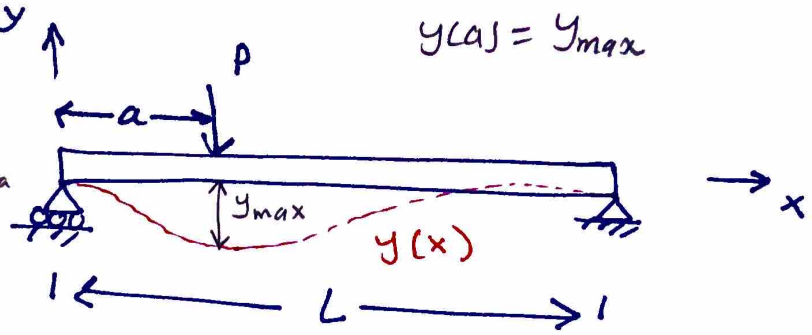
Chapter 9: Deflection of Beams

From ch 4

$$\frac{1}{\rho} = \frac{M(x)}{EI}$$

ρ: radius of curvature
E: Elastic mod.
I: moment of Inertia
M: moment



$y(a) = y_{max}$

$y(x)$ beam deflection

From calculus

$$\frac{d^2y}{dx^2} = \frac{1}{\rho}$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{M(x)}{EI}}$$ Elastic curve formula

$$EI\frac{d^2y}{dx^2} = M(x)$$ → integrate

$$EI\frac{dy}{dx} = \int_0^x M(x)\,dx = \int_0^x M(x)\,dx + C_1$$ Integrate, again

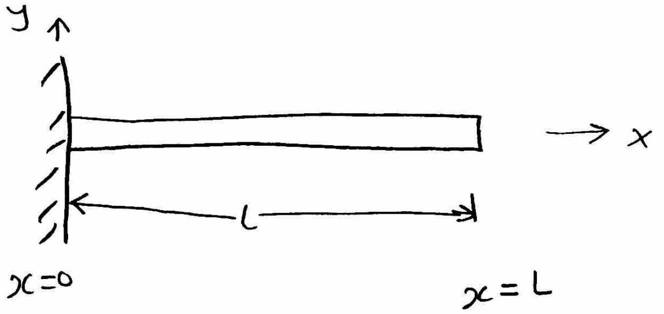
$$EIy = \int_0^x\left[\int_0^x M(x)\,dx + C_1\right]dx = \int_0^x\left(\int_0^x M(x)\,dx\right)dx + C_1x + C_2$$

$$\Rightarrow y(x) = \frac{1}{EI}\left[\int_0^x\left(\int_0^x M(x)\,dx\right)dx + C_1x + C_2\right]$$

C_1 and C_2 are constant to be determined from Boundary Conditions (BC's)

* Boundary conditions.

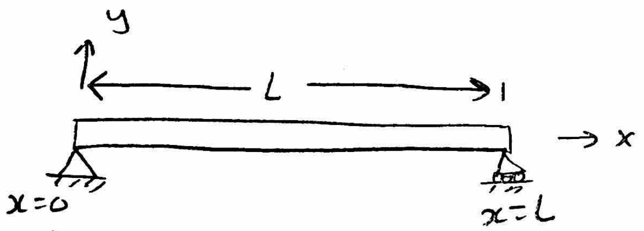
① Cantilevered Beam



$y(0) = 0$

$y'(0) = 0$

② Simply Supported Beam

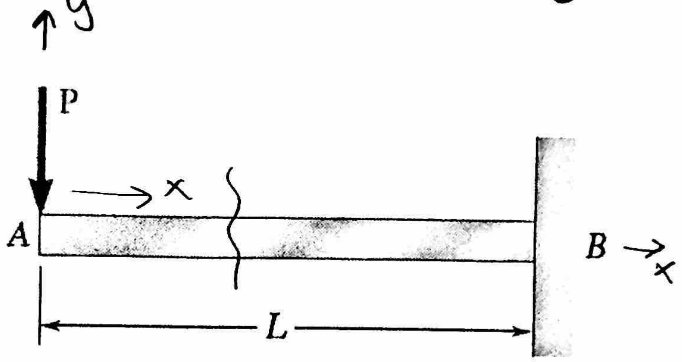


$y(0)=0$ $\qquad$ $y(L) = 0$

Slope $\theta(x) = y'(x) \Rightarrow \theta(x) = \frac{1}{EI}\left[\int_0^x M(x)\,dx + C_1\right]$

Example

For the beam and loading shown, (1) Find elastic curve formula and (2) slope at point A.

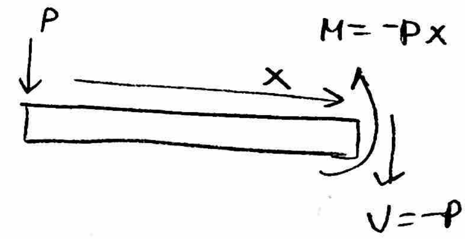


$y(L)=0$
$y'(L)=0$

Solution

$$y(x) = \frac{1}{EI}\left[\int_0^x \left(\int_0^x M(x)\,dx\right) dx + C_1 x + C_2\right]$$

$$y(x) = \frac{1}{EI}\int_0^x \left(\int_0^x -Px\,dx\right) dx + C_1 x + C_2$$



$$= \frac{1}{EI}\int_0^x -\frac{Px^2}{2}\,dx + C_1 x + C_2$$

$$= \frac{1}{EI}\left(-\frac{Px^3}{6} + C_1 x + C_2\right)$$

Apply BC's

$$y(L) = 0 = \frac{1}{EI}\left(\frac{-PL^3}{6} + C_1 L + C_2\right)$$

$$y'(L) = 0 \Rightarrow y'(x) = \frac{1}{EI}\left(-\frac{Px^2}{2} + C_1\right)$$

$$y'(L) = 0 = \frac{1}{EI}\left(-\frac{PL^2}{2} + C_1\right) \Rightarrow C_1 = \frac{1}{2}PL^2$$

$$C_2 = -\frac{1}{3}PL^3$$

$$\Rightarrow y(x) = \frac{P}{6EI}\left(-x^3 + 3L^2 x - 2L^3\right)$$

Slope $\theta(x) = \frac{P}{6EI}\left(-3x^2 + 3L^2\right)$, $\underset{\text{at}_A}{\theta(0)} = \frac{3PL^2}{6EI} = \frac{PL^2}{2EI}$