**Managerial Economics Chapter Seven Study Problems**

2. The following demand function for a price-setting firm selling good $X$ was estimated using standard regression analysis:

$$Q = a + bP + cM + dP_R$$

The estimation results are:

| DEPENDENT VARIABLE: | QX | | | |
|---|---|---|---|---|
| OBSERVATIONS: | 275 | | | |
| VARIABLE | PARAMETER ESTIMATE | STANDARD ERROR | T-RATIO | P-VALUE |
| INTERCEPT | 9500.4 | 3350.6 | 2.84 | 0.0049 |
| P | –12.75 | 4.30 | –2.87 | 0.0044 |
| M | –0.0163 | 0.0066 | –2.49 | 0.0135 |
| PR | 5.05 | 1.10 | 4.59 | 0.0001 |

a. Is the sign of $\hat{b}$ as we would have predicted? Why or why not?

b. Is this good normal or inferior good? Explain.

c. Are goods $X$ and $R$ substitutes or complements? Explain.

d. Which coefficients are significant at the 5 percent level of significance? Explain.

e. Using the values $P = \$20$, $M = \$50{,}000$, and $P_R = \$100$, calculate estimates of the following:

(i) The price elasticity of demand is _____.

(ii) The cross-price elasticity of demand is _____.

(iii) The income elasticity of demand is _____.

3. Consider the following nonlinear demand function, which is estimated for a price-setting firm. The method of least-squares is used to estimate the parameters.

$$Q = aP^b M^c P_R^d$$

The results of the estimation are:

| DEPENDENT VARIABLE: | LNQ | R-SQUARE | F-RATIO | P-VALUE ON F | |
|---|---|---|---|---|---|
| OBSERVATIONS: | 26 | 0.9248 | 90.18 | 0.0001 | |
| VARIABLE | | PARAMETER ESTIMATE | STANDARD ERROR | T-RATIO | P-VALUE |
| INTERCEPT | | 3.04 | 1.01 | 3.01 | 0.0064 |
| LNP | | –1.90 | 0.48 | –3.96 | 0.0007 |
| LNM | | 2.16 | 0.675 | 3.20 | 0.0041 |
| LNPR | | 0.78 | 0.169 | 4.62 | 0.0001 |

a. Before the nonlinear demand equation can be estimated using regression analysis, the demand equation must be transformed into the following linear form:

ln $Q$ = _____.

b. Are the parameter estimates statistically significant at the 5 percent level of significance?

c. The estimated value of $a$ is equal to _____.

d. Is this good a normal or inferior good?

e. Is this good a substitute or complement with respect to related good $R$?

f. Compute estimates of the following elasticities:

(i) The price elasticity of demand is _____.

(ii) The income elasticity of demand is _____.

(iii) The cross-price elasticity of demand is _____.

g. A 23.15 percent decrease in household income, holding all other things constant, will cause quantity demanded to _____ (increase, decrease) by _____ percent.

h. All else constant, a 4 percent increase in price causes quantity demanded to _____ (increase, decrease) by _____ percent.

i. A 12.82 percent decrease in the price of $R$, holding all other things constant, will cause quantity demanded to _____ (increase, decrease) by _____ percent.

4. For the past 12 months you have been the night manager of Dixie Fried Chicken. In order to evaluate your performance as a manager, your boss estimates the following linear trend equation for nighttime sales ($Q_t$) over the last 12 months ($t$ = 1,..,12):

$$Q_t = a + bt$$

where $Q_t$ is the number of pieces of chicken sold nightly. The results of the regression are as follows:

| DEPENDENT VARIABLE: | QT | R-SQUARE | F-RATIO | P-VALUE ON F | |
|---|---|---|---|---|---|
| OBSERVATIONS: | 12 | 0.8991 | 89.108 | 0.0001 | |
| VARIABLE | | PARAMETER ESTIMATE | STANDARD ERROR | T-RATIO | P-VALUE |
| INTERCEPT | | 175.0 | 38.88 | 4.50 | 0.0011 |
| T | | 16.0 | 6.4 | 2.50 | 0.0314 |

a. Evaluate the statistical significance of the estimated coefficients. Does this estimation indicate a significant trend, either upward or downward, in sales during your tenure as night manager?

b. Perform an *F*-test for significance of the trend equation at the 5 percent level of significance.

c. If your boss uses the estimated linear trend to forecast your sales for months 14 and 16, how many units does he expect you to sell in these months?

$\hat{Q}_{t=14}$ = _____ and $\hat{Q}_{t=16}$ = _____

d. Comment on the precision of these two forecasts.

5. Suppose you manage the pro shop at a golf club in Miami and would like to be able to forecast the number of golf cart rentals on a quarterly basis. A simple linear trend model must account for the fact that demand for golf carts is always higher in the winter quarters (quarters 1 and 4) as tourists from northern states vacation in Florida. You decide to estimate the following equation:

$$Q_t = a + bt + cD$$

where $D$ is a dummy variable equal to 1 for quarters 1 and 4, and zero otherwise. Using quarterly data from 2007–2010 ($t$ = 1,...,16), you obtain the following estimation results:

| DEPENDENT VARIABLE: | QT | R-SQUARE | F-RATIO | P-VALUE ON F | |
|---|---|---|---|---|---|
| OBSERVATIONS: | 16 | 0.9510 | 126.153 | 0.0001 | |
| VARIABLE | | PARAMETER ESTIMATE | STANDARD ERROR | T-RATIO | P-VALUE |
| INTERCEPT | | 3471.23 | 901.61 | 3.85 | 0.0020 |
| T | | –6.10 | 1.80 | –3.39 | 0.0048 |
| D | | 870.18 | 133.87 | 6.50 | 0.0001 |

a. Perform *t*- and *F*- tests to check for statistical significance (at the 99 percent confidence level) of the individual parameter estimates and the equation.

b. Is the downward trend in golf cart rentals statistically significant?

c. Calculate the intercept for winter quarters and summer quarters. What do the values imply?

d. Use the estimated equation to forecast golf cart rentals in the four quarters of 2011.

## **Answers**:

2. a. Yes, $Q$ should be inversely related to $P$ along a demand curve.

   b. Inferior. Since $\hat{c}$ is negative, $X$ is an inferior good.

   c. Substitutes. Since $\hat{d}$ $(= \Delta Q / \Delta P_R)$ is positive, goods $X$ and $R$ must be substitutes.

   d. The critical value of $t$ for $n \quad k = 275 - 4 = 271$ degrees of freedom and the 5 percent level of significance is (approximately) 1.96. Since $|t| > 1.96$ for all four $t$-ratios, all four parameters are statistically significant. Also, the $p$-values are all smaller than 0.05, which indicates exact levels of significance smaller than 5%.

   e. $\hat{Q} = 8{,}942$ at $P = 20$, $M = 50{,}000$, $P_R = 100$

      (i) $\hat{E} = -0.028$ $[= -12.35 \times (20/8{,}942)]$

      (ii) $\hat{E}_{XR} = 0.056$ $[= 5.05 \times \quad (100/8{,}942)]$

      (iii) $\hat{E}_M = -0.091$ $[= -0.0163 \times (50{,}000/8{,}942)]$

3. a. $\ln Q = \ln a + b \ln P + c \ln M + d \ln P_R$

   b. Yes, the absolute values of all $t$-ratios are greater than 2.074.

   c. $\hat{a} = e^{3.04} = 20.905$

   d. The estimated value of $c$ is positive, and significant, indicating this good is a normal good.

   e. The estimated value of $d$ is positive, and significant, indicating the two goods are substitutes.

      (i) $\hat{E} = \hat{b} = -1.90$, (ii) $\hat{E}_M = \hat{c} = 2.16$, (iii) $\hat{E}_{XR} = \hat{d} = 0.78$

   f. decrease; 50%

   g. decrease; 7.60%

   h. decrease; 10%

4. a. $\hat{a}$ : $p$-value is 0.0011, so $\hat{a}$ is statistically significant at better than the 1% level of significance (or 99% level of confidence). The probability that $a = 0$ (i.e., a Type I error) is quite small, about one-tenth of 1 percent.
   $\hat{b}$ : $p$-value is 0.0314, so $\hat{b}$ is statistically significant at the 5% level of significance (or 95% level of confidence). The probability that $b = 0$ is small, about a 3 percent chance.
   Conclusion: Sales exhibit a statistically significant positive trend over time (i.e., $\hat{b} > 0$ and its $p$-value is acceptably small). The model as a whole is also explaining a statistically significant amount of the total variation in sales, as indicated by the very small $p$-value for the $F$-ratio.

   b. From the $F$-table, $F_{1,10} = 4.96$. Since the calculated $F$-ratio is 89.108, the equation is significant at the 95 percent confidence level or 5 percent significance level.

c. $\hat{Q}_{14} = 175 + 16 \times 14 = 399$

$\hat{Q}_{16} = 175 + 16 \times 16 = 431$

d. We would expect the confidence interval to be smaller for the forecast of sales in the 14th month because 14 is closer to the sample mean value of $t$ ( $\overline{t} = 6.5$) than is 16.

5. a. $t_{\hat{a}} = 3.85 > 3.012 \Rightarrow \hat{a}$ is statistically significant.

$\left|t_{\hat{b}}\right| = \left|-3.39\right| > 3.012 \Rightarrow \hat{b}$ is statistically significant.

$t_{\hat{c}} = 6.5 > 3.012 \Rightarrow \hat{c}$ is statistically significant.

The calculated value of $F$ of 126.153 > $F_{2,13}$ = 6.70 $\Rightarrow$ the equation as a whole is statistically significant.

b. Yes; the exact level of significance is 0.48% $\Rightarrow$ virtually no chance of committing a Type I error (finding significance when there is none).

c. The winter intercept = $\hat{a} + \hat{c} = 4341.41$.

The summer (i.e., "other" months) intercept = $\hat{a} = 3471.23$ .

Since $\hat{c}$ is positive and statistically significant, the regression indicates that golf cart rentals increase in winter months (despite the overall downward trend in rentals).

d. First quarter 2011: $Q_{17} = 3{,}471.23 - 6.10(17) + 870.18(1) = 4{,}237.71$

Second quarter 2011: $Q_{18} = 3{,}471.23 - 6.10(18) + 870.18(0) = 3{,}361.43$

Third quarter 2011: $Q_{19} = 3{,}471.23 - 6.10(19) + 870.18(0) = 3{,}355.33$

Fourth quarter 2011: $Q_{20} = 3{,}471.23 - 6.10(20) + 870.18(1) = 4{,}219.41$