

Second Exam

Real Analysis I

April 30, 2013

Student Name:

Serial Number:.....

*** Show your work in details, no credit will be given for *answers* without details.**

1. (5 points) Use the ϵ - **definition** of the limit to prove that (show all details)

$$\lim_{n \rightarrow \infty} \frac{2n^2-1}{5n^2+4} = \frac{2}{5}.$$

2. (7 points) Let $x_n := 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2}$, for a $n \geq 1$.

- (a) Determine whether the sequence $X = \{x_n\}_{n \geq 1}$ is bounded or not.
- (b) Determine whether the sequence $X = \{x_n\}_{n \geq 1}$ is monotone or not.
- (c) Is the sequence X converge? Justify your answer.

3. (6 points)

- (a) State the definition of a “Cauchy sequence”.
- (b) Prove that “ If a sequence $X = (x_n)$ of real numbers converge, then it is a Cauchy sequence”.

4. (7 points) **State and prove the Bolzano-Weierstrass Theorem** for sequences.

Good Luck