

Suggested Problems from Chapter 1

9. Kinetic energy K (Chapter 7) has dimensions of $\text{kg} \cdot \text{m}^2/\text{s}^2$. It can be written in terms of the momentum p (Chapter 9) and mass m as:

$$K = \frac{p^2}{2m}$$

(a) Determine the proper units for momentum using dimensional analysis. (b) The unit of the force is the Newton N , where $1 N = 1 \text{ kg} \cdot \text{m}/\text{s}^2$. What are the units of momentum p in terms of a Newton and another fundamental SI units?

(a) We can rewrite the above equation as follows:

$$p = \sqrt{2 m K}$$

Units of K : $\text{kg} \cdot \text{m}^2/\text{s}^2$

Units of 2 : No units (just a number)

Units of m : kg

$$\begin{aligned} \text{Units of } p &= \sqrt{\text{kg} \cdot \text{kg} \cdot \frac{\text{m}^2}{\text{s}^2}} \\ &= \sqrt{\text{kg}^2 \cdot \left(\frac{\text{m}}{\text{s}}\right)^2} \\ &= \text{kg} \cdot \frac{\text{m}}{\text{s}} \end{aligned}$$

$$(b) \text{ Units of force : } = N = \text{kg} \cdot \frac{\text{m}}{\text{s}^2}$$

$$\text{Units of } p = \text{kg} \cdot \frac{\text{m}}{\text{s}} = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{s} = N \cdot \text{s}$$

10. Newton's law of universal gravitation is represented by:

$$F = G \frac{Mm}{r^2}$$

where **F** is the magnitude of the gravitational force exerted by one small object on another, **M** and **m** are the masses of the objects, and **r** is a distance. Force has the SI units **kg . m/s²**. What are the SI units of the proportionality constant **G**?

We can rewrite the above equation as shown below.

$$G = \frac{F \cdot r^2}{Mm}$$

$$\text{Units of } G: = \frac{\left(\text{kg} \cdot \frac{\text{m}}{\text{s}^2} \right) \cdot \text{m}^2}{\text{kg} \cdot \text{kg}} = \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$$

11. Which of the following equations are dimensionally correct?

- (a) $v_f = v_i + ax$ (b) $y = (2 \text{ m}) \cos(kx)$, where $k = 2 \text{ m}^{-1}$

(a) The equation is $v_f = v_i + a x$, where a is the acceleration, v is the velocity and x is the displacement.

Left side:

v_f : has units of m/s

Right side:

v_i : has units of m/s \Rightarrow has the same dimensions as the left side \Rightarrow OK

$a x$: has unit of $(\text{m/s}^2) \text{ m} = \text{m}^2/\text{s}^2 \Rightarrow$ not the same units of the left side \Rightarrow NOT OK

The left side and the right side do not have the same units. That is, the equation is dimensionally not correct.

- (b) $y = (2 \text{ m}) \cos(kx)$, where $k = 2 \text{ m}^{-1}$, x and y are displacements.

Left side:

y : has units of meter m.

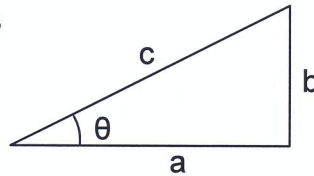
Right side:

The constant **2** : has units of m which is the same units of the left side \Rightarrow OK.

However this is multiplied by $\cos(kx)$. The question is what are the units of kx and $\cos(kx)$.

kx : has units of $m^{-1} \cdot m$. i.e. has no units.

$\cos(kx)$: has no units also since as shown in the following triangle, $\cos \theta = a/c$. i.e. m/m . The cosine has no units



⇒ The above equation is dimensionally correct since the left side and the right side have the same units.

13. A solid piece of lead (الرصاص) has a mass of 23.94 g (g means gram) and a volume of 2.10 cm^3 . From this data, calculate the density of lead in SI units (kilogram per cubic meter, kg/m^3).

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

The mass = 23.94 g = $23.94 \times 10^{-3} \text{ kg}$ ($1 \text{ g} = 10^{-3} \text{ kg}$)

The volume = $2.10 \text{ cm}^3 = 2.10 \times 10^{-6} \text{ m}^3$ ($1 \text{ cm} = 10^{-2} \text{ m}$, $1 \text{ cm}^3 = (10^{-2})^3 = 10^{-6} \text{ m}^3$)

$$\text{Density of lead} = \frac{23.94 \times 10^{-3}}{2.10 \times 10^{-6}} = 11.40 \times 10^3 \text{ kg/m}^3$$