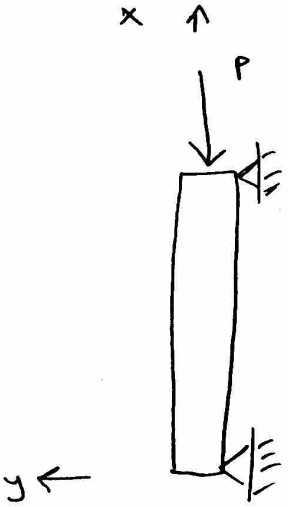
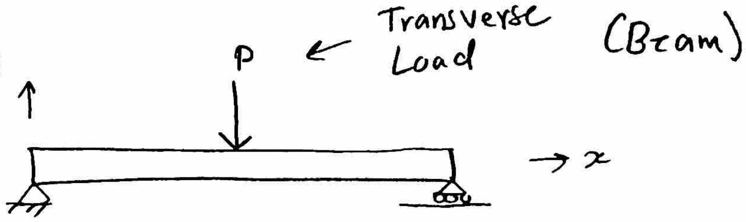
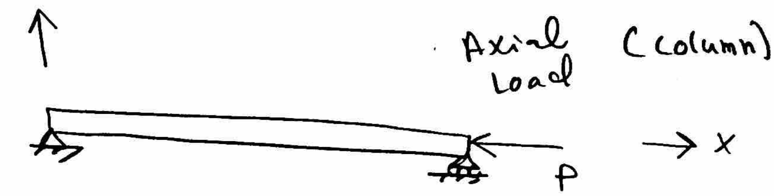
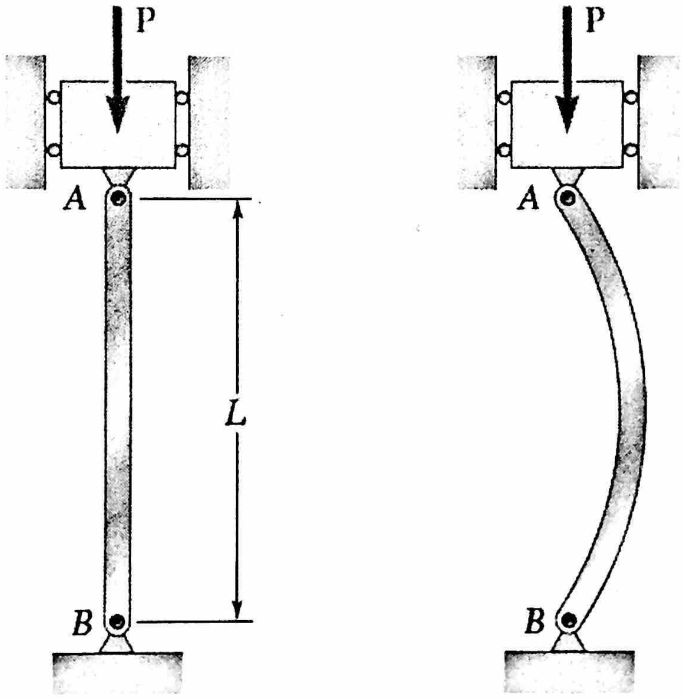
Chapter 10: Columns









Buckling

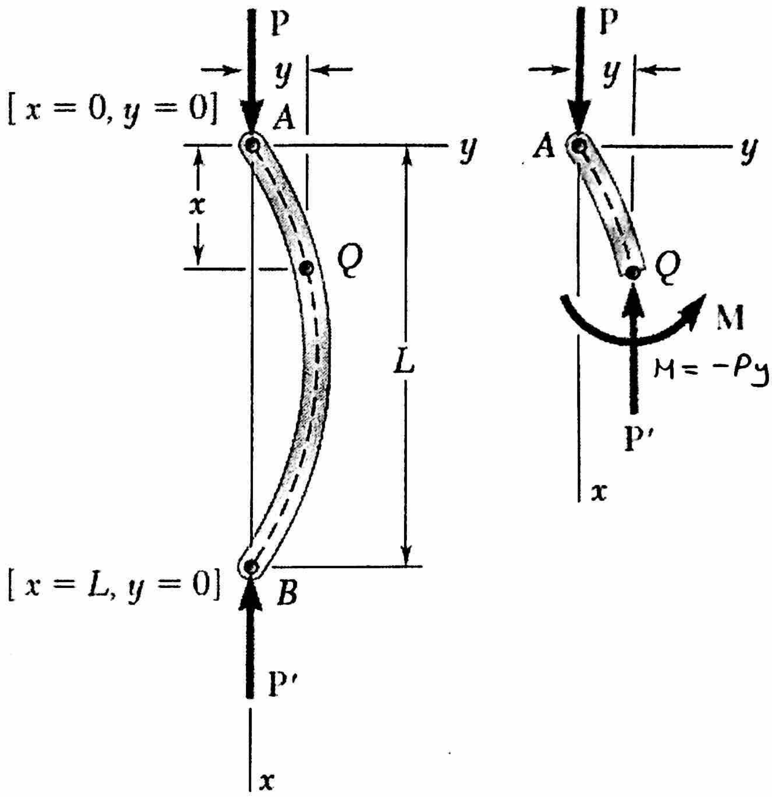
Increase Load (p) $\Rightarrow$ Buckling

P at buckling $\Rightarrow$ Critical buckling force

P_{cr}

From ch.9

$$\frac{d^2y}{dx^2} = \frac{+M(y)}{EI} = \frac{-Py}{EI}$$



$$\frac{d^2y}{dx^2} + \underbrace{\left(\frac{P}{EI}\right)}^{\lambda^2} y = 0$$

$y'' + \lambda^2 y = 0$ ← Differential equation

$y(x) = c\,e^{rx}$ c: constant

$y''(x) = r^2 c\,e^{rx}$

$$r^2 \cancel{c}\cancel{e^{rx}} + \lambda^2 \cancel{c}\cancel{e^{rx}} = 0$$

$$r^2 + \lambda^2 = 0 \Rightarrow r_{1,2} = \pm i\lambda \qquad i = \sqrt{-1}$$

$$y(x) = A\cos\lambda x + B\sin\lambda x$$

A and B are constants to be determined from Boundary constants

For Simple Supports $y(0) = 0$, $y(L) = 0$

Apply $y(0) = 0 \Rightarrow \boxed{A = 0}$ $y(x) = B\sin\lambda x$

$y(L) = 0 \Rightarrow B\sin\lambda L = 0$, $B \neq 0 \Rightarrow \sin\lambda L = 0$

$$\lambda L = n\pi$$

$\lambda = \dfrac{n\pi}{L}$, $n = 1$ (Simplest case)

$$\lambda = \frac{\pi}{L}$$

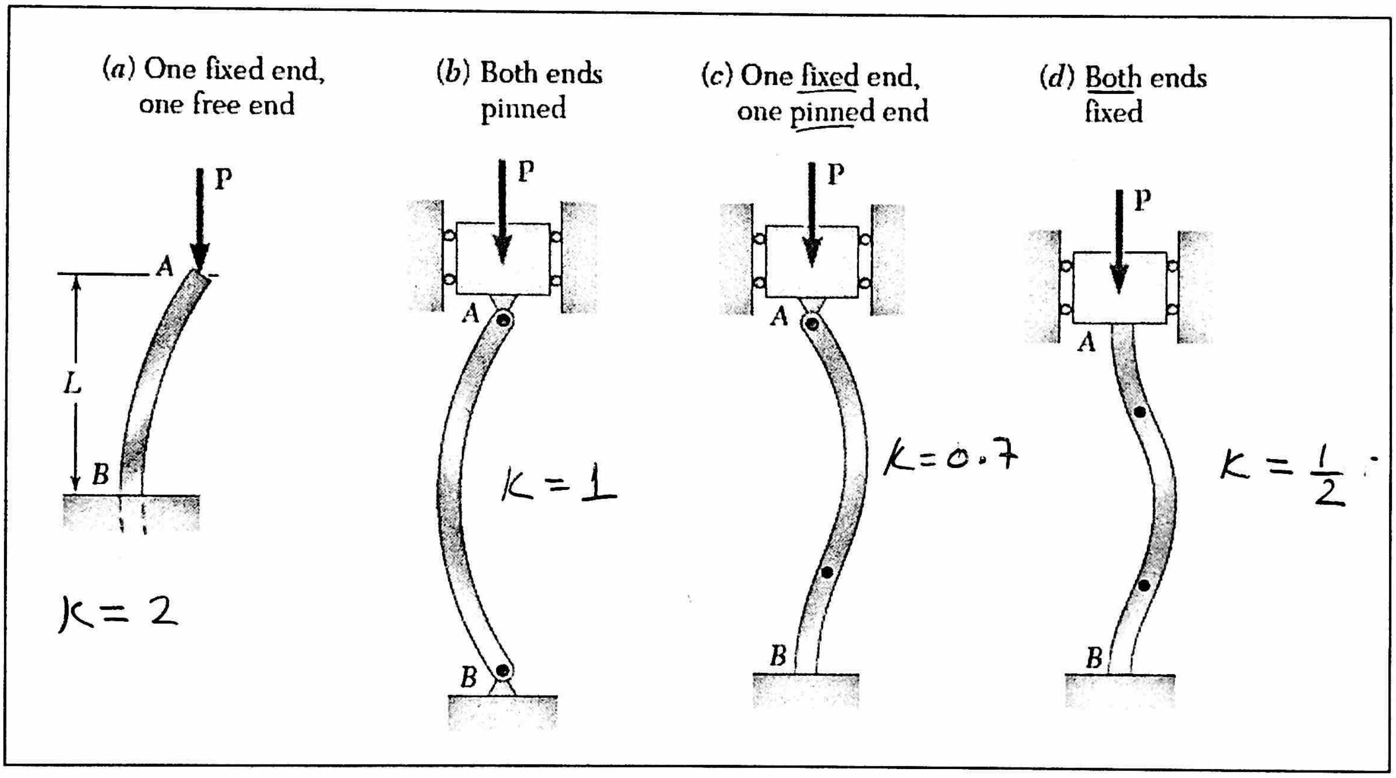
Remember $\lambda^2 = \dfrac{P}{EI}$ $\Rightarrow \dfrac{\pi^2}{L^2} = \dfrac{P}{EI} \Rightarrow \boxed{P_{cr} = \dfrac{\pi^2 EI}{L^2}}$

In General

$$P_{cr} = \frac{\pi^2 EI}{K^2 L^2}$$

K : Buckling factor [-]

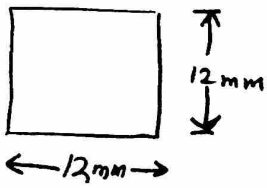
Depends on support type



Example

For a Fixed – Fixed Column, $E = 200$ GPa

$L = 2$ m



Find P_{cr}

Solution

$$P_{cr} = \frac{\pi^2 EI}{K^2 L^2} = \frac{\pi^2 (200)(10^9)\left[\frac{1}{12}(12\times10^{-3})(12\times10^{-3})^3\right]}{\left(\frac{1}{2}\right)^2 \quad (2)^2}$$

$$\boxed{P_{cr} = 23.7 \text{ N}}$$