Phys 761
Quantum Mechanics
Final Exam, Fall 2016

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1. Consider an electron in the Hydrogen atom that occupies the combined spin and position state

$$\psi = R_{32}\ (\sqrt{1/4}\ Y_2^0\ \chi_+\ + \sqrt{3/4}\ Y_2^{-1}\ \chi_-)$$

   (a) What is the probability of finding the electron in the state n=1 (2 points)

   (b) If you measured L^2, what values might you get, and what is the probability of each (2 points)

   (c) If you measured L_z, what values might you get, and what is the probability of each (4 points)

   (d) If you measured S_z, what values might you get, and what is the probability of each (4 points)

2. Consider a real hydrogen atom in its ground state and ignoring the hyperfine structure,

   (a) Calculate the fine structure correction (in eV) of the ground state (5 points)

   (b) If the atom is placed in a magnetic field of strength 10 T, calculate the Zeeman splitting (in eV) of the ground state. Does the Zeeman splitting dominate the fine structure, explain your answer

   Hint: $\mu_B = 9.27 \times 10^{-24}\ T.J^{-1}$ and $1\ eV = 1.6 \times 10^{-19} J$ (5 points)

3. Consider a particle of mass m moving in the potential $V(x) = g|x|$, with $g > 0$. Use the variational method to estimate the ground state energy using the trial wave function $\psi(x) = Ae^{-\alpha|x|}$. Compare your result with the exact value $0.809(\frac{g^2\hbar^2}{m})^{1/3}$

   Hint: $\int_0^\infty dr\ r^n\ e^{-\alpha r} = \frac{n!}{\alpha^{n+1}}$ (8 points)

4. Consider a particle of mass m moving in a 1D harmonic oscillator potential $V(x) = \frac{1}{2}m\omega_0^2 x^2$. The particle is initially in the ground state $|0\rangle$. At $t = 0$, a perturbation $H'(t) = 2\ \alpha\ x\ cos(\omega t)$, where α is a real constant, is turned on.

   (a) Calculate the transition probability $P_{n0}(t)$ from the ground state $|0\rangle$ to the nth excited state $|n\rangle$ after a sufficiently long time (i.e. $t \to \infty$) (8 points)

   (b) What is the transition probability from the ground state $|0\rangle$ to the second exited state (2 points)

Good Luck