Question (1): (9 points)
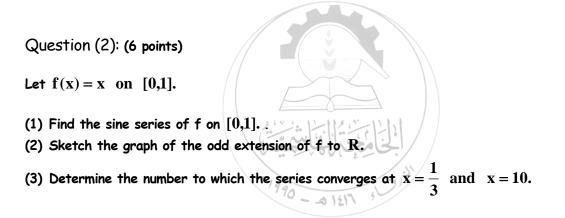
Find the eigenvalues and eigefunctions of the following Sturm Liouville problem:

$$x^2 y'' + x y' + \lambda y = 0, \qquad 1 \le x \le e,$$
$$y(1) = 0, \quad y(e) = 0.$$

Question (2): (6 points)

Let $f(x) = x$ on $[0,1]$.

(1) Find the sine series of f on $[0,1]$.

(2) Sketch the graph of the odd extension of f to $\mathbf{R}$.

(3) Determine the number to which the series converges at $x = \dfrac{1}{3}$ and $x = 10$.

Question (3): (10 points)
Find the general solution to the following IBVP

$$u_t - 9 u_{yy} = 0 \qquad\qquad 0 < y < 10$$

$$u(t,0) = 0; \quad u(t,10) = 0; \qquad 0 < y < 10$$
$$u(0, y) = \sin(y).$$