Chapter 2

Motion in One Dimension

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Kinematics

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Describes motion while ignoring the external agents that might have caused or modified the motion

For now, will consider motion in one dimension

- Along a straight line

Motion represents a continual change in an object's position.

Types of Motion

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Translational

- An example is a car traveling on a highway.

Rotational

- An example is the Earth’s spin on its axis.

Vibrational

- An example is the back-and-forth movement of a pendulum.

Particle Model

We will use the particle model.

- A particle is a point-like object; has mass but infinitesimal size

Position

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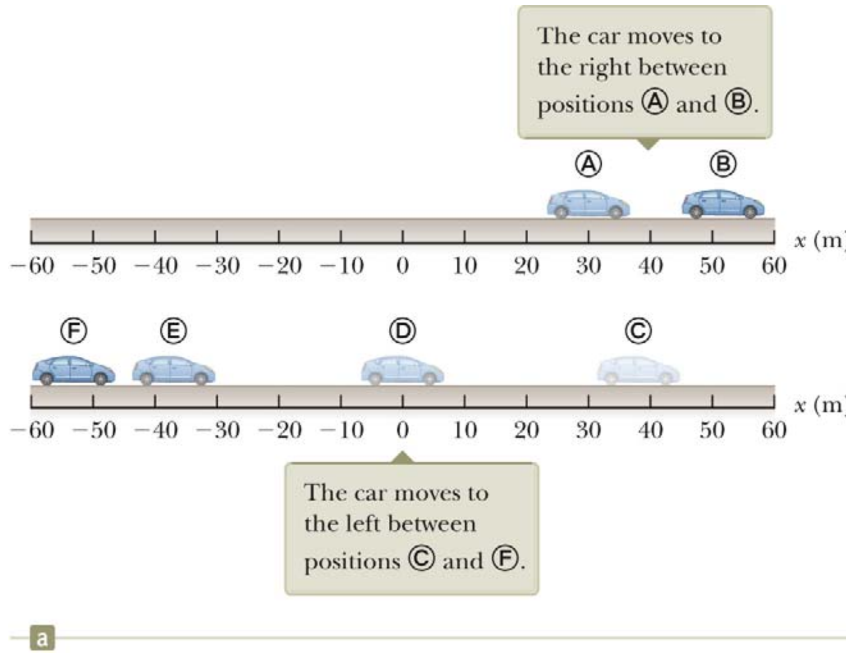
The object's position is its location with respect to a chosen reference point.

- Consider the point to be the origin of a coordinate system.

Only interested in the car's translational motion, so model as a particle



Position-Time Graph

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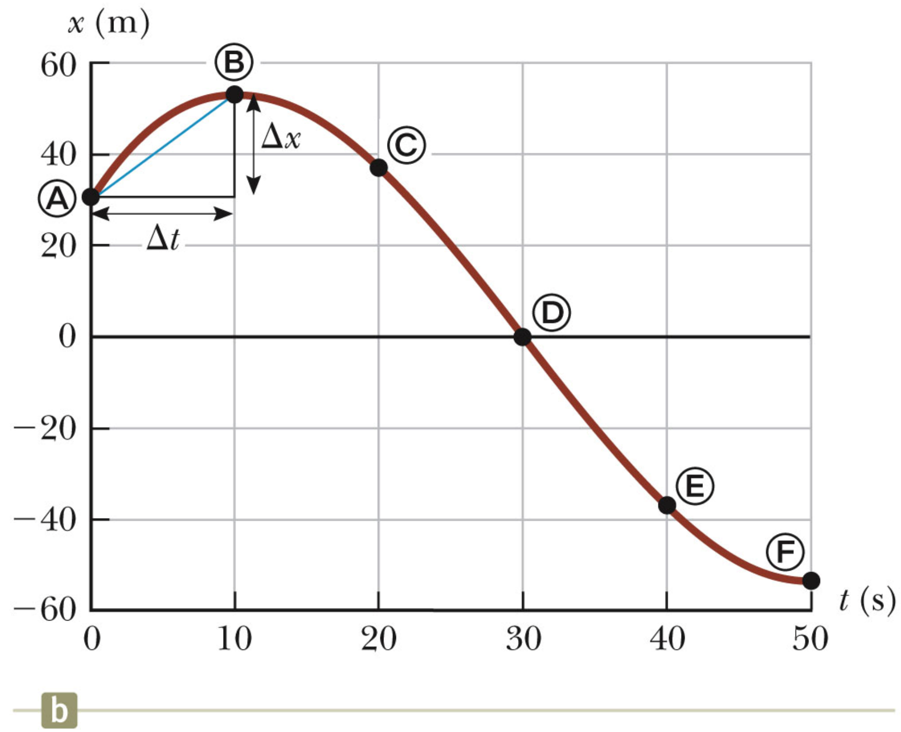
The position-time graph shows the motion of the particle (car).

The smooth curve is a guess as to what happened between the data points.



Data Table

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The table gives the actual data collected during the motion of the object (car).

Positive is defined as being to the right.

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Representations of the Motion of Car

Various representations include:

- Pictorial
- Graphical
- Tablular
- Mathematical
  - The goal in many problems

Using alternative representations is often an excellent strategy for understanding the situation of a given problem.

- For example, compare the different representations of the motion.

Alternative Representations

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Using alternative representations is often an excellent strategy for understanding a problem.

- For example, the car problem used multiple representations.
  - Pictorial representation
  - Graphical representation
  - Tabular representation

Goal is often a mathematical representation

Displacement

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Displacement is defined as the change in position during some time interval.

- Represented as Δx

  $\Delta x \equiv x_f - x_i$

- SI units are meters (m)
- Δx can be positive or negative

Different than distance

- Distance is the length of a path followed by a particle.

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Distance vs. Displacement – An Example

Assume a player moves from one end of the court to the other and back.

Distance is twice the length of the court

- Distance is always positive

Displacement is zero

- $\Delta x = x_f - x_i = 0$ since $x_f = x_i$

Vectors and Scalars

Vector quantities need both magnitude (size or numerical value) and direction to completely describe them.

- Will use + and – signs to indicate vector directions in this chapter

Scalar quantities are completely described by magnitude only.

Average Velocity

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The **average velocity** is rate at which the displacement occurs.

$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The x indicates motion along the x-axis.

The dimensions are length / time [L/T]

The SI units are m/s

Is also the slope of the line in the position – time graph

Average Speed

Speed is a scalar quantity.

- Has the same units as velocity
- Defined as total distance / total time: $v_{avg} \equiv \frac{d}{t}$

The speed has no direction and is always expressed as a positive number.

Neither average velocity nor average speed gives details about the trip described.

Average Speed and Average Velocity

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The average speed is not the magnitude of the average velocity.

- For example, a runner ends at her starting point.
- Her displacement is zero.
- Therefore, her velocity is zero.
- However, the distance traveled is not zero, so the speed is not zero.

Instantaneous Velocity

The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero.

The instantaneous velocity indicates what is happening at every point of time.

Instantaneous Velocity, graph

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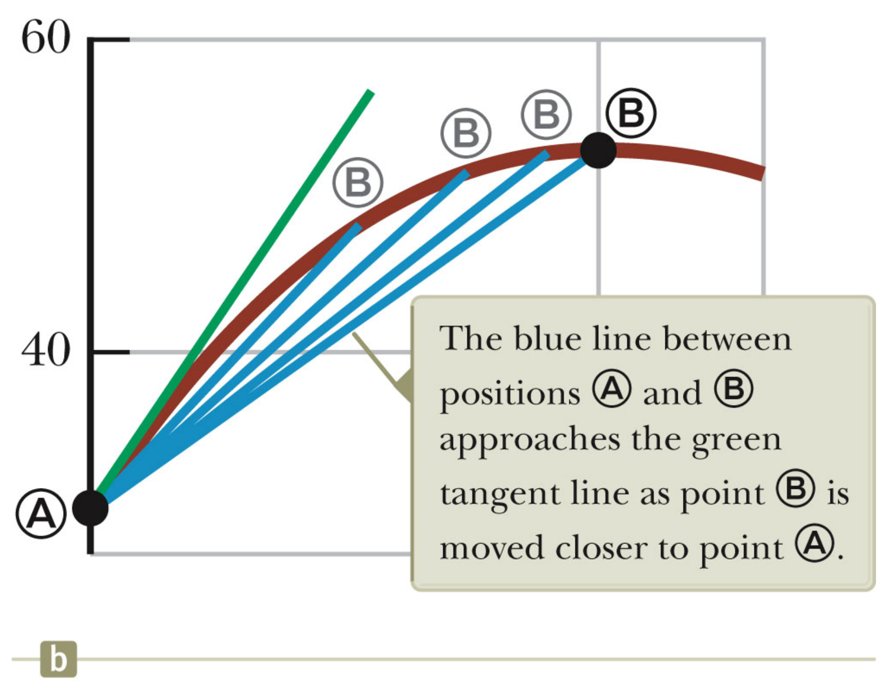
The instantaneous velocity is the slope of the line tangent to the *x* vs. *t* curve.

- This would be the green line.

The light blue lines show that as Δt gets smaller, they approach the green line.



A Note About Slopes

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The slope of a graph of physical data represents the ratio of change in the quantity represented on the vertical axis to the change in the quantity represented by the horizontal axis.

The slope has units

- Unless both axes have the same units

Instantaneous Velocity, equations

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The general equation for instantaneous velocity is:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous velocity can be positive, negative, or zero.

Instantaneous Speed

The instantaneous speed is the magnitude of the instantaneous velocity.

The instantaneous speed has no direction associated with it.

Vocabulary Note

“Velocity” and “speed” will indicate *instantaneous* values.

Average will be used when the average velocity or average speed is indicated.

Analysis Models

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Analysis models are an important technique in the solution to problems.

An analysis model is a description of:

- The behavior of some physical entity, or
- The interaction between the entity and the environment.

Try to identify the fundamental details of the problem and attempt to recognize which of the types of problems you have already solved could be used as a model for the new problem.

Analysis Models, cont

Based on four *simplification models*

- Particle model
- System model
- Rigid object
- Wave

Problem approach

- Identify the analysis model that is appropriate for the problem.
- The model tells you which equation to use for the mathematical representation.

Model: A Particle Under Constant Velocity

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Constant velocity indicates the instantaneous velocity at any instant during a time interval is the same as the average velocity during that time interval.

- $v_x = v_{x,\ avg}$
- The mathematical representation of this situation is the equation.

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \quad or \quad x_f = x_i + v_x \Delta t$$

- Common practice is to let $t_i = 0$ and the equation becomes: $x_f = x_i + v_x\, t$ (for constant v_x)

Particle Under Constant Velocity, Graph

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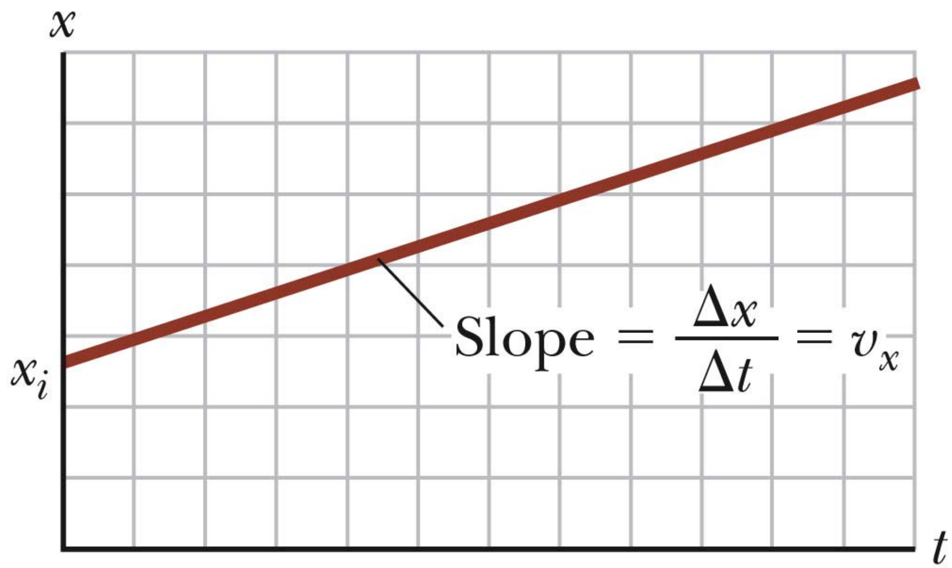
The graph represents the motion of a particle under constant velocity.

The slope of the graph is the value of the constant velocity.

The y-intercept is x_i.



Model: A Particle Under Constant Speed

A particle under constant velocity moves with a constant speed along a straight line.

A particle can also move with a constant speed along a curved path.

This can be represented with a model of a **particle under constant speed.**

The primary equation is the same as for average speed, with the average speed replaced by the constant speed.

$$v = \frac{d}{\Delta t}$$

Average Acceleration

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Acceleration is the rate of change of the velocity.

$$a_{x,avg} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

Dimensions are L/T^2

SI units are m/s^2

In one dimension, positive and negative can be used to indicate direction.

Instantaneous Acceleration

The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0.

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

The term acceleration will mean instantaneous acceleration.

- If average acceleration is wanted, the word average will be included.

Instantaneous Acceleration – graph

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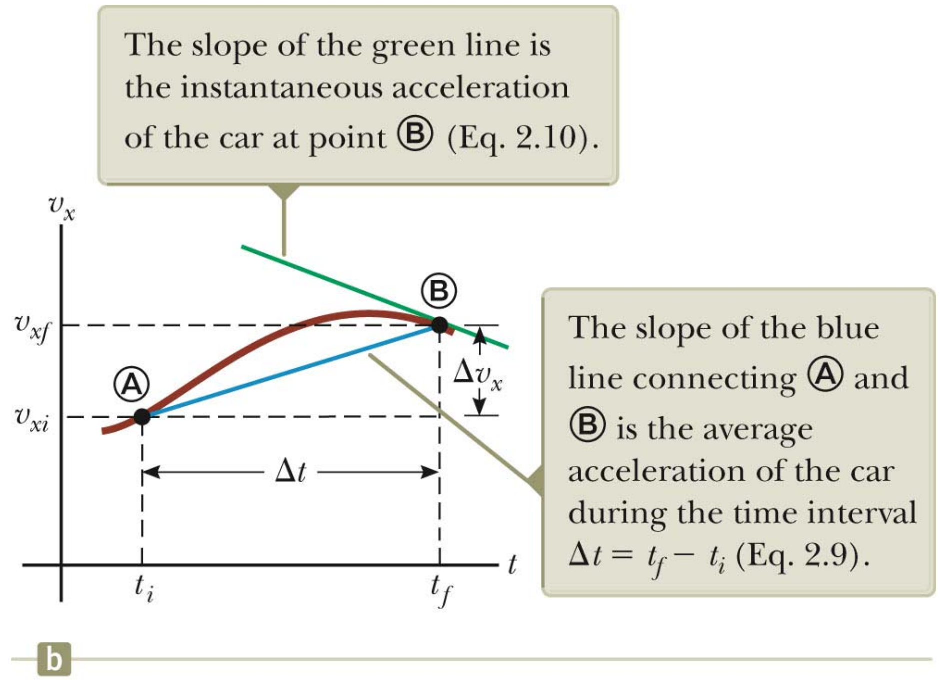
The slope of the velocity-time graph is the acceleration.

The green line represents the instantaneous acceleration.

The blue line is the average acceleration.



Graphical Comparison

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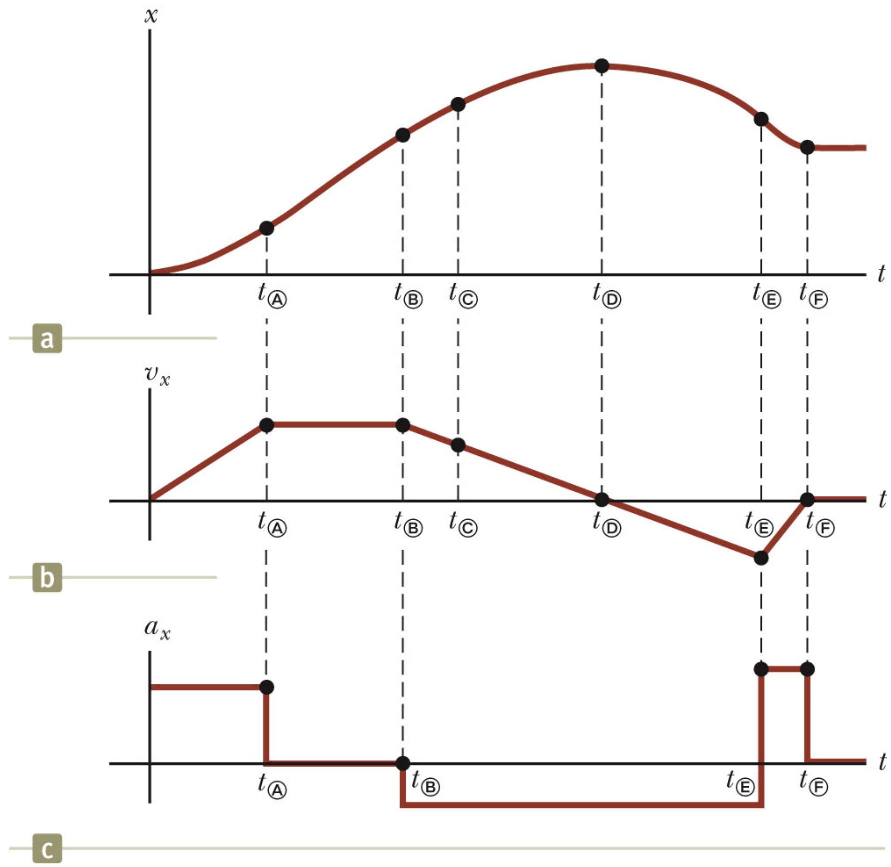
Given the displacement-time graph (a)

The velocity-time graph is found by measuring the slope of the position-time graph at every instant.

The acceleration-time graph is found by measuring the slope of the velocity-time graph at every instant.



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Acceleration and Velocity, Directions

When an object’s velocity and acceleration are in the same direction, the object is speeding up.

When an object’s velocity and acceleration are in the opposite direction, the object is slowing down.

Acceleration and Force

The acceleration of an object is related to the total force exerted on the object.

- The force is proportional to the acceleration, $F_x \propto a_x$.
- Assume the velocity and acceleration are in the same direction.
  - The force is in the same direction as the velocity and the object speeds up.
- Assume the velocity and acceleration are in opposite directions.
  - The force is in the opposite direction as the velocity and the object slows down.

Notes About Acceleration

Negative acceleration does not necessarily mean the object is slowing down.

- If the acceleration and velocity are both negative, the object is speeding up.

The word *deceleration* has the connotation of slowing down.

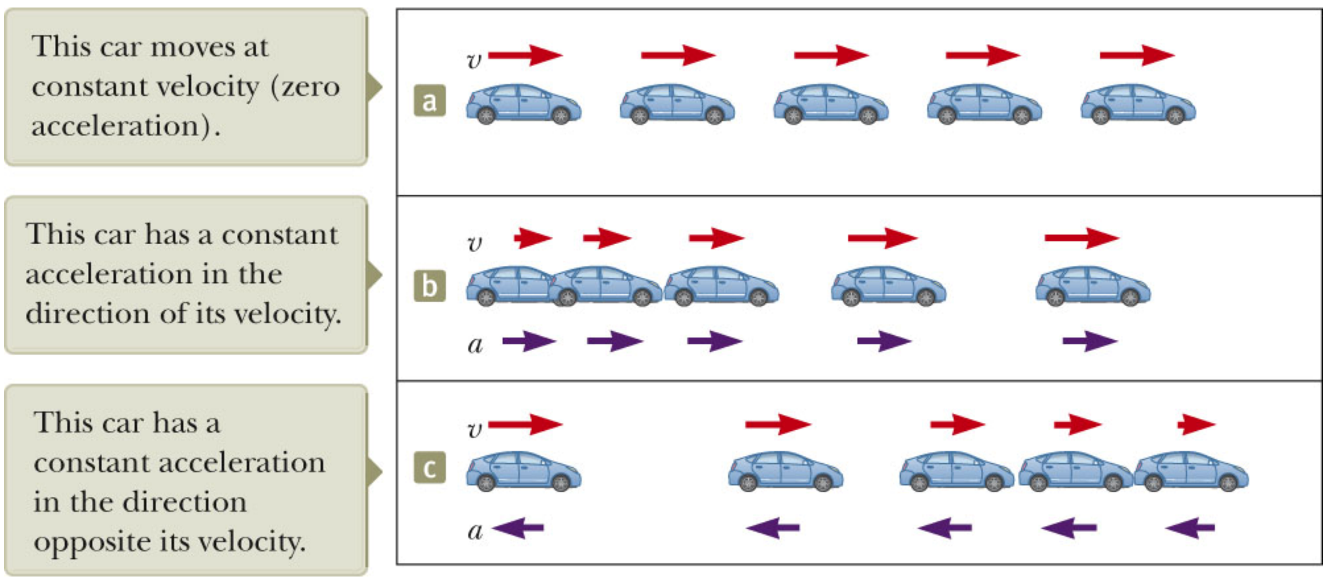
- This word will not be used in the text.

Motion Diagrams

A motion diagram can be formed by imagining the stroboscope photograph of a moving object.

Red arrows represent velocity.

Purple arrows represent acceleration.



Constant Velocity

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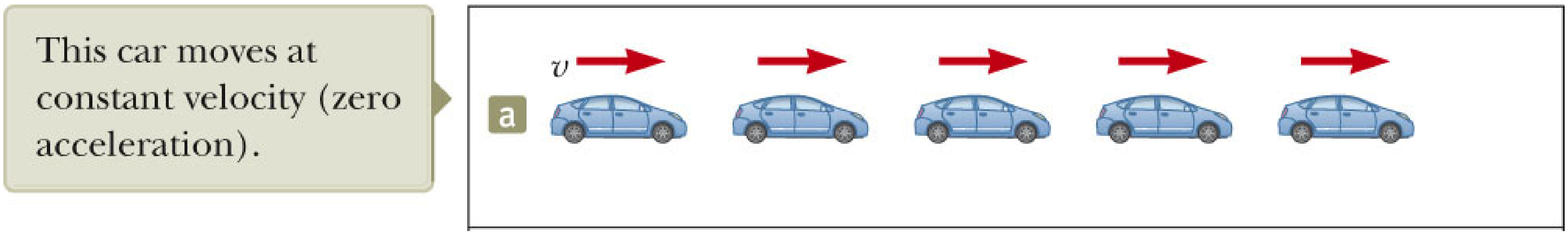

Images are equally spaced.

The car is moving with constant positive velocity (shown by red arrows maintaining the same size).

Acceleration equals zero.

Acceleration and Velocity, 3

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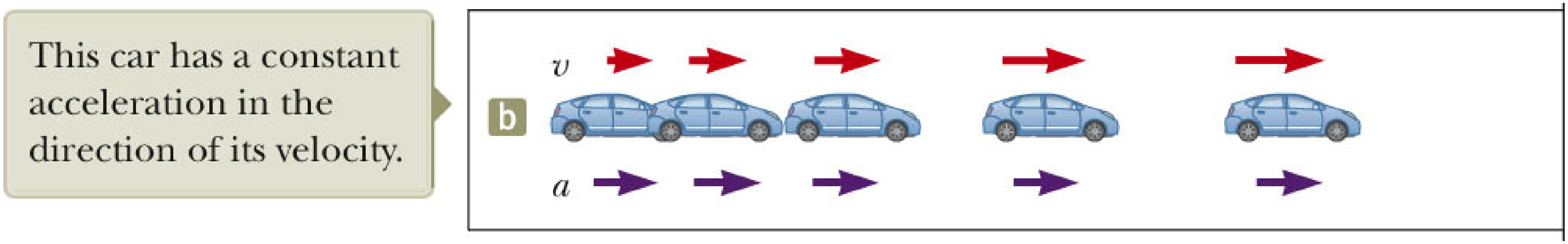

Images become farther apart as time increases.

Velocity and acceleration are in the same direction.

Acceleration is uniform (violet arrows maintain the same length).

Velocity is increasing (red arrows are getting longer).

This shows positive acceleration and positive velocity.

Acceleration and Velocity, 4

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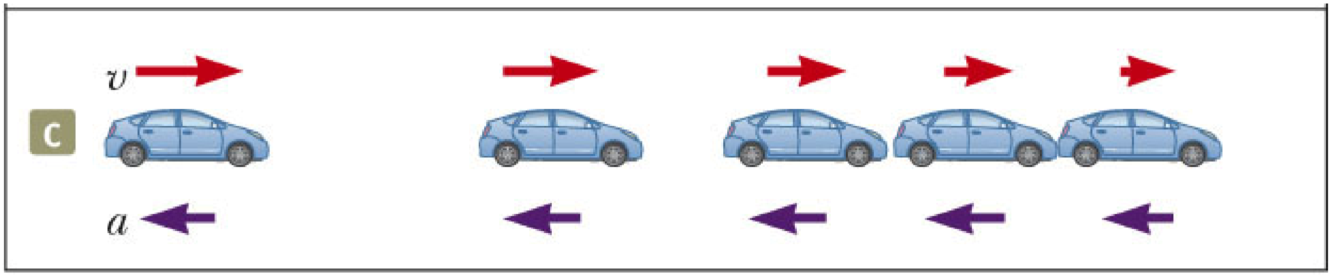
This car has a constant acceleration in the direction opposite its velocity.



Images become closer together as time increases.

Acceleration and velocity are in opposite directions.

Acceleration is uniform (violet arrows maintain the same length).

Velocity is decreasing (red arrows are getting shorter).

Positive velocity and negative acceleration.

Acceleration and Velocity, final

In all the previous cases, the acceleration was constant.

- Shown by the violet arrows all maintaining the same length

The diagrams represent motion of a particle under constant acceleration.

A particle under constant acceleration is another useful analysis model.

Kinematic Equations

The kinematic equations can be used with any particle under uniform acceleration.

The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration.

You may need to use two of the equations to solve one problem.

Many times there is more than one way to solve a problem.

Kinematic Equations, 1

For constant a_x,

$$v_{xf} = v_{xi} + a_x t$$

Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration

- Assumes $t_i = 0$ and $t_f = t$

Does not give any information about displacement

Kinematic Equations, 2

For constant acceleration,

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}$$

The average velocity can be expressed as the arithmetic mean of the initial and final velocities.

- This applies only in situations where the acceleration is constant.

Kinematic Equations, 3

For constant acceleration,

$$x_f = x_i + v_{x,avg}\, t = x_i + \frac{1}{2}\left(v_{xi} + v_{fx}\right) t$$

This gives you the position of the particle in terms of time and velocities.

Doesn't give you the acceleration

Kinematic Equations, 4

For constant acceleration,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Gives final position in terms of velocity and acceleration

Doesn’t tell you about final velocity

Kinematic Equations, 5

For constant *a*,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Gives final velocity in terms of acceleration and displacement

Does not give any information about the time

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When a = 0

When the acceleration is zero,

- $v_{xf} = v_{xi} = v_x$
- $x_f = x_i + v_x t$

The constant acceleration model reduces to the constant velocity model.

Kinematic Equations – summary

TABLE 2.2 *Kinematic Equations for Motion of a Particle Under Constant Acceleration*

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

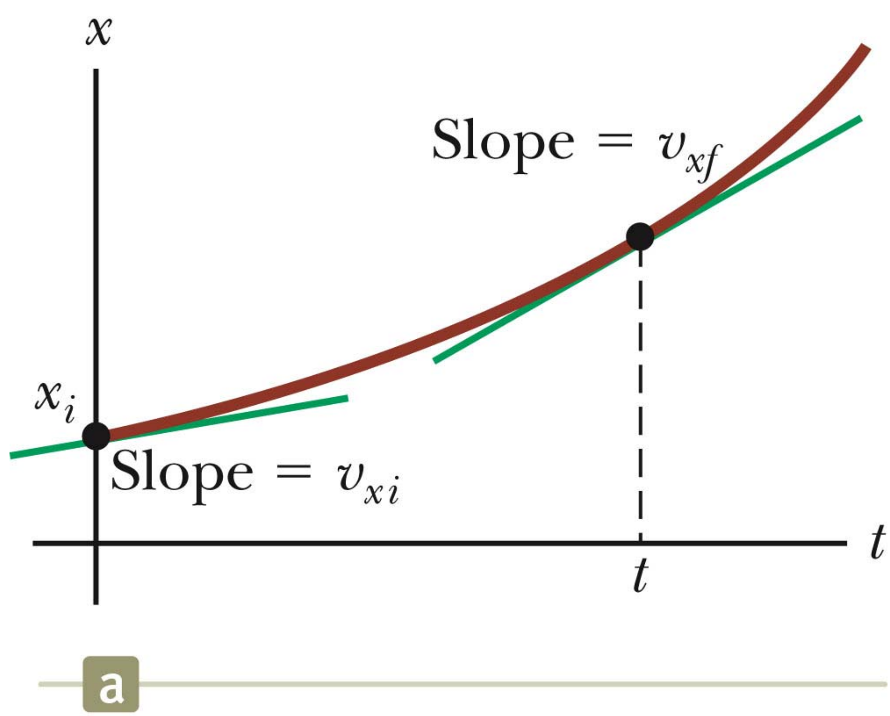
Note: Motion is along the *x* axis.

Graphical Look at Motion: Displacement – Time curve

The slope of the curve is the velocity.

The curved line indicates the velocity is changing.

- Therefore, there is an acceleration.



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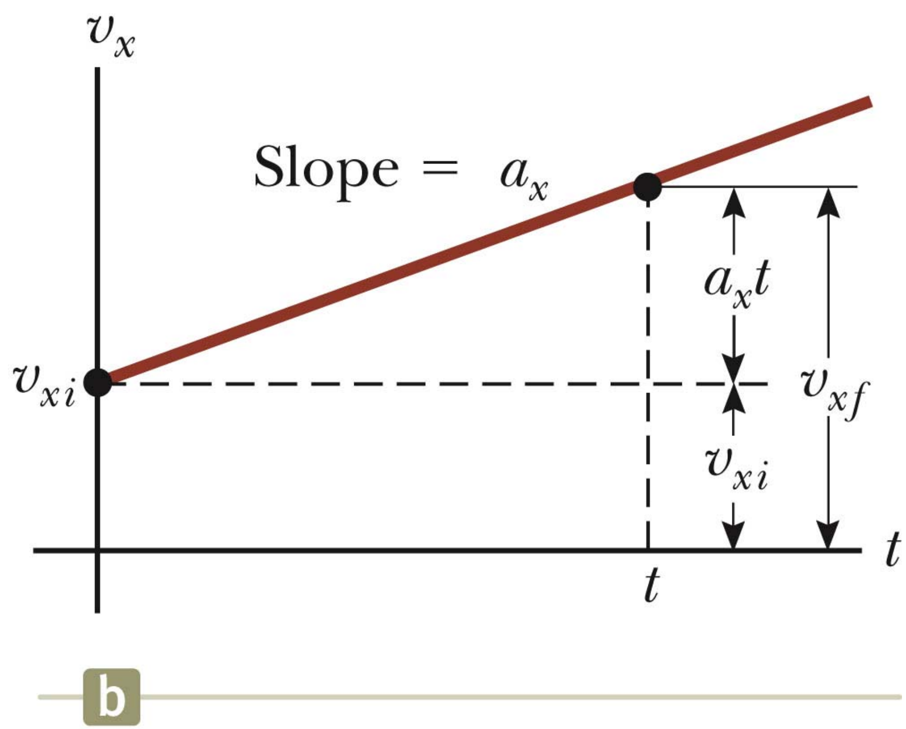
Graphical Look at Motion: Velocity – Time curve

The slope gives the acceleration.

The straight line indicates a constant acceleration.



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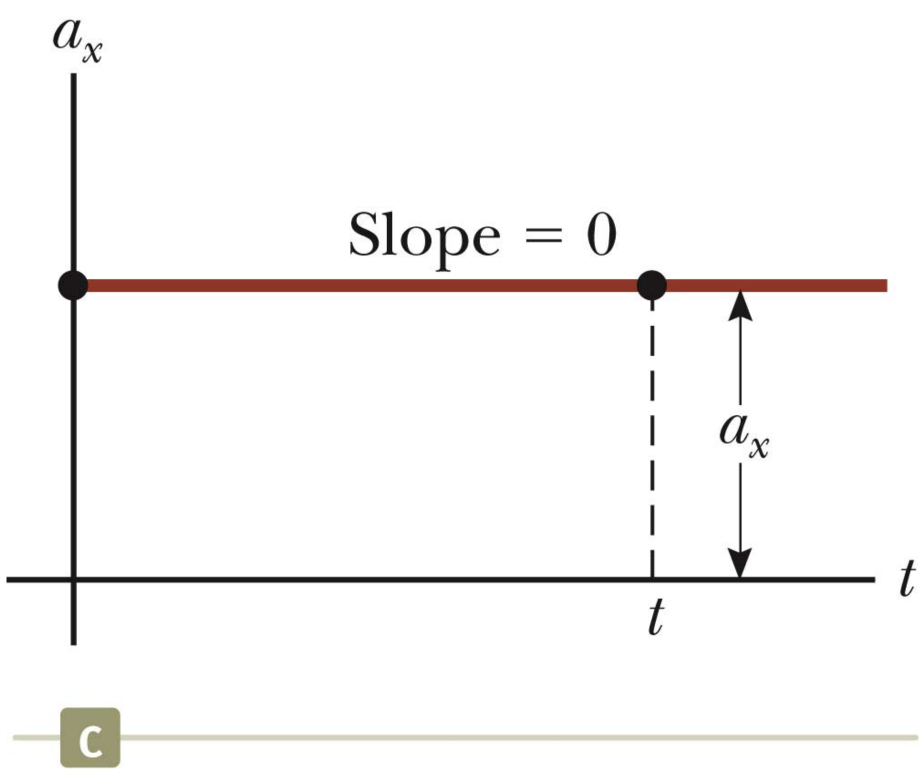
Graphical Look at Motion: Acceleration – Time curve

The zero slope indicates a constant acceleration.



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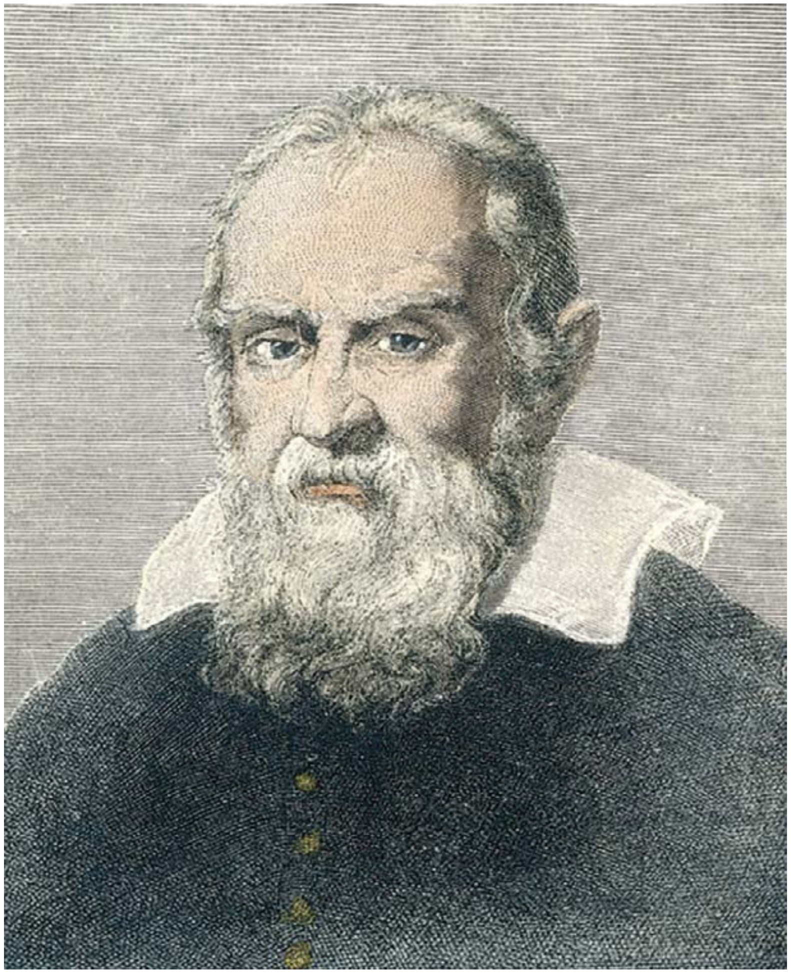
Galileo Galilei

1564 – 1642

Italian physicist and astronomer

Formulated laws of motion for objects in free fall

Supported heliocentric universe



Freely Falling Objects

A ***freely falling object*** is any object moving freely under the influence of gravity alone.

It does not depend upon the initial motion of the object.

- Dropped – released from rest
- Thrown downward
- Thrown upward

Acceleration of Freely Falling Object

The acceleration of an object in free fall is directed downward, regardless of the initial motion.

The magnitude of free fall acceleration is $g = 9.80\ \text{m/s}^2$.

- g decreases with increasing altitude
- g varies with latitude
- $9.80\ \text{m/s}^2$ is the average at the Earth's surface
- The italicized g will be used for the acceleration due to gravity.
  - Not to be confused with g for grams

Acceleration of Free Fall, cont.

We will neglect air resistance.

Free fall motion is constantly accelerated motion in one dimension.

- Use model of a particle under constant acceleration

Let upward be positive

Use the kinematic equations

- With $a_y = -g = -9.80 \text{ m/s}^2$
- Note displacement is in the vertical direction

Free Fall – An Object Dropped

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Initial velocity is zero

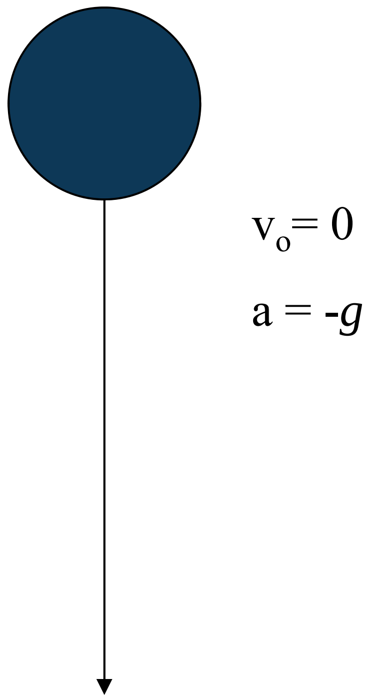
Let up be positive

Use the kinematic equations

- Generally use y instead of x since vertical

Acceleration is

- $a_y = -g = -9.80\ m/s^2$



Free Fall – An Object Thrown Downward

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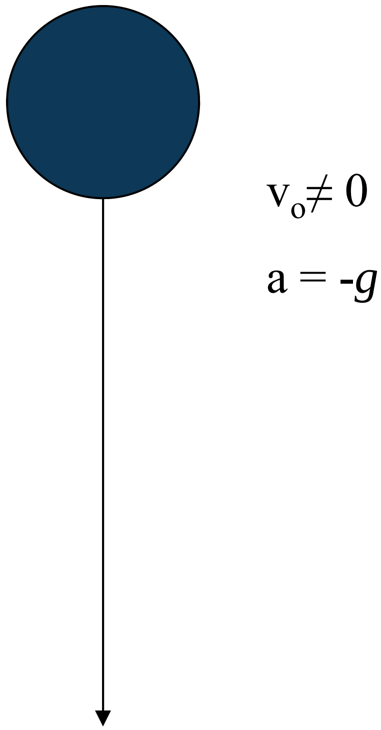
$a_y = -g = -9.80 \text{ m/s}^2$

Initial velocity $\neq 0$

- With upward being positive, initial velocity will be negative.



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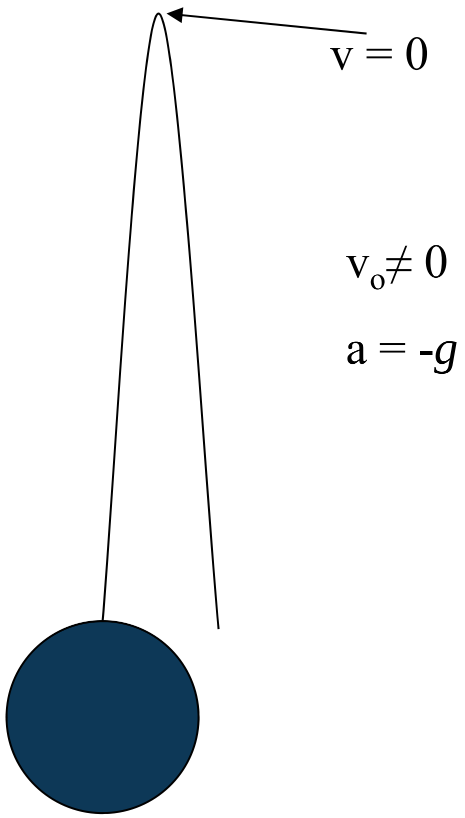
Free Fall – Object Thrown Upward

Initial velocity is upward, so positive

The instantaneous velocity at the maximum height is zero.

$a_y = -g = -9.80\ \text{m/s}^2$ everywhere in the motion



Thrown upward, cont.

The motion may be symmetrical.

- Then $t_{up} = t_{down}$
- Then $v = -v_o$

The motion may not be symmetrical.

- Break the motion into various parts.
  - Generally up and down

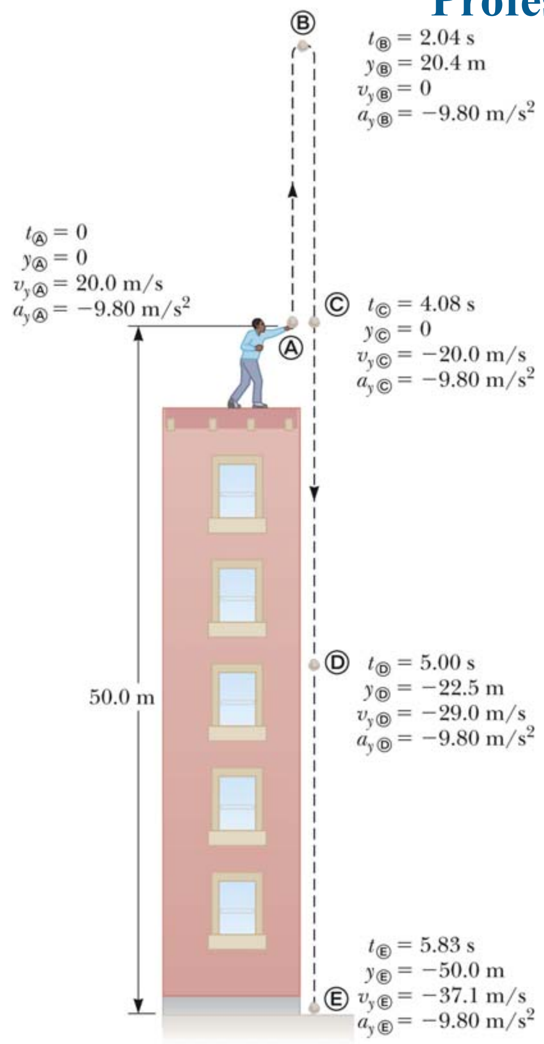
Free Fall Example

Initial velocity at A is upward (+) and acceleration is -*g* (-9.8 m/s^2).

At B, the velocity is 0 and the acceleration is -*g* (-9.8 m/s^2).

At C, the velocity has the same magnitude as at A, but is in the opposite direction.

The displacement is –50.0 m (it ends up 50.0 m below its starting point).



Kinematic Equations from Calculus

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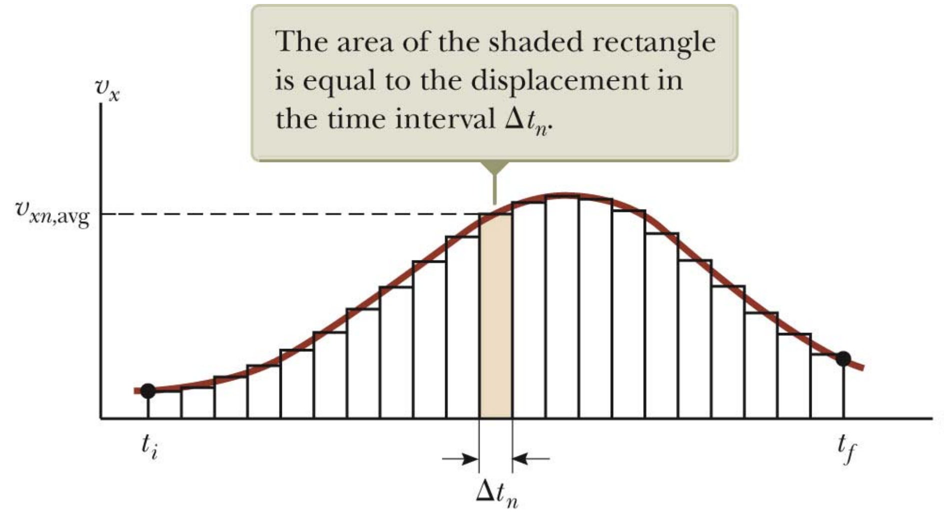
Displacement equals the area under the velocity – time curve

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

The limit of the sum is a definite integral.



Kinematic Equations – General Calculus Form

$$a_x = \frac{dv_x}{dt}$$

$$v_{xf} - v_{xi} = \int_0^t a_x dt$$

$$v_x = \frac{dx}{dt}$$

$$x_f - x_i = \int_0^t v_x dt$$

Kinematic Equations – From Integration

The integration form of $v_f - v_i$ gives

$$v_{xf} - v_{xi} = a_x t$$

The integration form of $x_f - x_i$ gives

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$

General Problem Solving Strategy

In addition to basic physics concepts, a valuable skill is the ability to solve complicated problems.

Steps in a general problem solving approach:

- Conceptualize
- Categorize
- Analyze
- Finalize

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Problem Solving – Conceptualize

Think about and understand the situation.

Make a quick drawing of the situation.

Gather the numerical information.

- Include algebraic meanings of phrases.

Focus on the expected result.

- Think about units.

Think about what a reasonable answer should be.

Problem Solving – Categorize

Simplify the problem.

- Can you ignore air resistance?
- Model objects as particles

Classify the type of problem.

- Substitution
- Analysis

Try to identify similar problems you have already solved.

- What analysis model would be useful?

Problem Solving – Analyze

Select the relevant equation(s) to apply.

Solve for the unknown variable.

Substitute appropriate numbers.

Calculate the results.

- Include units

Round the result to the appropriate number of significant figures.

Problem Solving – Finalize

Check your result.

- Does it have the correct units?
- Does it agree with your conceptualized ideas?

Look at limiting situations to be sure the results are reasonable.

Compare the result with those of similar problems.

Problem Solving – Some Final Ideas

When solving complex problems, you may need to identify sub-problems and apply the problem-solving strategy to each sub-part.

These steps can be a guide for solving problems in this course.

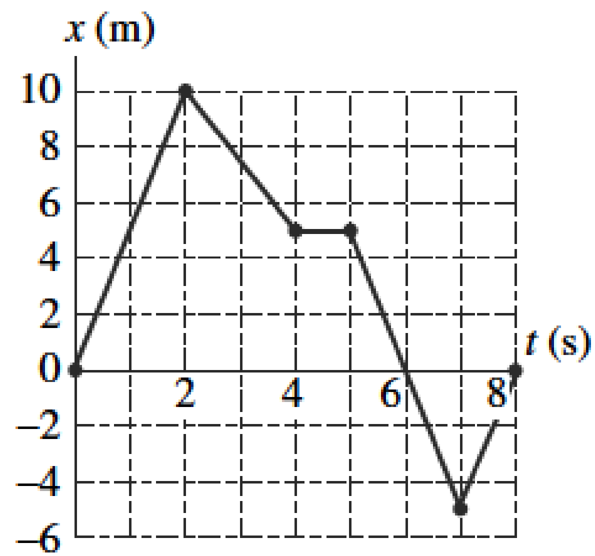
Problem 2. 8

(a) $v = \dfrac{(5-0)\ \text{m}}{(1-0)\ \text{s}} = \boxed{5\ \text{m/s}}$

(b) $v = \dfrac{(5-10)\ \text{m}}{(4-2)\ \text{s}} = \boxed{-2.5\ \text{m/s}}$

(c) $v = \dfrac{(5\ \text{m}-5\ \text{m})}{(5\ \text{s}-4\ \text{s})} = \boxed{0}$

(d) $v = \dfrac{0-(-5\ \text{m})}{(8\ \text{s}-7\ \text{s})} = \boxed{+5\ \text{m/s}}$



ANS FIG. P2.8