

Suggested Problems from Chapter 3

2. The rectangular coordinates of a point are given by $(2, y)$, and its polar coordinates are $(r, 30^\circ)$. Determine (a) the value of y and (b) the value of r .

a) From class and equations

b) 3.1 and 3.2 in the textbook
we have:

The projection of the vector
 $\vec{r}$ on the x -axis gives x .

The projection of the vector
 $\vec{r}$ on the y -axis gives y .

$x = 2$, y is unknown.

From geometry

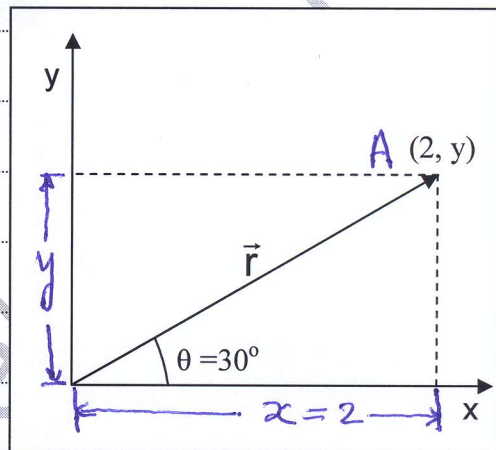
$$x = r \cos 30 = 2 \quad (1)$$

$$y = r \sin 30 = y \quad (2)$$

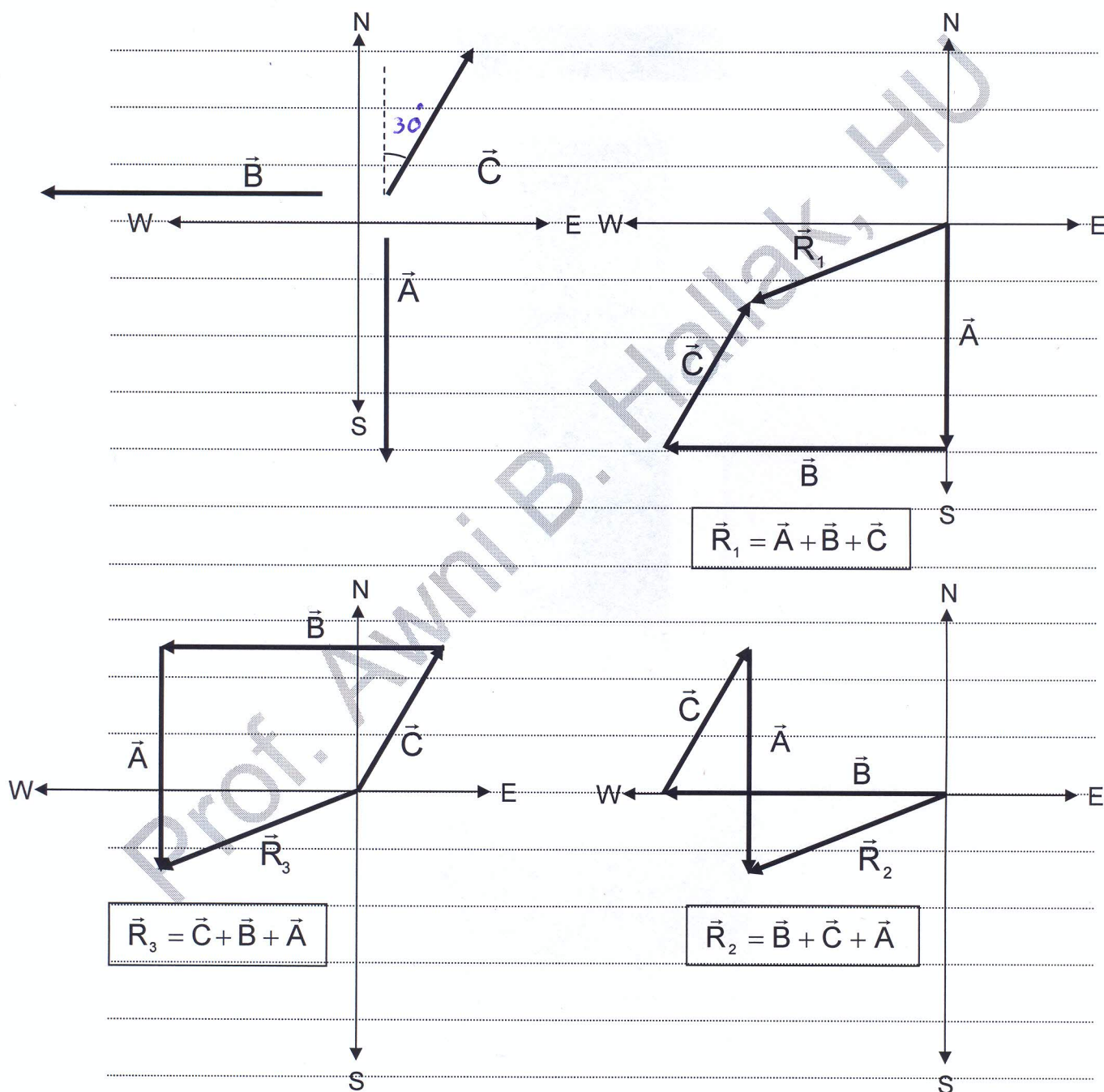
$$\text{From (1)} \Rightarrow r = \frac{2}{\cos 30} = 2.31 \text{ m} \quad (3)$$

Using (3) in (2) $\Rightarrow$

$$y = 2.31 \sin 30 = 1.15 \text{ m}.$$



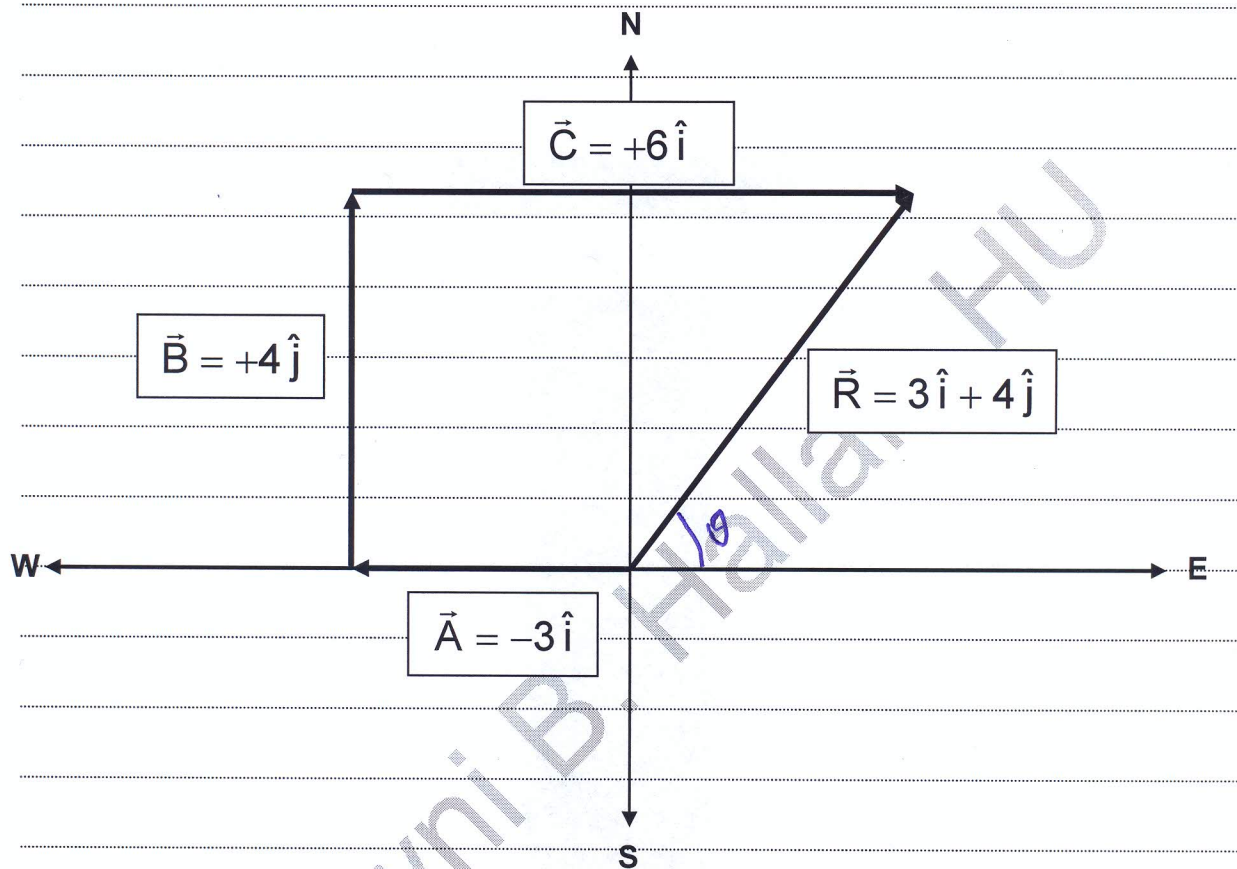
8. Three displacements are $\vec{A} = 200 \text{ m}$ due south, $\vec{B} = 250 \text{ m}$ due west, and $\vec{C} = 150 \text{ m}$ at 30.0° east of north. (a) Construct a separate diagram for each of the following possible ways of adding these vectors: $\vec{R}_1 = \vec{A} + \vec{B} + \vec{C}$; $\vec{R}_2 = \vec{B} + \vec{C} + \vec{A}$; $\vec{R}_3 = \vec{C} + \vec{B} + \vec{A}$. (b) Explain what you can conclude from comparing the diagrams.



$$\Rightarrow \vec{R}_1 = \vec{R}_2 = \vec{R}_3.$$

i.e. the order of **adding** vectors is not important. The result in this case is always the same.

14. A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, and then 6.00 blocks east. (a) What is her resultant displacement? (b) What is the total distance she travels?



a) The resultant can be found in two ways:

- 1) Graphically as shown in Figure
- 2) Using unit vectors

The result is the same in both ways.

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

Take East as positive x -direction where $\hat{i}$ is the unit vector and positive y -direction along North where $\hat{j}$ is the unit vector.

$$\Rightarrow \text{Vectors as shown. } \vec{R} = -3\hat{i} + 4\hat{j} + 6\hat{i}$$

$$\Rightarrow \vec{R} = 3\hat{i} + 4\hat{j} \Rightarrow R = \sqrt{3^2 + 4^2} = 5 \text{ block}$$

$$\tan \theta = \frac{4}{3} \Rightarrow \theta \approx 53.1^\circ \text{ North of east.}$$

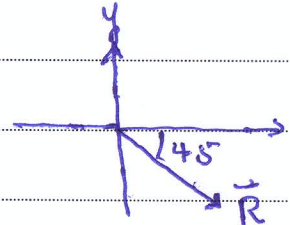
b) Total distance she travels = $3 + 4 + 6 = 13$ blocks

25. Consider the three displacement vectors $\vec{A} = (3\hat{i} - 3\hat{j})\text{m}$, $\vec{B} = (\hat{i} - 4\hat{j})\text{m}$, and $\vec{C} = (-2\hat{i} + 5\hat{j})\text{m}$. Use the component method to determine (a) the magnitude and direction of the vector $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ and (b) the magnitude and direction of $\vec{E} = -\vec{A} - \vec{B} + \vec{C}$.

In the component method, all components (with $\hat{i}$ or $\hat{j}$ or $\hat{k}$) are treated individually.

$$\begin{aligned} \text{a) } \vec{D} &= \vec{A} + \vec{B} + \vec{C} \\ &= (3\hat{i} - 3\hat{j}) + (\hat{i} - 4\hat{j}) + (-2\hat{i} + 5\hat{j}) \\ &= 2\hat{i} - 2\hat{j} \quad (\text{x is positive, y is negative}) \\ \Rightarrow D &= \sqrt{2^2 + (-2)^2} = 2.83 \text{ m} \end{aligned}$$

$$\tan \theta_1 = \frac{-2}{2} = -1 \Rightarrow \theta_1 = -45^\circ$$



$$\begin{aligned} \text{b) } \vec{E} &= -\vec{A} - \vec{B} + \vec{C} \\ &= -(3\hat{i} - 3\hat{j}) - (\hat{i} - 4\hat{j}) + (-2\hat{i} + 5\hat{j}) \\ &= -6\hat{i} + 12\hat{j} \quad (\text{x is negative, y is positive}) \\ \Rightarrow E &= \sqrt{(-6)^2 + (12)^2} = 13.42 \text{ m} \end{aligned}$$

$$\tan \theta_2 = \frac{+12}{-6} = -2 \Rightarrow \theta_2 = -63.43^\circ \text{ or } 116.57^\circ$$

Since x is negative and y is positive

$$\Rightarrow \theta_2 = 116.57^\circ$$

37. (a) Taking $\vec{A} = (6.00\hat{i} - 8.00\hat{j})$ units, $\vec{B} = (-8.00\hat{i} + 3.00\hat{j})$ units, and $\vec{C} = (26.0\hat{i} + 19.0\hat{j})$ units, determine a and b such that $a\vec{A} + b\vec{B} + \vec{C} = 0$. (b) A student has learned that a single equation cannot be solved to determine values for more than one unknown in it. How would you explain to him that both a and b can be determined from the single equation used in part (a)?

(a) $\vec{A} = 6\hat{i} - 8\hat{j}$, $\vec{B} = -8\hat{i} + 3\hat{j}$, $\vec{C} = 26\hat{i} + 19\hat{j}$.

We require that $a\vec{A} + b\vec{B} + \vec{C} = 0$

$$\Rightarrow a(6\hat{i} - 8\hat{j}) + b(-8\hat{i} + 3\hat{j}) + (26\hat{i} + 19\hat{j}) = 0$$

arranging components $\Rightarrow$

$$(6a - 8b + 26)\hat{i} + (-8a + 3b + 19)\hat{j} = 0$$

For any vector that has magnitude zero, all components must equal to zero.

$$\Rightarrow 6a - 8b + 26 = 0 \quad \text{--- (1)}$$

$$-8a + 3b + 19 = 0 \quad \text{--- (2)}$$

$\Rightarrow$ Two equations with two unknowns

To eliminate b :

$$3 / 6a - 8b + 26 = 0$$

$$8 / -8a + 3b + 19 = 0$$

$\Rightarrow$

$$18a - 24b + 78 = 0$$

$$-64a + 24b + 152 = 0$$

$$-46a + 0 + 230 = 0$$

$$\Rightarrow a = \frac{230}{46} = 5 \text{ units}$$

Substitute in (1) to get b

$$6 \times 5 - 8b + 26 = 0$$

$$\Rightarrow b = \frac{56}{8} = 7 \text{ units}$$

- (b) For two vectors to be equal, all their components must be equal.