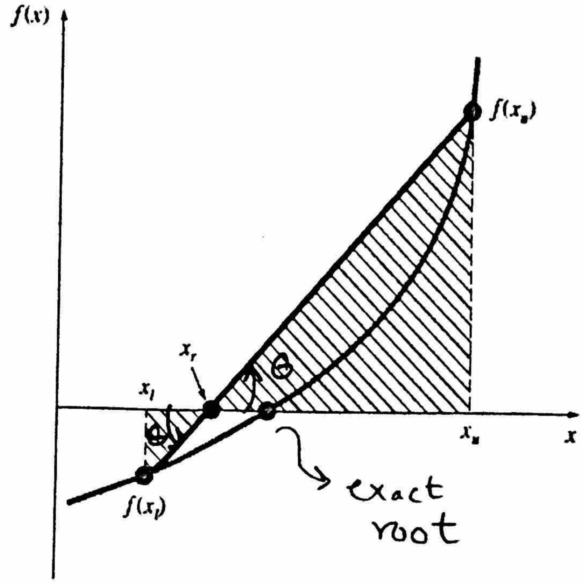
Chapter 05

- False position Method (FPM)

FPM is very similar to Bisection method but faster and little more accurate. However, little more complicated in theory.



- If function $f(x)$ has a root (x_r) lies in the interval $[x_L, x_u]$ and $f(x_L) \cdot f(x_u) < 0$, then FPM calculates x_r, as:

$$x_r = x_u - \frac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)}$$

$$\tan\theta = \frac{f(x_u)}{x_u - x_r} = \frac{-f(x_L)}{x_r - x_L}$$

$$\Rightarrow x_r = x_u - \frac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)}$$

Solution Procedure

① Check Sign $f(x_L) \cdot f(x_u) < 0$, If not, we cannot use FPM "No Solution"

② $x_r = x_u - \dfrac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)}$

③ Find $f(x_r)$ and check sign of $f(x_L) \cdot f(x_r)$

- If $f(x_L) \cdot f(x_r) = 0$, x_r = exact root
- If $f(x_L) \cdot f(x_r) > 0$ (+ve), $x_L = x_r$ and back to step 2
- If $f(x_L) \cdot f(x_r) < 0$ (−ve), $x_u = x_r$ and back to step 2

$\Rightarrow$ Also, we need iterations in FPM

Example $f(x) = e^{x} - 2$ $\quad x_L = 0$, $x_u = 1$ Use FPM to find the root (x_r) for $\varepsilon_a < 5\%$

Solution

Iteration 1

- Check $f(x_L) \cdot f(x_u) < 0$ $\quad (-)(+) = (-)$

$f(x_L) = f(0) = -1$, $\quad f(x_u) = f(1) = 0.7183 \Rightarrow f(x_L) \cdot f(x_u) < 0$ ✓

$$- \; x_r = x_u - \frac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)}$$

$$= 1 - \frac{f(1)(0 - 1)}{f(0) - f(1)} = 1 - \frac{0.7183(-1)}{-1 - 0.7183} = 0.5819$$

$$f(x_L) = f(0) = e^{0} - 2 = -1$$
$$f(x_u) = f(1) = e^{1} - 2 = 0.7183$$

$$- \; f(x_r) = f(0.5819) = e^{0.5819} - 2 = -0.2106$$

$f(x_L) \cdot f(x_r) = (-ve)(-ve) = (+ve)$ $\quad x_L = x_r = 0.5819$

Iteration 2

$x_L = 0.5819$, $x_u = 1$

$$x_r = x_u - \frac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)} = 1 - \frac{f(1)(0.5819 - 1)}{f(0.5819) - f(1)}$$

$$= 1 - \frac{0.7183(0.5819 - 1)}{-0.2106 - 0.7183} = 0.6770$$

$f(0.5819) = -0.2106$

$$f(x_r) = f(0.677) = e^{0.677} - 2 = -0.032$$

$f(x_L) \cdot f(x_r) = (-)(-) = +ve$ $\quad x_L = x_r = 0.6770$

Iteration #	x_L	x_u	x_r	Sign $f(x_L) \cdot f(x_r)$	ϵ_a
1	0	1	0.5819	(+)	N/A
2	0.5819	1	0.6770	(+)	$\epsilon_a = \left\lvert \frac{0.677 - 0.5819}{0.6770} \right\rvert 100\% = 14\%$
3	0.6770	1	0.6879	+	$\epsilon_a = \left\lvert \frac{0.6879 - 0.677}{0.6879} \right\rvert 100\% = 1.6\%$

$$\epsilon_a = \left\lvert \frac{\text{Present } x_r - \text{Previous } x_r}{\text{present } x_r} \right\rvert 100\%$$

$x_r = 0.6879$

Iteration 3

$x_L = 0.6770 \quad , \quad x_u = 1$

$$x_r = x_u - \frac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)} = 1 - \frac{f(1)(0.677 - 1)}{f(0.677) - f(1)} = 0.6879$$

$f(0.677) = -0.032$

$f(1) = +0.7183$

Quick Comparison between Bisection and FPM

	# iterations	ϵ_a
Bisection	5	4.3 %
FPM	3	1.6 %

Faster
Accurate

needs more calculations in x_r.