Phys 741
Statistical Mechanics
Problem Set # 8

Dr. Gassem Alzoubi
The Hashemite University Department of Physics, Zarqa, Jordan

1. Consider a 2D system of an ideal Fermi gas confined to a plane of area A at $T = 0$
   (a) Calculate the density of states of the system
   (b) Calculate the Fermi energy
   (c) Calculate the total energy
   (d) Calculate the chemical potential
   (e) Show that the equation of state is given by $PA = E$
2. Consider a 3D system of an ultra-relativistic electron gas confined to a volume V at $T = 0$
   (a) Calculate the density of states of the system
   (b) Calculate the Fermi energy
   (c) Calculate the total energy
   (d) Calculate the pressure of the electron gas
3. Consider a 3D system of an ultra-relativistic electron gas confined to a volume V at $T \ll T_F$
   (a) Calculate the density of states of the system
   (b) Find $n = N/V$ and show that the chemical potential correction to the lowest order is given by $\delta\mu = -\frac{\pi^2}{3}\frac{(k_B T)^2}{\varepsilon_F}$
   (c) Find an expression for the total energy E of the system and show that $C_V \ \alpha \ T$
4. The Fermi integral is defined as
$$f_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1} dx}{z^{-1} e^x + 1}$$
   (a) By expanding the integrand in powers of z, show that
$$f_n(z) = \sum_{j=1}^{\infty} (-1)^{j-1} \frac{z^j}{j^n}$$
   and find $f_n(1)$
   (b) Show that for the case $z = 1$ ($\mu = 0$),
$$\int_0^\infty \frac{x^n dx}{e^x + 1} = (1 - \frac{1}{2^n})\ \Gamma(n+1)\ \zeta(n+1)$$
5. The low temperature specific heat of a typical metal is the sum of the electronic specific heat and the phonon specific heat $C = C_{ele} + C_{pho}$. At high T, the phonon specific heat dominates the electronic specific heat($C_{pho} \gg C_{ele}$). However, at very low T, the electronic specific heat dominates the phonon specific heat ($C_{ele} \gg C_{pho}$). Defining T_c as the crossover temperature at which $C_{ele} = C_{pho}$, estimate the value of T_c for Sodium (Na) and make a rough sketch of their behavior in the temperature range $0 < T < 2T_c$.

Good Luck