CH16: Clutches, Brakes, Couplings and Flywheels

These types of elements are associated with rotation and they have in common the function of dissipating, transferring and/or storing rotational energy.

Clutches and Brakes perform the same function where two elements having different velocities are forced to have the same velocity (*zero in the case of brake*) by applying an actuating force.

When analyzing the performance of clutches/brakes, we will look at:

- Actuating force.
- Transmitted torque.
- Energy loss.
- Temperature rise.

➢ The transmitted torque is related to the actuating force, the coefficient of friction and geometry of the brake.

➢ Temperature rise is related to energy loss and geometry of heat-dissipation surfaces.

The basic types of clutches/brakes are:

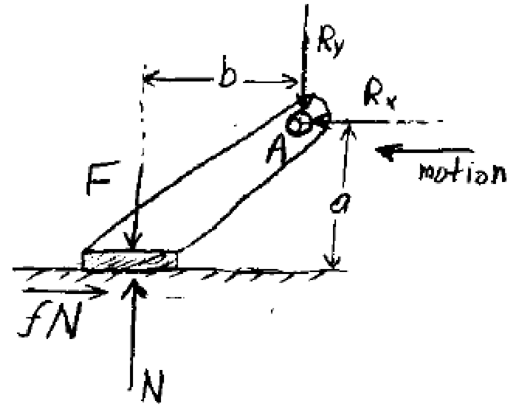
- Rim types with internal or external shoes (*Fig. 16-3 & 16-10*).
- Band types (*Fig. 16-13*).
- Disk or axial types (*Fig. 16-14*).
- Cone types (*Fig. 16-21*).

Static analysis of Clutches and Brakes

The following general procedure can be used for analyzing many types of clutches or brakes:

1- Estimate the pressure distribution on friction surfaces.
2- Find the relation between largest pressure and pressure at any point (*where the largest pressure will be set to be equal to the maximum allowable pressure for the frictional material*).
3- Use static analysis to find braking force or torque and reaction forces.

- Take for example the door-stop shown.
  Applying the analysis procedure:



1- For <u>simplicity</u> since the shoe is short, we assume the pressure to be <u>uniformly distributed</u> over the frictional area.

2- Since pressure is uniform, the pressure at any point is:

$$P = P_{all}$$

where P_{all} is the <u>max allowable</u> pressure for a given shoe material

3- Static analysis,

For uniform pressure, the normal force is:

$$N = P_{all} A$$

where A is the area of the shoe

$$\sum M_A = 0 \quad \rightarrow \quad Fb - Nb \pm fNa = 0$$

Substituting $N = P_{all} A$
and solving for the actuating force F we get:

if the motion in the opposite direction

$$F = \frac{P_{all} A \, (b \pm fa)}{b}$$

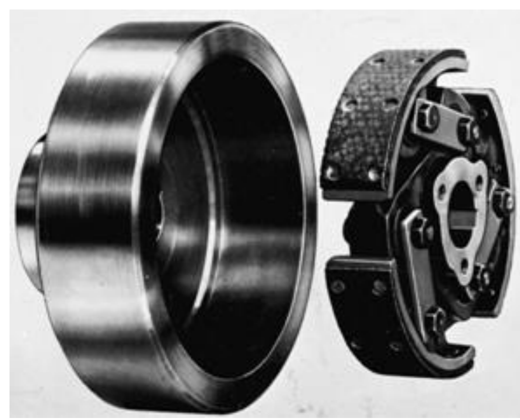
Solving for the reactions:

$$\sum F_x = 0 \quad \rightarrow \quad R_x = f P_{all} A$$
$$\sum F_y = 0 \quad \rightarrow \quad R_y = P_{all} A - F$$

- The actuating force "F" (*the maximum value of force that can be applied without exceeding P_{all} of the shoe material*) can be found from the equation above.
- The value of the term $(b \pm fa)$ needs to be <u>larger than zero</u>.
  - If the term is <u>zero</u> → $F = 0$ and the brake is called "<u>self locking</u>" (*i.e., no actuating force is needed*).
  - It is <u>not desired</u> to have a <u>negative value</u> for the actuating force since it means that a reversed force needs to be applied to prevent the maximum pressure from exceeding the material allowable pressure.
- The brake with the <u>direction of motion shown</u> is called "<u>self energizing</u>" (*i.e., the frictional force helps in reducing the needed actuating force* $(b - fa)$).

- If the motion is in the opposite direction, the brake is called "self de-energizing" (*i.e., the direction of the frictional force makes the needed actuating force to be larger* $(b + fa)$)
- In such example we made a good use of the max allowable pressure of the frictional material because we assumed the pressure to be uniform, thus we can reach the max allowable pressure at all points of contact. However, in reality the pressure distribution is not uniform.
- When designing a brake/clutch system, the designer has a required value of torque and makes the choice of the friction material to be used, the area of the friction surface, the geometry, and then finds the needed actuating force. Thus, an iterative approach will be needed.
- Also, the designer needs to ensure that the system is not "self locking" if that is not required.

Internal Expanding Rim Clutches and Brakes



An internal-shoe rim clutch/brake consists mainly of three elements:

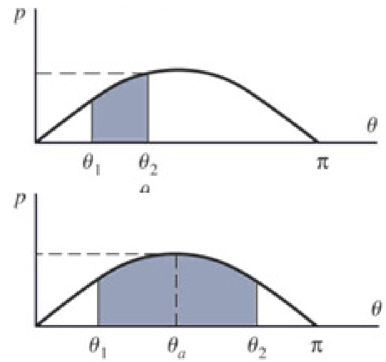
- The mating frictional surfaces.
- The means of transmitting the torque to and from the surfaces.
- The actuating mechanisms.

According to the actuating mechanism, clutches/brakes are further classified as:

- Expanding ring.
- Centrifugal.
- Magnetic.
- Hydraulic & Pneumatic.

- Consider the shown internal expanding rim brake with a single shoe and the rim rotating clockwise.
  - For such configuration we cannot make the assumption that the pressure distribution is uniform. But rather the pressure distribution has the following characteristics:

- ✓ Pressure distribution is <u>sinusoidal</u> with respect to angle θ.
- ✓ For <u>short shoe</u>, the largest pressure P_{all} occurs at the end of the shoe θ_2.
- ✓ For <u>long shoe</u>, the largest pressure P_{all} occurs at $\theta_a = 90^\circ$.



- Using static analysis, the characteristic equation for such brake configuration can be found to be *(see derivation in the text)*:

  - The <u>actuating force</u>:

For clockwise rotation (self-energizing)

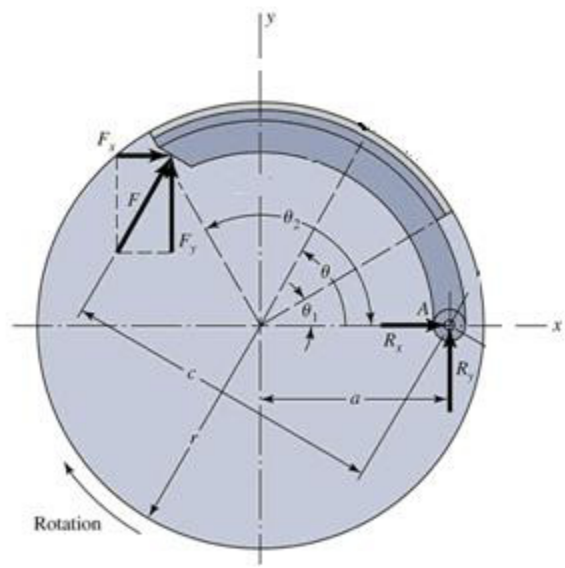
$$F = \frac{M_N - M_f}{c}$$

- ✓ To <u>avoid self locking</u> we should have $M_N > M_f$

<u>or</u>

For counterclockwise rotation (self-deenergizing)

$$F = \frac{M_N + M_f}{c}$$



Where:

c : is the moment arm for the actuating force.

M_N : is the moment of <u>normal forces</u>.

$$M_N = \frac{P_{all}\, b\, r\, a}{\sin\theta_a}\int_{\theta_1}^{\theta_2}(\sin\theta)^2\, d\theta = \frac{P_{all}\, b\, r\, a}{\sin\theta_a}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right|_{\theta_1}^{\theta_2}$$

M_f: is the moment of the <u>frictional force</u>.

$$M_f = \frac{f\, P_{all}\, b\, r}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin\theta\,(r - a\cos\theta)\, d\theta = \frac{f\, P_{all}\, b\, r}{\sin\theta_a}\left[-r\cos\theta - \frac{a}{2}\sin^2\theta\right|_{\theta_1}^{\theta_2}$$

where b: is the face width.

θ_a is the angle defining the location of max pressure "P_{all}".

- The <u>torque</u> applied to the drum by the brake shoe is:

$$T = \frac{f\, P_{all}\, b\, r^2\,(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

- The reaction forces are found as:

$$\begin{cases} R_x = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(A - fB) - F_x \\ R_y = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(B + fA) - F_y \end{cases}$$

Clockwise rotation (self-energizing)

$$\begin{cases} R_x = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(A + fB) - F_x \\ R_y = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(B - fA) - F_y \end{cases}$$

Counterclockwise rotation (self-deenergizing)

Where:

$$\begin{cases} A = \left(\frac{1}{2}(\sin\theta)^2\right|_{\theta_1}^{\theta_2} \\ B = \left(\frac{\theta}{2} - \frac{1}{4}(\sin 2\theta)^2\right|_{\theta_1}^{\theta_2} \end{cases}$$

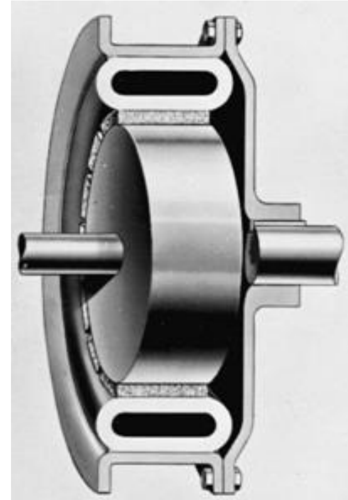
➢ It is important to note that these reactions equations can be used only when: the origin of the axis is at the center of the drum, the positive *x* axis passes through the hinge pin, and the positive *y* axis is in the direction of the shoe.

*See **Example 16-2** from text*

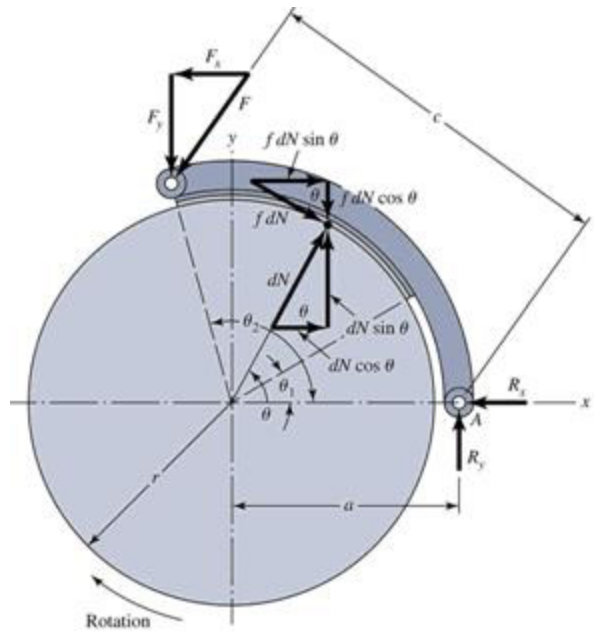
- It should be noted that in *Example 16-2*, the braking capacity of the right-hand shoe is much larger than that of the left-hand shoe (*because the right-hand shoe is self-energizing*).
- If the left hand shoe is turned over such that the hinge is at top, it will become self-energizing as well and the braking capacity will increase. However, if the rim is to turn in the opposite direction, both shoes will be self-deenergizing and the braking capacity will be small. If the same braking capacity is to be obtained, a larger actuating force needs to be applied.

External Contracting Rim Clutches and Brakes

- The figure shows a Pneumatic external contracting clutch/brake system.
  Self-energizing or not?



- It should be clear that self-energizing or deenergizing condition only applies to pivoted-shoe brakes where the summation of moments about the pivoting pin should be zero. If the moment of the frictional force is in the same direction of the moment of the actuating force the brake will be self-energizing and if it is in the opposite direction the brake will be self-deenergizing.



- The figure shows the notation for pivoted external contracting rim brake system.
  - For the shoe configuration shown, a clockwise rotation will be self- deenergizing while a counterclockwise will be self-energizing.
  - The exact same analysis procedure of the internally expanding shoe applies here.
  - The moment of the <u>normal and frictional</u> forces are found using the <u>same equations</u> used before.
  - <u>Torque</u> also is found using the <u>same equation</u> as before.
  - The <u>actuating force</u> and <u>pin reactions</u> are found using:

$$\boxed{F = \frac{M_N + M_F}{c}}$$

$$\begin{cases} R_x = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(A + fB) - F_x \\ R_y = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(fA - B) + F_y \end{cases}$$

(Self-deenergizing)

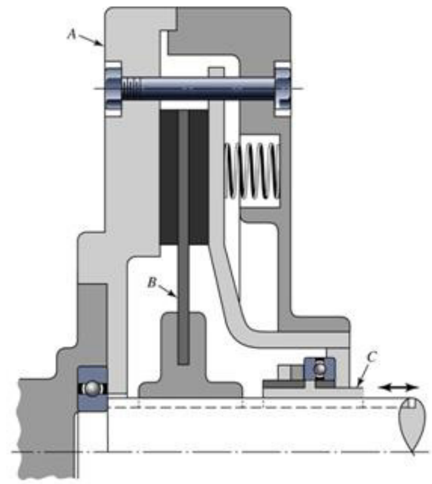
$$\boxed{F = \frac{M_N - M_F}{c}}$$

$$\begin{cases} R_x = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(A - fB) - F_x \\ R_y = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(-fA - B) + F_y \end{cases}$$

(Self-energizing)

- Fig. *16-12* shows a rim brake with symmetrical pivoted shoe.
  - A special case arise there where the moment of the frictional forces about the pivot is zero (*i.e., the same braking capacity will be obtained for clockwise or counterclockwise rotation*).

Frictional Contact Axial Clutches



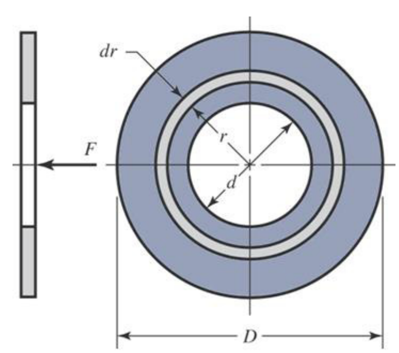
In axial clutches the mating frictional members are moved in a direction parallel to the shaft.

- The advantages of disk clutches over rim clutches include:
  - Freedom from centrifugal effects.
  - The large frictional area that can be installed in a small space.
  - More effective heat dissipation surfaces.
  - More uniform pressure distribution.

- Disk clutches can have single-plate (*friction on two surfaces*), *fig. 16-14*, or multiple-disks *fig. 16-15*.

- There are two methods for analyzing disk clutches, *uniform wear* and *uniform pressure*.
  - If the disk is rigid (*or if springs are used*) a uniform pressure will be applied over the frictional surfaces. This will cause more wear in the outer areas since more work is done at the outer areas. Uniform pressure is usually the case for new clutches.
  - After certain amount of wear has taken place (*more wear at the outer areas*), the pressure distribution will change (*less pressure at the outer areas*) and that makes the wear to become more uniform, this is usually the case with old clutches.

Uniform Wear (old clutches)



After uniform wear condition has been reached, the axial wear can be expressed as:

$$w = f_1 f_2 KPVt \quad \text{*from chapter 12*}$$

- Since wear is uniform and P & V are the only variables then (PV) needs to be constant;

$$PV = Pr\omega = constant, \qquad \text{since } \omega \text{ is constant}$$

$$\rightarrow Pr = constant = P_{max} r_i = P_{all} r_i = P_{all}\frac{d}{2}$$

- Taking a differential ring of radius "r" and thickness"dr"
- The force applied over this area is: $dF = PdA = P\,2\pi\,r\,dr$
- The actuating force over the whole area is:

$$F = \int dF = \int 2\pi\,P\,r\,dr = \int_{\frac{d}{2}}^{\frac{D}{2}} \pi\,P_{all}\,d\,dr = \boxed{\frac{\pi\,P_{all}\,d}{2}(D-d)} \qquad (1)$$

- The torque is found by integrating the product of frictional force and radius.

$$T = \int r\,f\,dF = \int 2\pi\,f\,P\,r^2\,dr = \pi\,f\,P_{all}\,d\int_{\frac{d}{2}}^{\frac{D}{2}} r\,dr = \frac{\pi\,f\,P_{all}\,d}{8}(D^2-d^2)$$

Substituting the value of F we get:

$$\boxed{T = \frac{F\,f}{4}(D+d)} \qquad (2)$$

- ➢ Equation (1) gives the actuating force required for the max pressure to reach the allowable pressure P_{all} and it holds true for any number of friction surfaces.
- ➢ Equation (2) gives the torque capacity associated with P_{all} for one friction surface.

Uniform pressure (new clutches)

When uniform pressure is assumed, the force is simply the product of pressure and area.

- For $P = P_{all}$, the actuating force is:

$$\boxed{F = \frac{\pi\,P_{all}}{4}(D^2-d^2)} \qquad \textit{for any number of frictional surfaces}$$

- The torque can be obtained as before and it is found to be:

$$\boxed{T = \frac{F\,f}{3}\left(\frac{D^3-d^3}{D^2-d^2}\right)} \qquad \textit{for one frictional surface only}$$

- From these equations the advantage of using multiple disks can be seen. For example if two disks are used (*four friction surfaces*), we can get four times the torque for the same actuating force compared to a single friction surface.

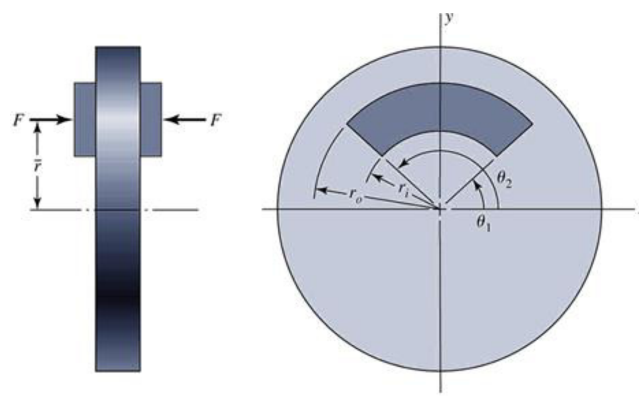
- Comparing the results obtained using uniform wear and uniform pressure equations (*see text*) it can be seen that the difference is not that big. Since new clutches get old anyway, it is suggested to use the uniform wear equations always.

Disk Brakes

There is no fundamental difference between clutches and disk brakes. The same analysis procedure applies to both. *Fig. 16-18* shows a typical automotive disk brake.

- We have seen that rim brakes can be design to be self-energizing (*of course disk brakes cannot*). While self-energization has an advantage in reducing the required braking force, it also has a big disadvantage when the coefficient of friction is reduced (*due to shoes getting wet for example*) where the braking torque will be decreased severely (*because the moment of the frictional forces will be decreased thus decreasing the actual force applied to the shoe*). This is not the case with disk brakes, where a reduction in the coefficient of friction will reduce the braking torque only by the same percentage by which the friction was reduced.

- The equations for disk brakes are developed the same way as for disk clutches, but since the pads are not full circles, double integration (*from r_i to r_o and from θ_1 to θ_2*) is used.
  - Note that the shoe is symmetric with respect to the *y* axis and that the angles θ_1 & θ_2 are measured from the positive *x* axis.



Uniform wear:

- Actuating force: $F = (\theta_2 - \theta_1) P_{all}\, r_i (r_o - r_i)$
- Braking torque: $T = \frac{1}{2}(\theta_2 - \theta_1) f\ P_{all}\, r_i (r_o{}^2 - r_i{}^2)$ *single friction surface*
- Torque equivalent radius:

  (*obtained from $T = f\ F\ r_e$*) $r_e = \frac{r_o + r_i}{2}$
- Location of actuating force: $\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} \frac{r_o + r_i}{2}$

Uniform pressure:

- Actuating force: $F = \frac{1}{2}(\theta_2 - \theta_1) P_{all}\, (r_o{}^2 - r_i{}^2)$
- Braking torque: $T = \frac{1}{3}(\theta_2 - \theta_1) f\ P_{all}\, (r_o{}^3 - r_i{}^3)$ *single friction surface*

- Torque equivalent radius: $$r_e = \frac{2}{3}\frac{r_o{}^3 - r_i{}^3}{r_o{}^2 - r_i{}^2}$$
- Location of "F" : $$\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}\frac{2}{3}\frac{r_o{}^3 - r_i{}^3}{r_o{}^2 - r_i{}^2}$$

➢ Note that the uniform wear or uniform pressure conditions in this case are controlled by controlling the location of the actuating force (i.e., by changing $\bar{r}$).

*See **Example 16-3** from text*

Energy Considerations

When rotating members are caused to stop (in the case of brake) or when members initially at rest are brought up to speed (in the case of clutch), slipping must occur between the mating surfaces causing some of the kinetic energy to transform into thermal energy (heat).

The capacity of a clutch/brake is limited by two factors, the characteristics of the material (*coefficient of friction and the max allowable pressure*) and the ability of the clutch or brake to dissipate heat.

Friction materials

The most important characteristic of friction materials used in brakes/clutches includes:

- High and reproducible coefficient of friction.
- Resistance to environmental conditions, such as moisture.
- The ability to withstand high temperatures and pressure.
- High resistance to wear.

❖ *Table 16-3* gives the important characteristics of some friction materials used in clutches/brakes.

Flywheels

A flywheel is a mechanical energy storage/delivery device that stores energy in the form of kinetic energy.

Flywheels are used to regulate rotational speed/torque for systems having non-uniform torque supply/demand. This happens because a flywheel resists changes in its rotational speed.

- The equation of motion for a flywheel is:

$$\sum M = 0 \quad \rightarrow \quad I\ddot{\theta} = T_i(\theta_i, \omega_i) - T_o(\theta_o, \omega_o)$$

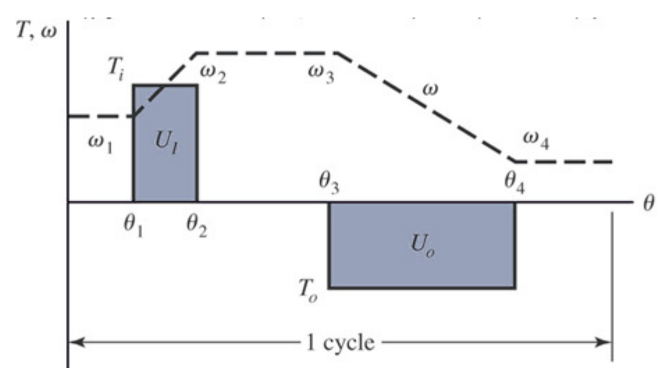
where: T_i& T_o are the input and output torque.
θ & ω are the angular position and angular velocity.

➢ In many cases, the torque depends on θ or ω or on both, for example electric motors torque depends on (ω) only.

- Assuming a rigid shaft $\theta_i = \theta_o = \theta$ and $\omega_i = \omega_o = \omega$, the equation of motion of the flywheel becomes:

$$I\ddot{\theta} = T_i(\theta, \omega) - T_o(\theta, \omega)$$

- This equation can be solved for the instantaneous values of , ω & $\ddot{\theta}$.
- However, those values are not of interest to us when designing a flywheel. We are interested in the overall performance of the flywheel:
  - ✓ What should the moment of inertia be?
  - ✓ How to match the power source to the load?
  - ✓ What are the resulting performance characteristics of the system?



- The figure shows the performance of a flywheel under a hypothetical situation *(friction is neglected)*.
  - The flywheel starts at angular velocity ω_1.
  - Between $\theta_1 \rightarrow \theta_2$ the power source supplies a constant torque T_i causing the flywheel to accelerate from ω_1 to ω_2.
  - Between $\theta_2 \rightarrow \theta_3$ the shaft rotates with zero torque, thus there will be no acceleration $\omega_2 = \omega_3$.
  - Between $\theta_3 \rightarrow \theta_4$, a constant output torque T_o is applied causing the flywheel to slow down from ω_3 to ω_4.

- The work input to the flywheel is: $U_i = T_i(\theta_2 - \theta_1)$
- And the work output is: $U_o = T_o(\theta_4 - \theta_3)$

  - If $U_o > U_i$ then $\omega_4 < \omega_1$.
  - If $U_o = U_i$ then $\omega_4 = \omega_1$.
  - If $U_o < U_i$ then $\omega_4 > \omega_1$

- The work done on the flywheel between θ_1 & θ_2 can also be found as the <u>difference in kinetic energy</u>.

$$U_i = E_2 - E_1 = \frac{1}{2} I\,\omega_2^{\,2} - \frac{1}{2} I\,\omega_1^{\,2}$$

$$\rightarrow \qquad E_2 - E_1 = \frac{1}{2} I(\omega_2^{\,2} - \omega_1^{\,2}) = T(\theta_2 - \theta_1)$$

  - Most of the torque-angular position functions encountered in engineering application are so complicated and they require using numerical integration to find the total work (*area under the curve*).
  - *Fig 16-28* shows the torque of a one cylinder engine for one cycle.
    - ✓ Integrating the curve gives the total energy supplied by the engine. Then dividing the result by the <u>length of one-cycle</u> (*4π*) gives the <u>mean torque</u> value T_m of the engine.

- The <u>allowable range of speed fluctuation</u> is usually defined using the "*<u>Coefficient of speed fluctuation</u>*":

$$\boxed{C_s = \frac{\omega_2 - \omega_1}{\omega}}$$

where ω is the <u>nominal</u> angular velocity: $\omega = \frac{\omega_1 + \omega_2}{2}$

- Substitute C_s in the energy difference equation we get:

$$\boxed{E_2 - E_1 = C_s I \omega^2 = T(\theta_2 - \theta_1)}$$

where $T(\theta_2 - \theta_1)$ is the area under the torque curve

➢ This equation can be used to obtain the flywheel inertia "I" needed to satisfy the required C_s value.

See ***Example 16-6*** *from text*