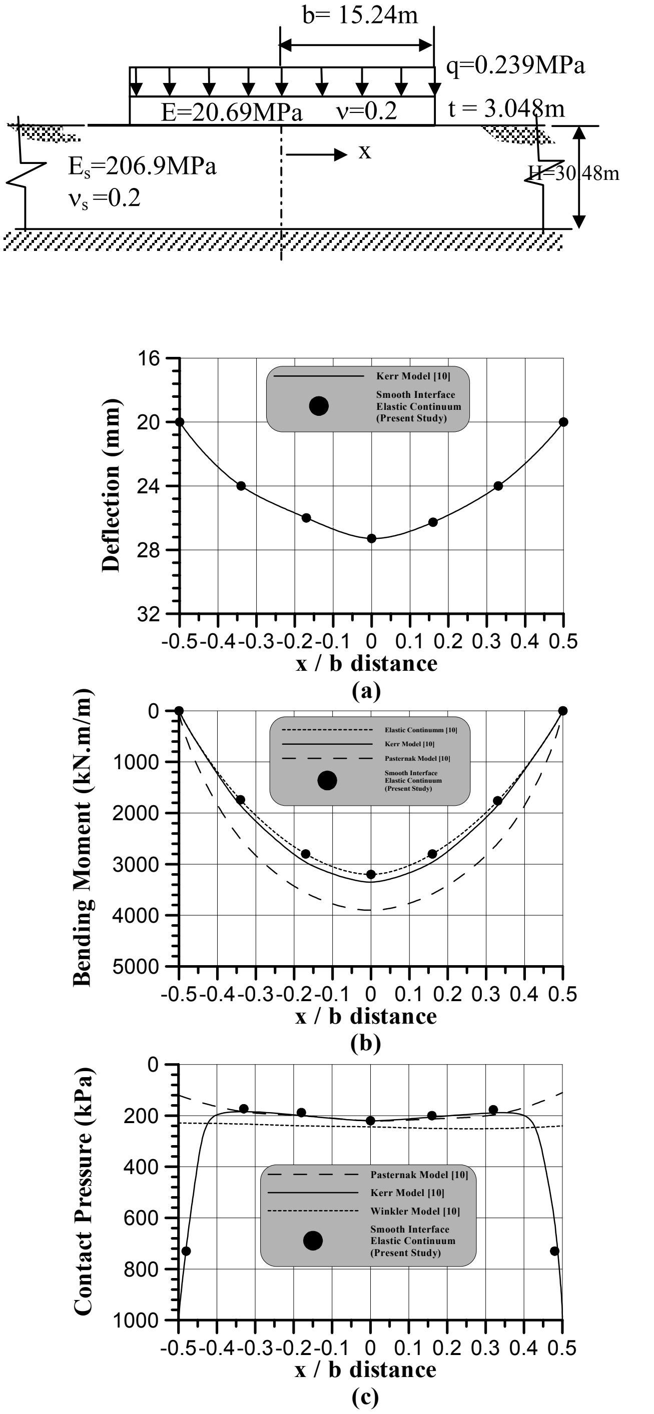

Fig. 9. Comparative results of uniformly-loaded infinitely long plate resting on different models.
(a): Deflection; (b): Bending moment; (c): Contact pressure

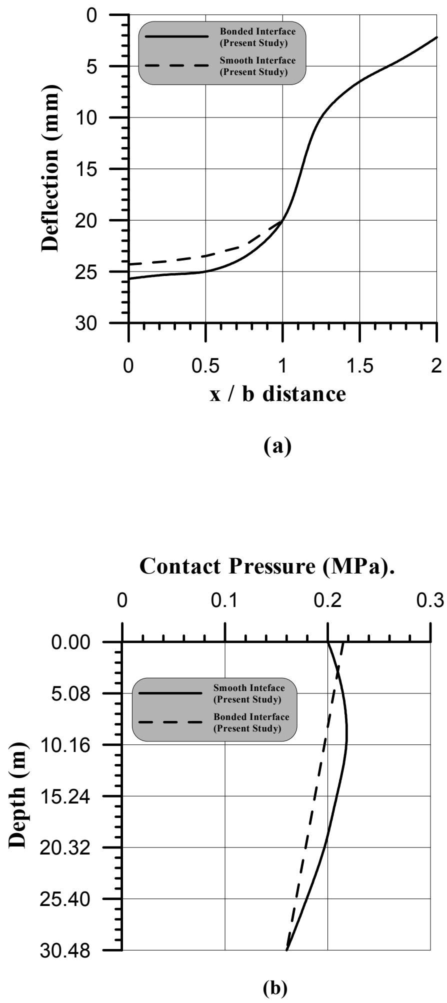

Fig. 10. Influence of interface conditions of uniformly loaded infinitely long plate underlain by horizontal rigid base
(a): Deflection
(b): Contact pressure distribution under the plate center.

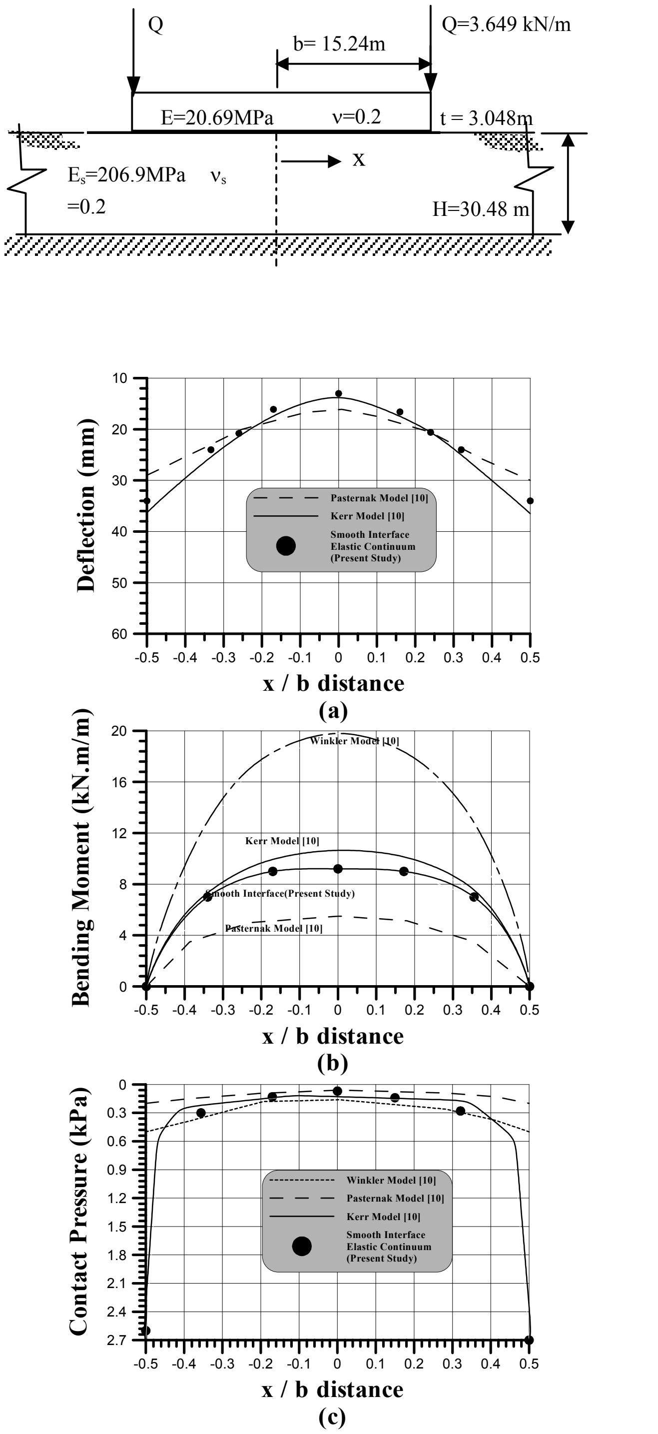

Fig. 11. Structural response of edge-loaded infinitely long plate resting on different models (a): Deflection; (b): Bending moment; (c): Contact pressure

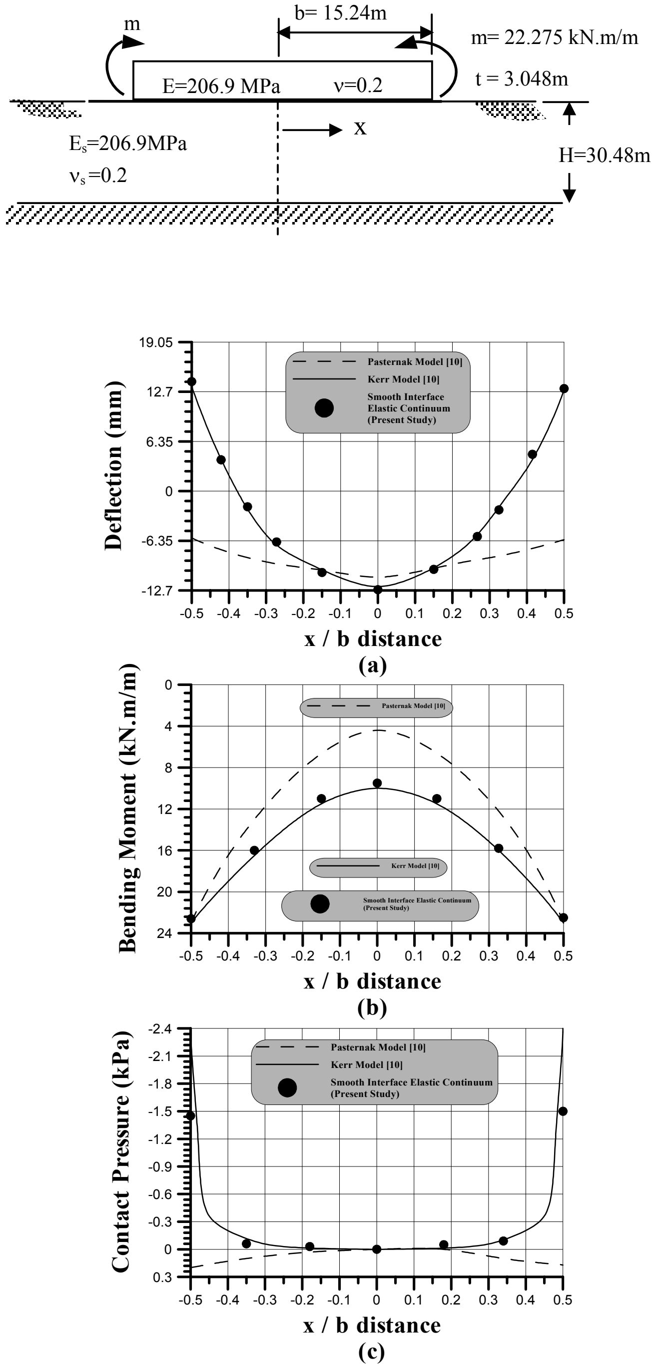

Fig. 12. Results of an infinitely long plate subjected to edge moment and resting on different models (a): Deflection; (b): Bending moment; (c): Contact pressure

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