Managerial Economics Study Problems Chapter Ten

1. Name the following empirical specifications of production and cost functions:
   a. $TVC = aQ + bQ^2 + cQ^3$ _____
   b. $SMC = a + 2bQ + 3cQ^2$ _____
   c. $Q = aK^3L^3 + bK^2L^2$ _____
   d. $AVC = a + bQ + cQ^2$ _____
   e. $Q = AL^3 + BL^2$ _____

2. What restrictions must be placed on the parameters in the empirical production and cost functions in question 1 above?

3. A firm estimates its long-run production function to be

$$Q = -0.008K^3L^3 + 10K^2L^2$$

Suppose the firm employs 15 units of capital.
   a. The equations for the product curves in the short run are:
      TP = _____
      AP = _____
      MP = _____
   b. At _____ units of labor, marginal product of labor begins to diminish.
   c. At _____ units of labor, average product of labor begins to diminish.
   d. Calculate the marginal product and average product of labor when 20 units of labor are employed.
      $MP_{L=20}$ = _____
      $AP_{L=20}$ = _____

4. A firm estimates its cubic production function of the following form

$$Q = AL^3 + BL^2$$

and obtains the following estimation results:

DEPENDENT VARIABLE:	Q	R–SQUARE	F–RATIO	P–VALUE ON F	
OBSERVATIONS:	62	0.7032	142.175	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T–RATIO	P–VALUE
INTERCEPT					
L3		-0.050	0.013	–3.85	0.0003
L2		0.600	0.250	2.40	0.0195

The firm pays $36 per unit for labor services.

a. The estimated total, average, and marginal product functions are:

$Q =$ _____

$AP =$ _____

$MP =$ _____

b. Are the parameters of the correct sign and are they significant? Discuss the *p*-values.

c. Average product reaches its maximum value at _____ units of labor.

d. Average product reaches its maximum value at _____ units of output.

e. At the output level for part *d*, $AVC =$ $ _____ and $SMC =$ $ _____.

f. When labor usage is 7 units, $AVC =$ $ _____ and $SMC =$ $ _____.

5. Consider a firm that estimates the following average variable cost function:

$$AVC = a + bQ + cQ^2$$

The computer printout for the regression analysis is:

DEPENDENT VARIABLE:	AVC	R–SQUARE	F–RATIO	P–VALUE ON F	
OBSERVATIONS:	16	0.9000	58.50	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T–RATIO	P–VALUE
INTERCEPT		75.00	25.00	3.00	0.0102
Q		–2.40	0.40	–6.00	0.0001
Q2		0.06	0.20	3.00	0.0102

a. Determine whether the estimate values of the coefficients indicate a - shaped *AVC* curve at the 5 percent level of significance.

b. The marginal cost function associated with this *AVC* function is

$SMC =$ _____.

c. The total variable cost function associated with this function is

$TVC =$ _____.

d. *AVC* reaches its minimum value at $Q_m =$ _____.

e. Minimum $AVC =$ $_____.

Answers:

1. a. short-run cubic cost function
   b. short-run cubic marginal cost function
   c. long-run cubic production function
   d. short-run cubic average variable cost function
   e. short-run cubic production function

2. a. $a > 0,\ b < 0,\ c > 0$
   b. same as part *a*
   c. $A = a\overline{K}^3 < 0$ and $B = b\overline{K}^2 > 0$
   d. same as part *a*
   e. $A < 0\ \ B > 0$

3. a. $TP = -0.008(15)^3L^3 + 10(15)^2L^2 = -27L^3 + 2{,}250L^2$
   $AP = -27L^2 + 2{,}250L$
   $MP = 3(-27)L^2 + 2(2{,}250)L = -81L^2 + 4{,}500L$
   b. $L_m = -B/3A = -2{,}250/3(-27) = 27.78$ units of labor
   c. $L_a = -B/2A = -2{,}250/2(-27) = 41.67$ units of labor
   d. $MP_{L=20} = -81(20)^2 + 4{,}500(20) = 57{,}600$
   $AP_{L=20} = -27(20)^2 + 2{,}250(20) = 34{,}200$

4. a. $Q = -0.05L^3 + 0.6L^2$
   $AP = -0.05L^2 + 0.6L$
   $MP = 3(-0.05)L^2 + 2(0.6)L = -0.15L^2 + 1.2L$
   b. The signs of both parameters are correct: A is negative, B is positive. The p-values indicate significance at better than the 2 percent level for both parameter estimates.
   c. $L_a = -B/2A = -0.6/-0.1 = 6$
   AP reaches its maximum value when 6 units of labor are employed.
   d. $Q = -0.05(6)^3 + 0.6(6)^2 = 10.8$
   At 10.8 units of output, AP reaches its maximum value.
   e. $AP_{max} = -0.05(6)^2 + 0.6(6) = 1.8$ (or $AP_{max} = Q/L = 10.8/6 = 1.8$)
   So, $AVC = w/AP = 36/1.8 = \$20$
   Since $AP = MP$ when AP is at its maximum value, $AVC = SMC = \$20$ at $L = 6$ and $Q = 10.8$.
   f. When $L = 7$, $AP = 1.75$ and $MP = 1.05$. Thus, $AVC = 36/1.75 = \$20.57$ and $SMC = 36/1.05 = \$34.29$.

5. a. The parameter restrictions are: $a > 0$, $b < 0$, and $c > 0$. In each case, the absolute value of the t-ratio is greater than the critical value of 2.160.
   b. $SMC = 75 - 4.8Q + 0.18Q^2$
   c. $TVC = 75Q - 2.4Q^2 + 0.06Q^3$
   d. $Q_m = -b/2c = 2.4/0.12 = 20$
   f. $AVC_{min} = 75 - 2.4(20) + 0.06(20)^2 = 51$