Solution for Ordinary Differential Equations (1) First Exam

Question one: Circle the letter that represents the correct answer (2 points each).

1. The largest interval on which the following initial value problem has a unique solution is

$$(x^2-1)\frac{dy}{dx}+\frac{1}{x-5}y=\frac{\ln(x-2)}{x-6}, \qquad y(4)=1.5$$

a. $(1,5)$ **b.** $(2,5)$ c. $(2,6)$ d. $(-1,5)$

2. A general solution to $y'+\frac{y}{2x}=\frac{x}{y}$, $x>0$, is

a. $y^2=\frac{2x^2}{3}+\frac{C}{x}$ b. $y^2=\frac{x^2}{3}+\frac{C}{x}$ c. $y^2=2x^2+\frac{C}{x}$ d. $y^2=3x^2+\frac{C}{x}$

3. If $\frac{dY}{dt}=\frac{Y^2+1}{t+1}$ with $Y(0)=0$, then $Y(2)=$

a. $\tan(\ln 5)$ b. $\tan(\ln 4)$ c. $\tan(\ln 2)$ **d.** $\tan(\ln 3)$

4. The differential equation $\frac{dw}{dt}=\frac{2w+t+2}{w-t+10}$ can be converted to a homogenous one by

a. $t=x+6, w=y-4$ b. $t=x+6, w=y+4$
c. $t=x-6, w=y+4$ d. $t=x-6, w=y-4$

5. If $(6x^my^2+e^x)dx+(2x^{(m+1)}y-\sin y)dy=0$ is an exact, then

a. $m=2$ b. $m=6$ **c.** $m=5$ d. $m=1$

6. An integrating factor of the following **linear** equation $(x^2+1)y'+2xy=\tan(x^2)$ is

a. $\mu(x)=\ln(1-x^2)$ b. $\mu(x)=\ln(x^2+1)$ c. $\mu(x)=1-x^2$ **d.** $\mu(x)=x^2+1$

Question two : Find a general solution to the following equations (2 points each)

(a) $y''-\alpha^2y=0$ (where α is a positive constant).
Aux.Equ. $r^2-\alpha^2=0\Rightarrow r=\pm\alpha$
$y(x)=c_1e^{\alpha x}+c_2e^{-\alpha x}$

(b) $y''-14y'+49y=0$.
Aux.Equ. $r^2-14r+49=0\Rightarrow r=7$ with multiplicity 2
$y(x)=c_1e^{7x}+c_2xe^{7x}$

(c) $2y''+2y'+5y=0$.
Aux.Equ. $2r^2+2r+5=0\Rightarrow \frac{-2\pm\sqrt{-36}}{4}\Rightarrow \frac{-1}{2}\pm\frac{3}{2}i$
$y(x)=c_1e^{-0.5x}\sin(1.5x)+c_2e^{-0.5x}\cos(1.5x)$

Question three : Consider the following differential equation (7 points)

$$(3x^2y+y^2+ye^x)dx+(2x^3+3xy+2e^x)dy=0 \qquad (1)$$

(a) Show that equation (1) is non-exact.
$M_y = 3x^2 + 2y + e^x,\ \ N_x = 6x^2 + 3y + 2e^x\ M_y \neq N_x$ so the equation is non-exact

(b) Find an integrating factor of equation (1).

$$\frac{N_x - M_y}{M} = \frac{6x^2 + 3y + 2e^x - (3x^2 + 2y + e^x)}{3x^2y + y^2 + ye^x} = \frac{3x^2 + y + e^x}{y(3x^2 + y + e^x)} = \frac{1}{y}$$

$\mu(y) = e^{\int \frac{1}{y} dy} = y$

(c) Find a general solution to equation (1).
Multiply equ. (1) by y, we get
$(3x^2y^2 + y^3 + y^2e^x)dx + (2x^3y + 3xy^2 + 2e^xy)dy = 0$
the new one is an exact
$f(x, y) = c,\ f(x, y) = \int (3x^2y^2 + y^3 + y^2e^x)dx = \int (2x^3y + 3xy^2 + 2e^xy)dy$
$x^3y^2 + y^3x + y^2e^x = c$