Managerial Economics Chapter Four Study Problems

1. A simple linear regression equation relates G and H as follows:

$$G = a + bH$$

   a. The explanatory variable is _____, and the dependent variable is _____.

   b. The slope parameter is _____, and the intercept parameter is _____.

   c. When H is zero, G equals _____.

   d. For each one unit increase in H, the change in G is _____ units.

4. Thirty data points on Y and X are employed to estimate the parameters in the linear relation

$$Y = a + bX$$

   The computer output from the regression analysis is shown at the top of the next page:

DEPENDENT VARIABLE:	Y	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	30	0.5223	8.747	0.0187	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		800.0	189.125	4.23	0.0029
X		–2.50	0.850	–2.94	0.0187

   a. The equation of the sample regression line is _____.

   b. Test the intercept and slope estimates for statistical significance at the 5 percent significance level. The critical t-value is _____. The parameter estimate for a is _____, which _____ (is, is not) statistically significant. The parameter estimate for b is _____, which _____ (is, is not) statistically significant.

   c. Interpret the p-values for the parameter estimates.

   d. Test the overall equation for statistical significance at the 5 percent significance level. Explain how you performed this test and present your results. Interpret the p-value for the F-statistic.

   e. If X equals 500, the fitted (or predicted) value of Y is _____.

   f. The fraction of the total variation in Y explained by the regression is _____ percent.

5. A manager wishes to determine the relation between a firm's sales and its level of advertising in the newspaper. The manager believes sales (S) and advertising expenditures (A) are related in a nonlinear way:

$$S = a + bA + cA^2 + dA^3$$

   Explain how to transform this nonlinear model into a linear regression model.

6. Suppose Y is related to X, W, and Z in the following nonlinear way:

$$Y = aX^{b}W^{c}Z^{d}$$

a. This nonlinear relation can be transformed into the linear regression model

_____ .

The computer output from the regression analysis is shown below.

DEPENDENT VARIABLE:	LNY	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	25	0.7360	19.52	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		3.1781	1.1010	2.89	0.0088
LNX		–2.173	0.6555	–3.32	0.0033
LNW		1.250	1.780	0.70	0.4902
LNZ		–0.8415	0.1525	–5.52	0.0001

b. At the 99 percent level of confidence, perform t-tests for statistical significance of $\hat{b}$, $\hat{c}$, and $\hat{d}$.

c. This regression leaves _____ percent of the variation in the dependent variable unexplained.

d. The estimated value of a is _____.

e. If $X = 10$, $W = 5$, and $Z = 2$, the expected value of Y is _____.

f. If Z decreases by 10 percent (all other things constant), Y will _____ (increase, decrease) by _____ percent.

g. If W decreases by 12 percent (all other things constant), Y will _____ (increase, decrease) by _____ percent.

7. A multiple regression model, $Y = a + bX + cX^2$, is estimated by creating a new variable named "X2" that equals X^2. A computer package produces the following output:

DEPENDENT VARIABLE:	Y	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	27	0.8766	85.25	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		8000.00	3524.0	2.27	0.0325
X		–12.00	4.50	–2.67	0.0135
X2		0.005	0.002	2.50	0.0197

a. The regression has _____ degrees of freedom.

b. Test to see if the estimates of a, b, and c are statistically significant at the 5 percent significance level.

c. The exact levels of significance of $\hat{a}, \hat{b}$, and $\hat{c}$ are _____, _____, and _____, respectively.

d. _____ percent of the total variation in Y is explained by the regression. _____ percent of the variation in Y is unexplained by the regression.

e. The critical value of the F-statistic at the 5 percent level of significance is _____. Is the overall regression equation statistically significant at the 5 percent level? The exact level of significance of the equation as a whole is _____ percent.

f. If X is equal to 1,200, then $Y =$ _____.

Answers:

1. a. H; G
   b. b; a
   c. a
   d. b

4. a. $Y = 800.0 - 2.50X$
   b. $t_{critical} = 2.048$; 800.0; is; –2.50; is
   c. The p-value gives the probability of committing a Type I error; that is rejecting the hypothesis that a parameter's true value is zero when the parameter value really is zero. For the intercept parameter estimate, there is only a 0.29% chance that $a = 0$, given the t-ratio of 4.23. For the slope parameter estimate, there is only a 1.87% chance that $b = 0$, given the t-ratio of –2.94.
   d. The critical value of the F-statistic is $F_{k-1,n-k} = F_{1,28} = 4.20$. Since the calculated F-statistic, 8.747, exceeds the critical value of F, the equation is statistically significant. The p-value for the F-ratio indicates there is only a 1.82% chance the equation is *not* truly significant when the F-ratio is as large as 8.747.
   e. $-450 = 800.0 + (-2.50)(500) = 800.0 - 1{,}250$
   f. 52.23%

5. Two new variables must be computed and substituted for A^2 and A^3. Let $X = A^2$ and $Z = A^3$ so that the nonlinear relation can be written in linear form as: $S = a + bA + cX + dZ$.

6. a. $\ln Y = \ln a + b \ln X + c \ln W + d \ln Z$
   b. The critical value of the t-statistic for $n - k = 25 - 4 = 21$ degrees of freedom and a 1 percent significance level is 2.831. When t-ratios are negative, their absolute values are used in their t-tests. Since $|t_{\hat{b}}|$ and $|t_{\hat{d}}|$ both exceed 2.831, the estimates of b and d are statistically significant. Since $t_{\hat{c}}$ is less than 2.831, the estimate of c is *not* statistically significant.
   c. 26.4% of the variation in $\ln Y$ is unexplained.
   d. The intercept estimate provides an estimate for $\log a$, (i.e., 3.1781 is an estimate of the *natural log* of a.) Therefore, $\hat{a}$ is found by taking the antilog: $\hat{a} = e^{3.1781} = 24.00$. If the intercept estimate (of $\ln a$) is statistically significant, then the estimate of a (24.00) is also statistically significant. Since the t-statistic for the intercept is 2.89 (= 3.1781 / 1.1010) exceeds 2.831 (barely), the estimate of a ($\hat{a} = 24$) is statistically significant.
   e. When $X = 10$, $W = 5$, and $Z = 2$, $Y = 0.6724$ $[= 24(10)^{-2.1730}(5)^{1.2500}(2)^{-0.8415}]$
   f. increase; 8.415 ($= 10 \times 0.8415$)
   g. decrease; 15 ($= 12 \times 1.2500$)

7. a. $24 = 27 - 3$
   b. The critical value of the t-statistic is 2.064 at the 5% level of significance with 24 degrees of freedom. Since all three calculated t-ratios (2.27, –2.67, and 2.50) exceed the critical t-value, all three parameter estimates are statistically significant at the 5% level of significance.
   c. 3.25%; 1.35%; 1.97%
   d. 87.66%; 12.34%
   e. 3.40; The equation is statistically significant at the 5% level. The p-value for the F-statistic indicates the exact level of significance is much better than 5%: there is less than a 0.01% chance the equation as a whole is not statistically significant.
   f. 800 ($= 8{,}000 - 12 \times 1{,}200 + 0.005 \times 1{,}200^2$)