Name: ....

Number: ....

MAXIMUM	SCORE
20	

The Hashemite University
Electrical Engineering Department

First Exam *Date: 20th Jan 2011* *Second Semester 2011* *Exam Duration: 1 hour*
Instructor: Ashraf ALI

Q1 Answer the following question in the answer sheet showing your full work: (12 points)

For questions 1 and 2 Let z1= 2+3j , and z2 = 4-5j, find in the form (x+jy) the following:

A) $\overline{\left(z1/z2 \right)}$

B) $\operatorname{Re}\left(\dfrac{1}{z_1^{2}}\right)$

C) Find the solutions of the following complex equation:
$8z^2-(36-6j)z+42-11j=0$

D) Find the roots of the following: $\sqrt[3]{3+j4}$

E) Solve for z in the following equation: $e^{z}=4-3j$

F) Find in the form x+jy the following : $(1-j)^{1+j}$

Q2) Solve the following IVP: (4 Points)

$e^{2x}\left[2\cos y\,dx-\sin y\,dy\right]=0$
$y(0)=0$

Q3) (4 Points) A balloon is equipped with temperature and altitude measuring instruments. The temperature (Tr) on the ground is 10°C.

$$T_r=\frac{10-0.05t}{1+15D}$$

Where Tr is the measured temperature, t is the time, D is the derivative operator.

If the balloon is released at time zero, and thereafter rises upwards. What temperature does the balloon report after 10 seconds?

A) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}} = \frac{2-3j}{4+5j}\cdot\frac{4-5j}{4-5j} = \frac{-7-22j}{41}$

B) $Re\left\{ 1/z_1^2 \right\} = Re\left\{ 1/(2+3j)^2 \right\} = Re\left\{ \frac{1}{-5+12j} \right\}$

$= Re\left\{ \frac{5-12j}{25+144} \right\} = 5/169$

C) $8z^2 - (36-6j)z + 42 - 11j = 0$

$z_{1,2} = \frac{36-6j}{16} \pm \frac{\sqrt{(36-6j)^2 - 4(8)(42-11j)}}{16} = \cdots$

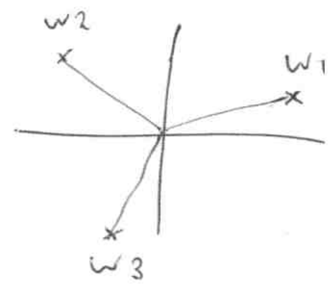
D) $\sqrt[3]{3+j4} = \sqrt[3]{r}\left[\cos\frac{\theta+2\pi k}{3} + j\sin\frac{\theta+2\pi k}{3}\right]$

$\theta = \tan^{-1} 4/3 = 0.3\pi \quad , \quad r = \sqrt{3^2+4^2} = 5$

$w_1 = \sqrt[3]{5}\left[\cos\frac{0.3\pi}{3} + j\sin\frac{0.3\pi}{3}\right]$

$w_2 = \sqrt[3]{5}\left[\cos\frac{0.3\pi+2\pi}{3} + j\sin\frac{0.3\pi+2\pi}{3}\right]$

$w_3 = \sqrt[3]{5}\left[\cos\frac{0.3\pi+4\pi}{3} + j\sin\frac{0.3\pi+4\pi}{3}\right]$



E) $e^z = 4-3j \Rightarrow e^x e^{jy} = 4-3j$

$e^x = |4-3j| = 5 \Rightarrow x = \ln 5 = 1.6$

$e^x[\cos y + j\sin y] = 4-3j \Rightarrow e^x\cos y = 4 \Rightarrow y = \cos^{-1} 4/5 = 0.2\pi$

$z = 1.6 + 0.2\pi j$

F) $(1-j)^{1+j} = e^{1+j[\ln\sqrt{2} \pm j(\pi/4 + 2\pi n)]} = e^{1+j(0.34 \pm j\pi/4)}$

$= e^{(0.34 + 0.34j + \pi/4\, j - \pi/4)} = e^{0.34-\pi/4 + j(0.34+\pi/4)}$

$= e^{0.34-\pi/4}\left(\cos(0.34+\pi/4) + j\sin(0.34+\pi/4)\right)$

Q2 $2e^{2x}\cos y\,dx - e^{2x}\sin y\,dy = 0$

$M(x,y) = 2e^{2x}\cos y \Rightarrow M_y = -2e^{2x}\sin y$

$N(x,y) = -e^{2x}\sin y \Rightarrow N_x = -e^{2x}\sin y$ Exact

$u = \int M(x,y)\,dx + L(y) = k$

$= \int 2e^{2x}\cos y\,dx + L(y) = e^{2x}\cos y + L(y)$

But $\frac{du}{dy} = N(x,y) \Rightarrow -e^{2x}\sin y + L'(y) = -e^{2x}\sin y$

$\Rightarrow L'(y) = 0 \Rightarrow L(y) = \text{Constant} = C$

$u = e^{2x}\cos y + C = k \Rightarrow e^{2x}\cos y + c_1 = 0$

$y(0) = 0 \Rightarrow c_1 = -1 \Rightarrow e^{2x}\cos y - 1 = 0$

Q3 $\dfrac{10 - 0.05t}{1 + 15D} = T_r$ $\quad T(0) = 10$, $T(10) = ?$

$10 - 0.05t = T_r + 15\frac{dT_r}{dt}$

$T_r' + \frac{1}{15}T_r = \dfrac{10 - 0.05t}{15}$ linear FODE (non-homogeneous)

$p(t) = \frac{1}{15}$, $r(t) = \dfrac{10 - 0.05t}{15}$

$h = -\int p(t)\,dt = -t/15$

$T_r = e^{+h}\left(\int e^{-h} r(t)\,dt + k\right)$ general Solution

$= e^{-t/15}\left(\int e^{+t/15}\left(\frac{10 - 0.05t}{15}\right)dt + k\right)$ ----

$= k e^{-t/15} + 10 - 0.05(t - 15)$

$T_r(0) = 10 \Rightarrow k = -0.75$

$T_r = -0.75\,e^{-t/15} + 10 - 0.05(t - 15)$

$T_r(10) = 9.86\ C^{\circ}$