EXAMPLE 6-12

A steel bar undergoes cyclic loading such that $\sigma_{\text{max}} = 420 \text{ MPa}$ and $\sigma_{\text{min}} = -140 \text{ MPa}$. For the material, $S_{nt} = 560 \text{ MPa}$, $S_y = 455 \text{ MPa}$, a fully corrected endurance limit of $S_c = 280 \text{ MPa}$, and f = 0.9. Estimate the number of cycles to a fatigue failure using:

(a) Modified Goodman criterion.

(b) Gerber criterion.

Solution From the given stresses,

$$\sigma_a = \frac{420 - (-140)}{2} = 280 \text{ MPa}$$
 $\sigma_m = \frac{420 + (-140)}{2} = 140 \text{ MPa}$

From the material properties, Eqs. (6-14) to (6-16), p. 277, give

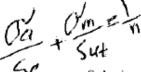
$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.9(560)]^2}{280} = 907 \text{ MPa}$$

$$b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3} \log \left[\frac{0.9(560)}{280}\right] = -0.0851$$

$$N = \left(\frac{S_f}{a}\right)^{1/b} = \left(\frac{S_f}{907}\right)^{-1/0.0851}$$
(1)

where S_f replaced σ_a in Eq. (6–16).

(a) The modified Goods γ_f line is given by Eq. (6-45), p. 298, where the endurance limit S_e is used for infined life. For finite life at $S_f > S_e$, replace S_e with S_f in Eq. (6-45) and rearrange giving



$$S_f = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} = \frac{280}{1 - \frac{140}{560}} = 373 \text{ MPa}$$

Substituting this into Eq. (1) yields

Answer

$$N = \left(\frac{373}{907}\right)^{-1/0.0851} \doteq 3.4(10^4) \text{ cycles}$$

(b) For Gerber, similar to part (a), from Eq. (6–46).

$$S_f = \frac{\sigma_a}{1 - \left(\frac{\sigma_m}{S_{ut}}\right)^2} = \frac{280}{1 - \left(\frac{140}{280}\right)^2} = 299 \text{ MPa}$$

Again, from Eq. (1),

Answer

$$N = \left(\frac{299}{907}\right)^{-1/0.0851} \doteq 4.6(10^5) \text{ cycles}$$

Comparing the answers, we see a large difference in the results. Again, the modified Goodman criterion is conservative as compared to Gerber for which the moderate difference in S_f is then magnified by a logarithmic S_f , N relationship.