

Suggested Problems from Chapter 2

1. The position versus time for a certain particle moving along the x axis is shown in Figure P2.1. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, and (e) 0 to 8 s.

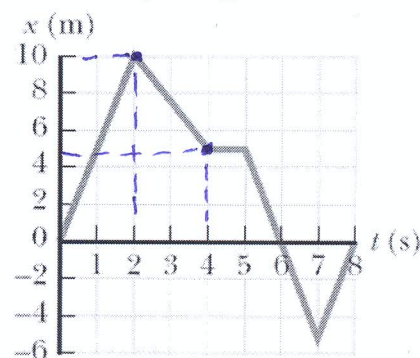


Figure P2.1 Problems 1 and 8.

The average velocity is given by:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

where

$$\Delta x = x_f - x_i$$

$$\Delta t = t_f - t_i$$

We do not care about how x

changes from x_i to x_f . What is important is only the initial and final values.

(a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$. We have: $x_i = 0$, $x_f = 10 \text{ m}$
 $\Rightarrow \bar{v} = \frac{10 - 0}{2 - 0} = 5 \text{ m/s}$ $t_i = 0$, $t_f = 2 \text{ s}$

(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$. We have: $t_i = 0$, $t_f = 4 \text{ s}$
 $\Rightarrow \bar{v} = \frac{5 - 0}{4 - 0} = 1.25 \text{ m/s}$ $x_i = 0$, $x_f = 5 \text{ m}$

(c) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$. We have: $t_i = 2 \text{ s}$, $t_f = 4 \text{ s}$
 $\Rightarrow \bar{v} = \frac{5 - 10}{4 - 2} = -2.5 \text{ m/s}$ $x_i = 10 \text{ m}$, $x_f = 5 \text{ m}$

(d) $\bar{v} = \frac{-5 - 5}{7 - 4} = -3.3 \text{ m/s}$

(e) $\bar{v} = \frac{0 - 0}{8 - 0} = 0$

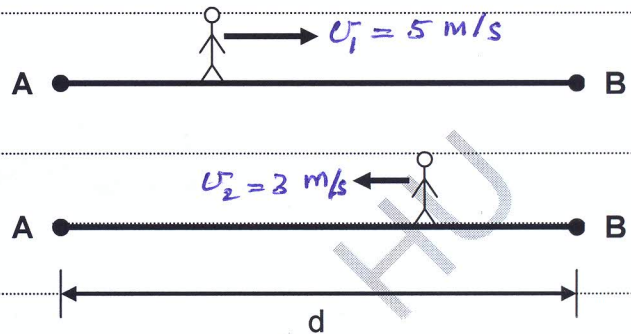
3. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. (a) What is the average speed over the entire trip? (b) What is her average velocity over the entire trip?

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$
 Let: d = distance between A

and B

t_1 = time to go from A to B

t_2 = time to go from B to A



(a) Total distance $A \rightarrow B \rightarrow A$ is: $d_{\text{total}} = d + d = 2d$

Also $t_1 = \frac{d}{u_1} = \frac{d}{5} \text{ s}$, $t_2 = \frac{d}{u_2} = \frac{d}{3} \text{ s}$

\Rightarrow Total time $t = t_1 + t_2 = \frac{d}{5} + \frac{d}{3} = \frac{8d}{15} \text{ s}$

\Rightarrow Average speed = $\frac{2d}{\frac{8d}{15}} = \frac{2 \times 15d}{8d} = 3.75 \text{ m/s}$

(b) Average $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

We have a round trip $A \rightarrow B \rightarrow A$

$x_i = 0$, $x_f = 0$, $t_i = 0$, $t_f = \frac{8d}{15}$

$\Rightarrow \bar{v} = \frac{0 - 0}{\frac{8d}{15}} = 0$

8. Find the instantaneous velocity of the particle described in Figure P2.1 at the following times: (a) $t = 1.0$ s, (b) $t = 3.0$ s, (c) $t = 4.5$ s, and (d) $t = 7.5$ s.

The instantaneous velocity is given by the slope of the curve between x and t at a certain time t , i.e.

$v = \frac{dx}{dt}$ For a straight line x versus t , the slope can be positive, zero or negative. Examples are:

OA \Rightarrow slope is positive

AB \Rightarrow slope is negative

BC \Rightarrow slope is zero

(a) At $t = 1$ s, we need to find the slope of line OA

$$\Rightarrow v = \frac{10 - 0}{2 - 0} = 5 \text{ m/s}$$

(b) At $t = 3$ s, we need to find the slope of line AB

$$\Rightarrow v = \frac{5 - 10}{4 - 2} = -2.5 \text{ m/s}$$

(c) At $t = 4.5$ s, we need to find the slope of line BC

$$\Rightarrow v = \frac{5 - 5}{5 - 4} = 0$$

(d) At $t = 7.5$ s, we need to find the slope of line DE

$$\Rightarrow v = \frac{0 - (-5)}{8 - 7} = 5 \text{ m/s}$$

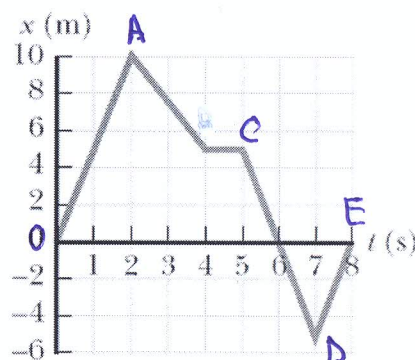


Figure P2.1 Problems 1 and 8.

17. A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

$$x(t) = (2 + 3t - t^2) \text{ m}$$

(a) Find x at $t = 3$ s

$$\Rightarrow x(3) = (2 + 3 \times 3 - 3^2) = 2 \text{ m}$$

(b) Find v at $t = 3$ s

$$v(t) = \frac{dx}{dt} = 0 + 3 - 2t$$

\Rightarrow

$$v(3) = 3 - 2 \times 3 = -3 \text{ m/s}$$

(c) Find a at $t = 3$ s

$$a(t) = \frac{dv}{dt} = 0 - 2 = -2 \text{ m/s}^2$$

\Rightarrow

$$a(3) = -2 \text{ m/s}^2$$

21. An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is - 5.00 cm, what is its acceleration?

The first thing to do is to write all quantities in SI units.

\Rightarrow

$$U_i = 12 \text{ cm/s} = 0.12 \text{ m/s}$$

$$x_i = 3 \text{ cm} = 0.03 \text{ m}$$

$$t = 2 \text{ s}$$

$$x_f = -5 \text{ cm} = -0.05 \text{ m}$$

$$a_x = ?$$

There are 3 equations for motion under constant acceleration. These are:

$$U_f = U_i + a_x t \quad \text{--- (1)}$$

$$x_f = x_i + U_i t + \frac{1}{2} a_x t^2 \quad \text{--- (2)}$$

$$U_f^2 = U_i^2 + 2 a_x (x_f - x_i) \quad \text{--- (3)}$$

Use equation (2) to find a_x

$$x_f = x_i + U_i t + \frac{1}{2} a_x t^2$$

$$-0.05 = 0.03 + 0.12 \times 2 + \frac{1}{2} a_x \times 2^2$$

\Rightarrow

$$a_x = -0.16 \text{ m/s}^2 = -16 \text{ cm/s}^2$$

28. A particle moves along the x axis. Its position is given by the equation $x = 2 + 3t - 4t^2$, with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

$$(a) \quad x = 2 + 3t - 4t^2 \quad (1)$$

$$v = \frac{dx}{dt} = 0 + 3 - 8t$$

or

$$v = 3 - 8t \quad (2)$$

The particle changes direction when its velocity becomes zero. The time then can be found by putting $v = 0$ in the velocity equation (2)

$$0 = 3 - 8t \Rightarrow t = \frac{3}{8} \text{ s}$$

The position then can be found from equation (1)

$$x = 2 + 3\left(\frac{3}{8}\right) - 4\left(\frac{3}{8}\right)^2$$

$$\Rightarrow x = 2.56 \text{ m}$$

(b) Find $v_f = ?$ when $x_f = x_i$

From (2) we can find v_i by putting $t = 0$

$$\Rightarrow v_i = 3 \text{ m/s} \quad (3)$$

We can then use $v_f = v_i + a_x t$ to find v_f

Since $x_f = x_i \Rightarrow$ We need to find $a_x t$

$$x_f = x_i + v_i t + \frac{1}{2} a_x t^2$$

$$x_f - x_i = 0 = 3t + \frac{1}{2} a_x t^2$$

$$\Rightarrow t(3 + \frac{1}{2} a_x t) = 0$$

$$\text{Since } t \neq 0 \Rightarrow 3 - \frac{1}{2} a_x t = 0 \Rightarrow a_x t = -6 \text{ m/s}$$

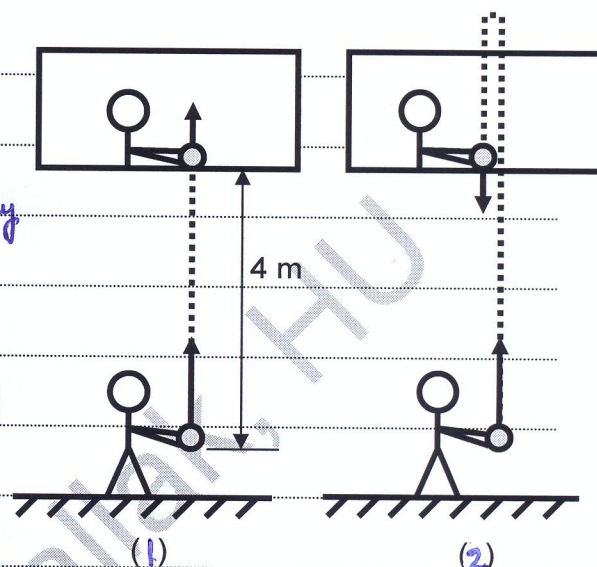
$$\Rightarrow \text{In the equation } v_f = v_i + a_x t \Rightarrow$$

$$v_f = 3 - 6 = -3 \text{ m/s}$$

43. A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The second student catches the keys 1.50 s later. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

There are two ways to catch the keys

1. Catch the keys on their way up.
2. Catch the keys after they reach maximum height and then come down.



Which case occurs in this

problem is determined by the time (1.5 s) needed to catch the keys. If the time needed to reach maximum height is t_h and $t_h < 1.5$ s, then case (1) is valid. If time needed = 1.5 s exactly, then case (2) is valid.

(a) $v_{yi} = ?$ We have $y_f - y_i = 4$ m, $t = 1.5$ s

But $y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$

$$\Rightarrow y_f - y_i = 4 = v_{yi} \cdot 1.5 - \frac{1}{2} \cdot 9.8 \cdot (1.5)^2$$

$$\Rightarrow v_{yi} = \boxed{10 \text{ m/s}}$$

(b) $v_{yf} = ?$ We have $v_{yi} = 10$ m/s, $t = 1.5$ s

But

$$v_{yf} = v_{yi} - gt$$

$$= 10 - 9.8 \cdot 1.5$$

$$= \boxed{-4.68 \text{ m/s}}$$

\Rightarrow Case (2) is our case.

60. Two students are on a balcony a distance h above the street. One student throws a ball vertically downward at a speed v_i ; at the same time, the other student throws a ball vertically upward at the same speed. Answer the following symbolically in terms of v_i , g , h , and t . (a) What is the time interval between when the first ball strikes the ground and the second ball strikes the ground? (b) Find the velocity of each ball as it strikes the ground.

(a) Let t_1 = time for case 1

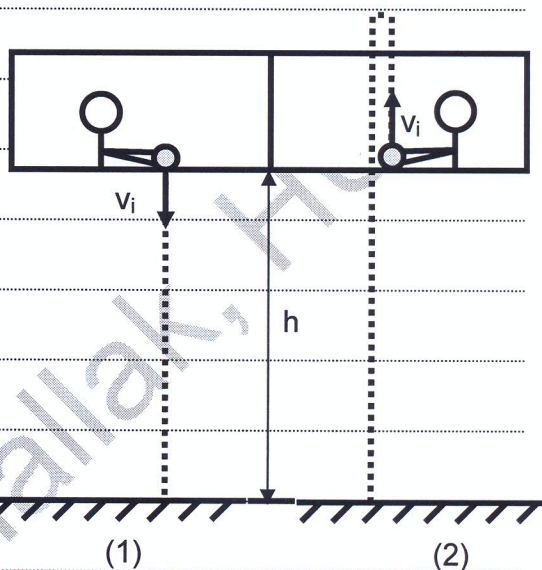
t_2 = time for case 2

$\Delta t = t_2 - t_1$ = time t_m needed for case 2 to reach maximum height and come back to the same level.

The time t_m was calculated

in class and $= \frac{2v_i}{g}$

$$\Rightarrow \Delta t = \frac{2v_i^2}{g}$$



(b) $v_{1f} = v_{2f} = v_f$

To find v_f , use $v_f^2 = v_i^2 - 2g(y_f - y_i)$

$$\Rightarrow v_f^2 = v_i^2 - 2g(0 - h)$$

$$\Rightarrow v_f = -\sqrt{v_i^2 + 2gh}$$