Procedures are developed for expressing certain types of signals as mather ical functions. Nonsinusoidal periodic functions are introduced, and a technique writing equations for these periodic signals is presented.

A general technique is then given for expressing the output of a continu time system that is an interconnection of systems. An example is given of a feedb control system. Several important properties of systems are defined, and procedu are given to determine whether a system possesses these properties.

This chapter is devoted to continuous-time signals and systems. In Chapte the same topics are developed as they relate to discrete-time signals and syste Many of the topics are identical; however, in some cases, there are significant dif ences. (See Table 2.4.)

PROBLEMS

2.1. The signals in Figure P2.1 are zero except as shown.

(a) For the signal $x(t)$ of Figure P2.1(a), plot

(i) $x(-t/3)$ **(ii)** $x(-t)$
(iii) $x(3 + t)$ **(iv)** $x(2 - t)$

Verify your results by checking at least two points.

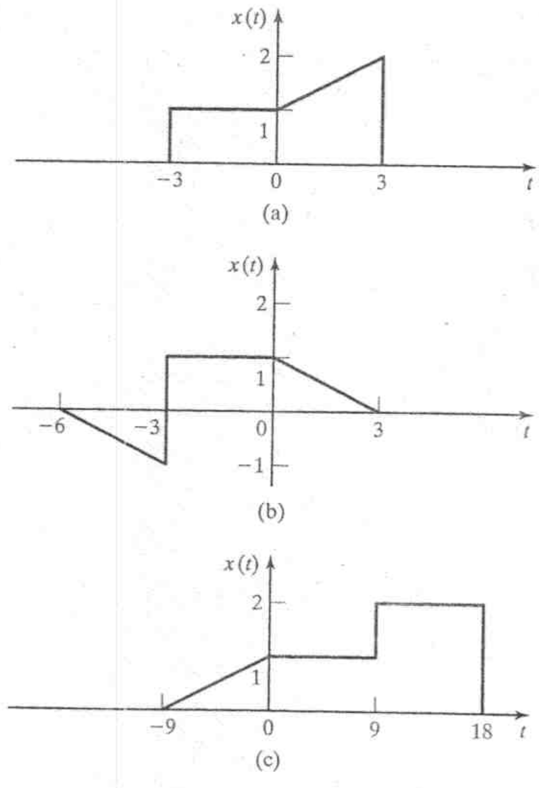


Figure P2.1

(b) Repeat (a) for the signal $x(t)$ of Figure P2.1(b).

(c) Repeat (a) for the signal $x(t)$ of Figure P2.1(c).

2.2. The signals in Figure P2.1 are zero except as shown.

(a) For the signal $x(t)$ of Figure P2.1(a), plot

(i) $4x(t) - 2$ **(ii)** $2x(t) + 2$

(iii) $2x(2t) + 2$ **(iv)** $-4x(t) + 2$

Verify your results by checking at least two points.

(b) Repeat part (a) for the signal $x(t)$ of Figure P2.1(b).

(c) Repeat part (a) for the signal $x(t)$ of Figure P2.1(c).

2.3. You are given the two signals in Figure P2.3.

(a) Express $y(t)$ as a function of $x(t)$.

(b) Verify your result by checking at least three points in time.

2.4. You are given the signals $x(t)$ and $y(t)$ in Figure P2.4.

(a) Express $y(t)$ as a function of $x(t)$.

(b) Verify your results by checking at least three points in time.

(c) Express $x(t)$ as a function of $y(t)$.

(d) Verify the results of part (c) by checking at least three points in time.

2.5. Given

$$x(t) = 4(t + 2)u(t + 2) - 4tu(t) - 4u(t - 2) - 4(t - 4)u(t - 4) + 4(t - 5)u(t - 5),$$

find and sketch $x(2t - 4)$.

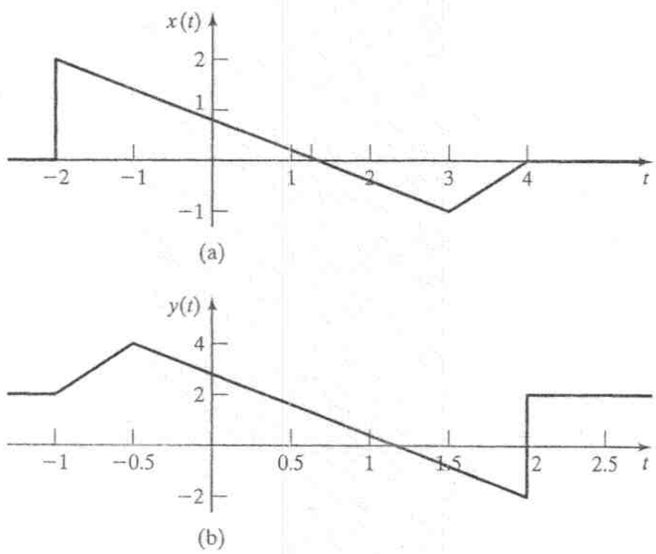


Figure P2.3

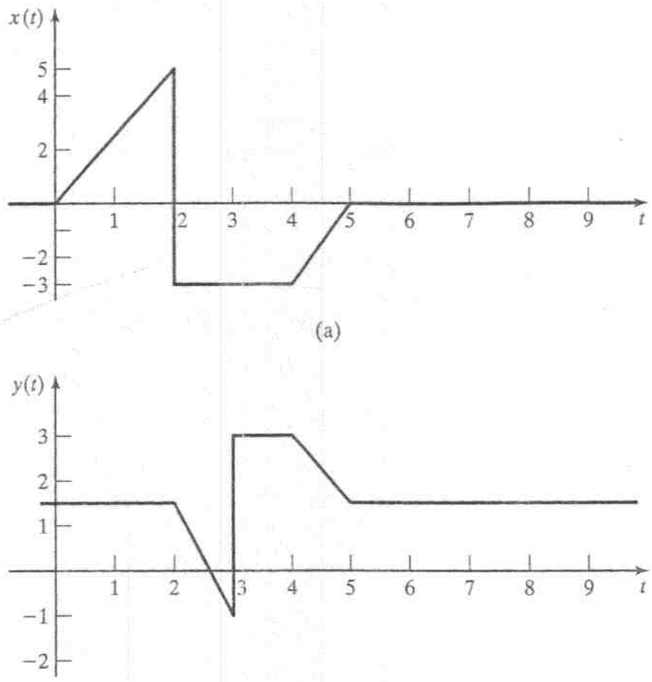


Figure P2.4

2.6. Given

$$x(t) = 5u(t + 2) - u(t) + 3u(t - 2) - 7u(t - 4),$$

find and sketch $x(-2t - 4)$.

2.7. Plot the even and odd parts of the signal of

(a) Figure P2.1(a)
(b) Figure P2.1(b)
(c) Figure P2.1(c)
(d) Figure P2.4(a)
(e) Verify your results, using (2.11).

2.8. For each of the signals given, determine mathematically whether the signal is even, odd, or neither. Sketch the waveforms to verify your results.

(a) $x(t) = -4t$
(b) $x(t) = e^{-|t|}$
(c) $x(t) = 5\cos 3t$
(d) $x(t) = \sin(3t - \frac{\pi}{2})$
(e) $x(t) = u(t)$

2.9. The average value A_x of a signal $x(t)$ is given by

$$A_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt.$$

Let $x_e(t)$ be the even part and $x_o(t)$ be the odd part of $x(t)$.

(a) Show that

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_o(t)dt = 0.$$

(b) Show that

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_e(t)dt = A_x.$$

(c) Show that $x_o(0) = 0$ and $x_e(0) = x(0)$.

2.10. Give proofs of the following statements:

(a) The sum of two even functions is even.
(b) The sum of two odd functions is odd.
(c) The sum of an even function and an odd function is neither even nor odd.
(d) The product of two even functions is even.
(e) The product of two odd functions is even.
(f) The product of an even function and an odd function is odd.

2.11. Given in Figure P2.11 are the parts of a signal $x(t)$ and its odd part $x_o(t)$, for $t \geq 0$ only; that is, $x(t)$ and $x_o(t)$ for $t < 0$ are not given. Complete the plots of $x(t)$ and $x_e(t)$, and give a plot of the even part, $x_e(t)$, of $x(t)$. Give the equations used for plotting each part of the signals.

2.12. Prove mathematically that the signals given are periodic. For each signal, find the fundamental period T_0 and the fundamental frequency ω_0.

(a) $x(t) = 7\sin 3t$
(b) $x(t) = \sin(8t + 30°)$

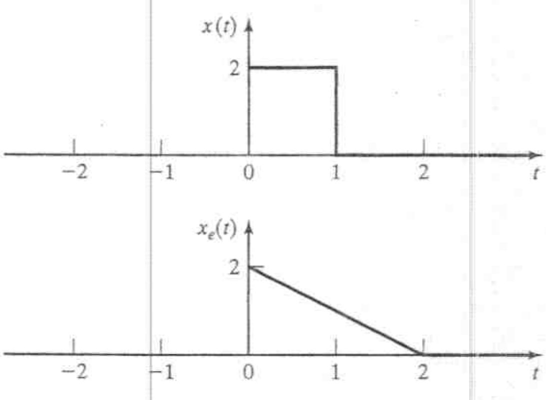


Figure P2.11

(c) $x(t) = e^{j2t}$
(d) $x(t) = \cos t + \sin 2t$
(e) $x(t) = e^{j(5t+\pi)}$
(f) $x(t) = e^{-j10t} + e^{j15t}$

2.13. For each signal, if it is periodic, find the fundamental period T_0 and the fundamental frequency ω_0. Otherwise, prove that the signal is not periodic.

(a) $x(t) = \cos 3t + \sin 5t$.
(b) $x(t) = \cos 6t + \sin 8t + e^{j2t}$.
(c) $x(t) = \cos t + \sin \pi t$.
(d) $x(t) = x_1(t) + x_2(3t)$ where $x_1(t) = \sin(\frac{\pi t}{6})$ and $x_2(t) = \sin(\frac{\pi t}{9})$.

2.14. (a) Consider the signal

$$x(t) = 4\cos(12t + 40°) + \sin 16t.$$

If this signal is periodic, find its fundamental period T_0 and its fundamental frequency ω_0. Otherwise, prove that the signal is not periodic.

(b) Repeat Part (a) for the signal

$$x(t) = \cos 4t + 3e^{-j12t}$$

(c) Repeat Part (a) for the signal

$$x(t) = \cos 2\pi t + \sin 6t.$$

(d) Repeat Part (a) for the signal $x_4(t) = x_1(t) + x_2(t) + x_3(t)$, where

$$x_1(t) = \cos(\pi t),\ x_2(t) = \sum_{n=-\infty}^{\infty} rect\left(\frac{t+n}{0.2}\right), \text{ and } x_3(t) = 4\sin\left(\frac{5\pi}{6}t + \frac{\pi}{4}\right).$$

2.15. Suppose that $x_1(t)$ is periodic with period T_1 and that $x_2(t)$ is periodic with period T_2.

(a) Show that the sum

$$x(t) = x_1(t) + x_2(t)$$

is periodic only if the ratio T_1/T_2 is equal to a ratio of two integers k_2/k_1.

(b) Find the fundamental period T_0 of $x(t)$, for $T_1/T_2 = k_2/k_1$.

2.16. Find

$$\int_{-\infty}^{\infty} \delta(at - b)\sin^2(t - 4)dt,$$

where $a > 0$. (*Hint*: Use a change of variables.)

2.17. Express the following in terms of $x(t)$:

$$y(t) = \frac{1}{2}\int_{-\infty}^{\infty} x(\tau)[\delta(\tau - 2) + \delta(\tau + 2)]d\tau.$$

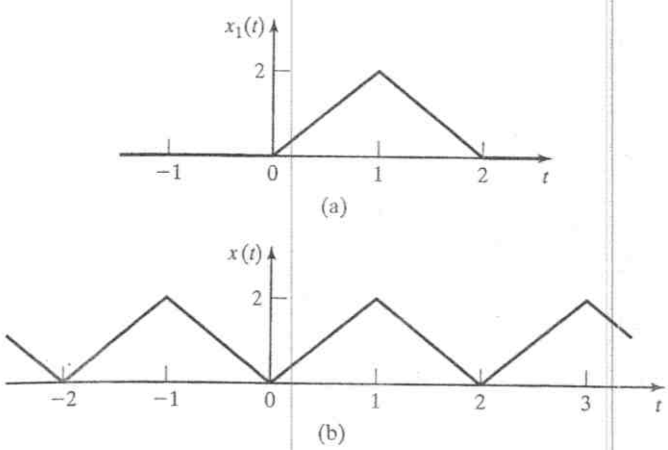


Figure P2.18

2.18. Consider the triangular pulse of Figure P2.18(a).

(a) Write a mathematical function for this waveform.

(b) Verify the results of Part (a), using the procedure of Example 2.12.

(c) Write a mathematical function for the triangular wave of Figure P2.18(b), using the results of Part (a).

2.19. Consider the trapezoidal pulse of Figure P2.19(a).

(a) Write a mathematical function for this waveform.

(b) Verify the results of Part (a), using the procedure of Example 2.12.

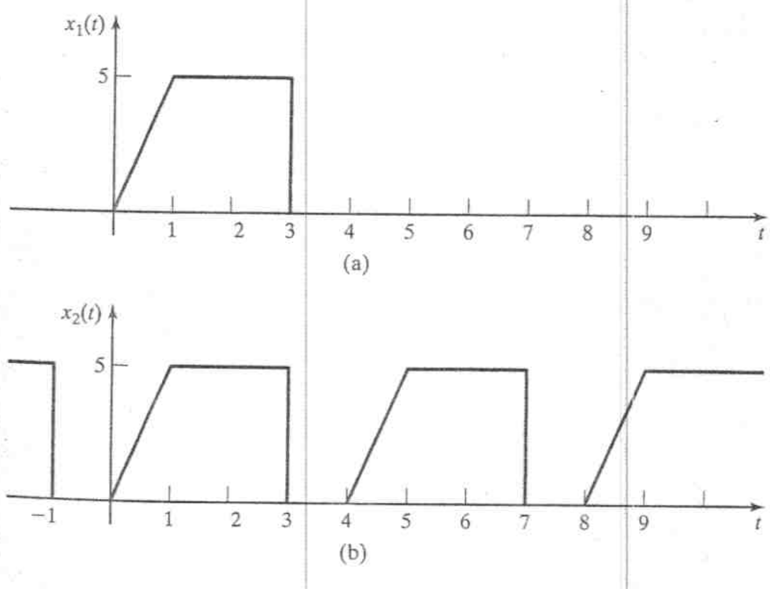


Figure P2.19

(c) Write a mathematical function for the waveform of Figure P2.19(b), using the results of Part (a).

2.20. (a) Prove the time-scaling relation in Table 2.3:

$$\int_{-\infty}^{\infty} \delta(at)dt = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta(t)dt.$$

(*Hint*: Use a change of variable.)

(b) Prove the following relation from Table 2.3:

$$u(t - t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)d\tau.$$

(c) Evaluate the following integrals:

(i) $\int_{-\infty}^{\infty} \cos(2t)\delta(t)dt$

(ii) $\int_{-\infty}^{\infty} \sin(2t)\delta(t - {}^{\pi}/_{4})dt$

(iii) $\int_{-\infty}^{\infty} \cos[2(t - {}^{\pi}/_{4})]\delta(t - {}^{\pi}/_{4})dt$

(iv) $\int_{-\infty}^{\infty} \sin[(t - 1)]\delta(t - 2)dt$

(v) $\int_{-\infty}^{\infty} \sin[(t - 1)]\delta(2t - 4)dt$

2.21. Express the following functions in the general form of the unit step function $u(\pm t - t_0)$:

(a) $u(2t + 6)$
(b) $u(-2t + 6)$
(c) $u(\frac{t}{4} + 2)$
(d) $u(\frac{t}{4} - 2)$

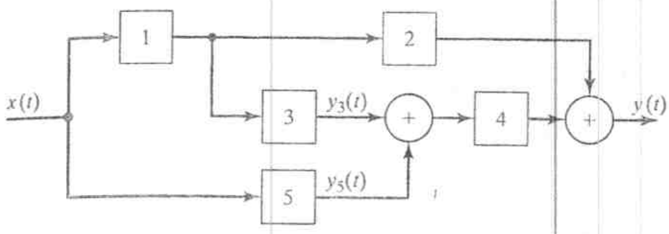
In each case, sketch the function derived.

2.22. Express each given signal in terms of $u(t - t_0)$. Sketch each expression to verify the results.

(a) $u(-t)$
(b) $u(3 - t)$
(c) $tu(-t)$
(d) $(t - 3)u(3 - t)$

2.23. (a) Express the output $y(t)$ as a function of the input and the system transformations, in the form of (2.56), for the system of Figure P2.23(a).

(b) Repeat Part (a) for the system of Figure P2.23(b).



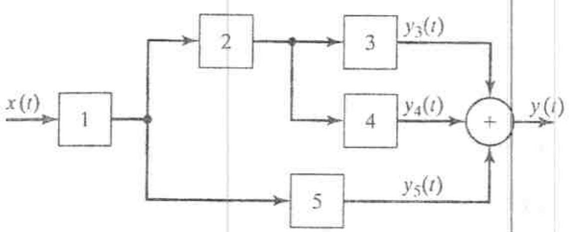


Figure P2.23

(c) Repeat Part (a) for the case that the summing junction with inputs $y_3(t)$ and $y_5(t)$ is replaced with a multiplication junction, such that its output is the product of these two signals.

(d) Repeat Part (b) for the case that the summing junction with inputs $y_3(t)$, $y_4(t)$, and $y_5(t)$ is replaced with a multiplication junction, such that its output is the product of these three signals.

2.24. Consider the feedback system of Figure P2.24. Express the output signal as a function of the transformation of the input signal, in the form of (2.58).

2.25. Consider the feedback system of Figure P2.25. Express the output signal as a function of the transformation of the input signal, in the form of (2.58). The minus sign at the summing junction indicates that the signal is subtracted.

2.26. **(a)** Determine whether the system described by

$$y(t) = \int_{t}^{t+1} x(\tau-\alpha)d\tau$$

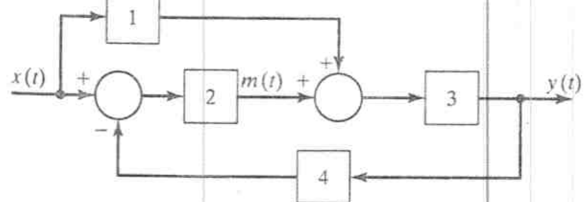


Figure P2.24

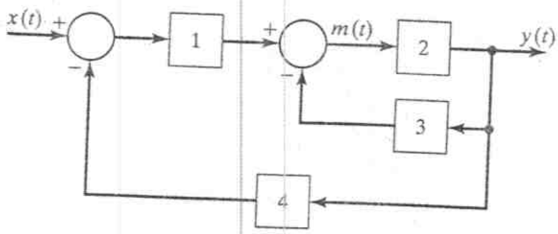


Figure P2.25

(where α is a constant) is

(i) memoryless,
(ii) invertible,
(iii) stable,
(iv) time invariant, and
(v) linear.

(b) For what values of the constant α is the system causal?

2.27. **(a)** Determine whether the system described by

$$y(t) = \cos[x(t-1)]$$

is

(i) memoryless,
(ii) invertible,
(iii) causal,
(iv) stable,
(v) time invariant, and
(vi) linear.

(b) Repeat Part (a) for

$$y(t) = 3x(3t+3).$$

(c) Repeat Part (a) for

$$y(t) = \ln[x(t)].$$

(d) Repeat Part (a) for

$$y(t) = e^{tx(t)}.$$

(e) Repeat Part (a) for

$$y(t) = 7x(t) + 6.$$

(f) Repeat Part (a) for

$$y(t) = \int_{-\infty}^{t} x(5\tau)d\tau.$$

(g) Repeat Part (a) for

$$y(t) = e^{-j\omega t}\int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau.$$

(h) Repeat Part (a) for

$$y(t) = \int_{t-1}^{t} x(\tau)d\tau.$$

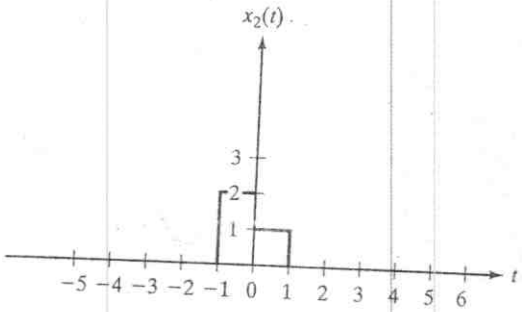


Figure P2.28

2.28. **(a)** You are given an LTI system. The response of the system to an input

$$x_1(t) = u(t) - u(t-1)$$

is a function $y_1(t)$. What is the response of the system to the input $x_2(t)$ in Figure P2.28 in terms of $y_1(t)$?

(b) You are given another LTI system with the input shown in Figure P2.28. Find the output $y_2(t)$ in terms of the system's output, $y_1(t)$, if $y_1(t)$ is in response to the input $x_1(t) = 2u(t-1) - u(t-2) - u(t-3)$.

2.29. Determine whether *the ideal time delay*

$$y(t) = x(t - t_0)$$

is

(i) memoryless, **(ii)** invertible,
(iii) causal, **(iv)** stable,
(v) time invariant, and **(vi)** linear.

2.30. Let $h(t)$ denote the response of a system for which the input signal is the unit impulse function $\delta(t)$. Suppose that $h(t)$ for a *causal* system has the given even part $h_e(t)$ for $t > 0$:

$$h_e(t) = t[u(t) - u(t-1)] + u(t-1), t > 0.$$

Find $h(t)$ for all time, with your answer expressed as a mathematical function.

2.31. **(a)** Sketch the characteristic y versus x for the system $y(t) = |x(t)|$. Determine whether this system is

(i) memoryless, **(ii)** invertible,
(iii) causal, **(iv)** stable,
(v) time invariant, and **(vi)** linear.

(b) Repeat Part (a) for

$$y(t) = \begin{cases} x(t), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(c) Repeat Part (a) for

$$y(t) = \begin{cases} -10, & x < -1 \\ 10x(t), & |x| \leqq 1. \\ 10, & x > 1 \end{cases}$$

(d) Repeat Part (a) for

$$y(t) = \begin{cases} 2, & 2 < x \\ 1, & 1 < x \leqq 2 \\ 0, & 0 < x \leqq 1 \\ -1, & -1 < x \leqq 0 \\ -2, & x \leqq -1 \end{cases}.$$

2.1 iV
2.2 iii
2.3
2.4
2.5
2.7 a
2.8 b
2.11
2.13 b
2.14 a
2.14 b
2.18
2.19
2.22
2.23
2.27
2.30