First Exam

Real Analysis I — March 14, 2013

Student Name: ..... Serial Number:.....

*** Show your work in details, no credit will be given for *answers* without details.**

1. (5 points) Prove that there does **not** exist a rational number r such that $r^2 = 7$.
2. (5 points) If $a, b \in \mathbb{R}$, prove that $a^2 + b^2 = 0$ ***if and only if*** $a = 0$ and $b = 0$.
3. (5 points) **State** and **prove** the "Density Theorem" of the rational numbers.
4. (5 points) Let D be a nonempty subset of $\mathbb{R}$, and let f and g be defined on D and have bounded ranges in $\mathbb{R}$. Show that
$$\sup_{x\in D}\{f(x) + g(x)\} \leq \sup_{x\in D}\{f(x)\} + \sup_{x\in D}\{g(x)\}$$
5. (5 points) Define $I_n := (-\frac{1}{n}, \frac{1}{n})$, $n \in \mathbb{N}$. Prove that
$$\bigcap_{n\in\mathbb{N}} I_n = \{0\}.$$

Good Luck