The Hashemite University — Real Analysis I, Second Exam — December 1, 2013

1^{st} semester 2013/2014 — Time: 55 minutes

Student Name: .... — Serial Number:.........

**** Show your work in details, no credit will be given for *answers* without details.**

1. (15 points) Determine whether these statements are true or false, **and justify why a statement is false**.

   1. (T F) If all of the subsequences of a sequence $\{x_n\}$ converge, then the sequence $\{x_n\}$ converges.

   2. (T F) Any bounded sequence converges.

   3. (T F) If a sequence $\{x_n\}$ diverges then every subsequence of $\{x_n\}$ diverges.

   4. (T F) If a sequence $\{x_n\} \subseteq \mathbb{R}$ has an unbounded subsequence, then $\{x_n\}$ is unbounded.

   5. (T F) Let X' and X'' be subsequences of $X = \{x_n\}$, such that X' converges to x' and X'' converges to x''. If $x' = x''$, then the sequence X converges to the common value.

   6. (T F) If $X = \{x_n\}$ is a sequence, then it contains a subsequence which is monotone.

   7. (T F) If $\{x_n\}$ is decreasing and bounded below by $M > 0$, then $\lim_{n\to\infty} x_n = L$, for some L and $L \leq M$.

   8. (T F) If $\{x_n\}$ converges to 0, then it is not necessary that the sequence $\{(-1)^n x_n\}$ converge to 0.

   9. (T F) Every Cauchy sequence is monotone.

   10. (T F) Let $f : \mathbb{R} \to \mathbb{R}$ be a function, and $\{x_n\}, \{y_n\}$ be two sequences each converges to the real number c. If $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} f(y_n) = L$, then $\lim_{x\to c} f(x)$ exist.

2. (5 points) Prove that " If $\{x_n\}$ is a Cauchy sequence then it is bounded". (*Don't use any other theorems in your proof*)

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3. (5 points) Let $\{x_n\}, \{y_n\}$ be two sequences of real numbers, such that $\lim_{n\to\infty} x_n = x$. If $\forall \epsilon > 0$, $\exists K \in \mathbb{N}$ such that $|x_n - y_n| < \epsilon$, for all $n \geq K$, show that $\{y_n\}$ converges.

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4. (**5 points Bonus**) Give an example of a sequence $\{x_n\} \subseteq \mathbb{R}$ that is **not** a Cauchy sequence, but that satisfies $|x_{n+1} - x_n| < \epsilon$ for all $n \in \mathbb{N}$ and all $\epsilon > 0$. Justify your answer.

Good Luck