EXAMPLE 16–2 The brake shown in Fig. 16–8 is 300 mm in diameter and is actuated by a mechanism that exerts the same force F on each shoe. The shoes are identical and have a face width of 32 mm. The lining is a molded asbestos having a coefficient of friction of 0.32 and a pressure limitation of 1000 kPa. Estimate the maximum

(*a*) Actuating force F.
(*b*) Braking capacity.
(*c*) Hinge-pin reactions.

Solution (*a*) The right-hand shoe is self-energizing, and so the force F is found on the basis that the maximum pressure will occur on this shoe. Here $\theta_1 = 0°$, $\theta_2 = 126°$, $\theta_a = 90°$, and $\sin\theta_a = 1$. Also,

$$a = \sqrt{(112)^2 + (50)^2} = 122.7 \text{ mm}$$

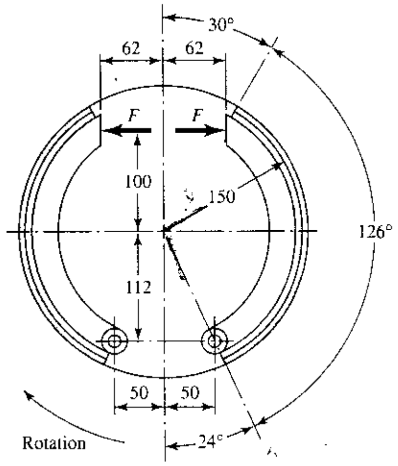
Integrating Eq. (16–2) from 0 to θ_2 yields

$$M_f = \frac{fp_a br}{\sin\theta_a}\left[\left(-r\cos\theta\right)_0^{\theta_2} - a\left(\frac{1}{2}\sin^2\theta\right)_0^{\theta_2}\right]$$

$$= \frac{fp_a br}{\sin\theta_a}\left(r - r\cos\theta_2 - \frac{a}{2}\sin^2\theta_2\right)$$

Figure 16–8

Brake with internal expanding shoes; dimensions in millimeters.



Changing all lengths to meters, we have

$$\begin{aligned} M_f &= (0.32)[1000(10)^3](0.032)(0.150) \\ &\quad\times\left[0.150 - 0.150\cos 126^\circ - \left(\frac{0.1227}{2}\right)\sin^2 126^\circ\right] \\ &= 304\ \text{N}\cdot\text{m} \end{aligned}$$

The moment of the normal forces is obtained from Eq. (16–3). Integrating from 0 to θ_2 gives

$$\begin{aligned} M_N &= \frac{p_a bra}{\sin\theta_a}\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_0^{\theta_2} \\ &= \frac{p_a bra}{\sin\theta_a}\left(\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2\right) \\ &= [1000(10)^3](0.032)(0.150)(0.1227)\left\{\frac{\pi}{2}\frac{126}{180} - \frac{1}{4}\sin[(2)(126^\circ)]\right\} \\ &= 788\ \text{N}\cdot\text{m} \end{aligned}$$

From Eq. (16–4), the actuating force is

Answer

$$F = \frac{M_N - M_f}{c} = \frac{788 - 304}{100 + 112} = 2.28\ \text{kN}$$

(*b*) From Eq. (16–6), the torque applied by the right-hand shoe is

$$\begin{aligned} T_R &= \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} \\ &= \frac{0.32[1000(10)^3](0.032)(0.150)^2(\cos 0^\circ - \cos 126^\circ)}{\sin 90^\circ} = 366\ \text{N}\cdot\text{m} \end{aligned}$$

The torque contributed by the left-hand shoe cannot be obtained until we learn its maximum operating pressure. Equations (16–2) and (16–3) indicate that the frictional and normal moments are proportional to this pressure. Thus, for the left-hand shoe,

$$M_N = \frac{788 p_a}{1000} \qquad M_f = \frac{304 p_a}{1000}$$

Then, from Eq. (16–7),

$$F = \frac{M_N + M_f}{c}$$

or

$$2.28 = \frac{(788/1000)p_a + (304/1000)p_a}{100 + 112}$$

Solving gives $p_a = 443$ kPa. Then, from Eq. (16–6), the torque on the left-hand shoe is

$$T_L = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

Since $\sin\theta_a = \sin 90^\circ = 1$, we have

$$T_L = 0.32[443(10)^3](0.032)(0.150)^2(\cos 0^\circ - \cos 126^\circ) = 162\ \text{N}\cdot\text{m}$$

The braking capacity is the total torque:

Answer
$$T = T_R + T_L = 366 + 162 = 528\ \text{N}\cdot\text{m}$$

(*c*) In order to find the hinge-pin reactions, we note that $\sin\theta_a = 1$ and $\theta_1 = 0$. Then Eq. (16–8) gives

$$A = \frac{1}{2}\sin^2\theta_2 = \frac{1}{2}\sin^2 126° = 0.3273$$

$$B = \frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2 = \frac{\pi(126)}{2(180)} - \frac{1}{4}\sin[(2)(126°)] = 1.3373$$

Also, let

$$D = \frac{p_a b r}{\sin\theta_a} = \frac{1000(0.032)(0.150)}{1} = 4.8\ \text{kN}$$

where $p_a = 1000$ kPa for the right-hand shoe. Then, using Eq. (16–9), we have

$$R_x = D(A - fB) - F_x = 4.8[0.3273 - 0.32(1.3373)] - 2.28\sin 24° = -1.410\ \text{kN}$$

$$R_y = D(B + fA) - F_y = 4.8[1.3373 + 0.32(0.3273)] - 2.28\cos 24° = 4.839\ \text{kN}$$

The resultant on this hinge pin is

Answer
$$R = \sqrt{(-1.410)^2 + (4.839)^2} = 5.04\ \text{kN}$$

The reactions at the hinge pin of the left-hand shoe are found using Eqs. (16–10) for a pressure of 443 kPa. They are found to be $R_x = 0.678$ kN and $R_y = 0.538$ kN. The resultant is

Answer
$$R = \sqrt{(0.678)^2 + (0.538)^2} = 0.866\ \text{kN}$$

The reactions for both hinge pins, together with their directions, are shown in Fig. 16–9.

Figure 16–9