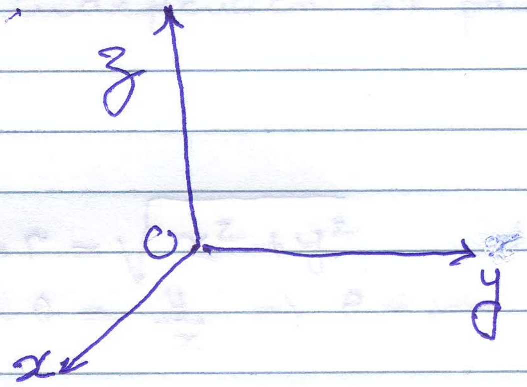
Chapter 3 : Vectors

Define Reference Frames or coordinate System ⇒ محاور إحداثيات ⇒

* One-dimensional

* Two-dimensional

* Three-dimensional

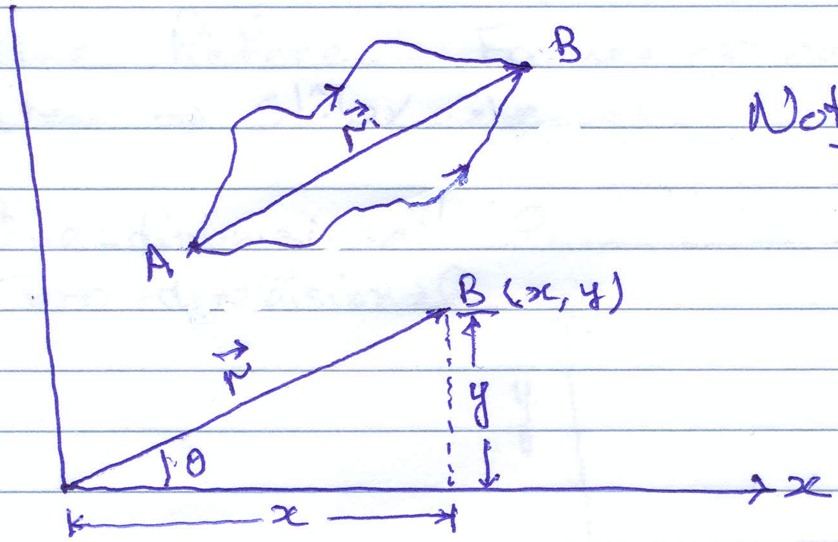


Quantities { Vectors
Scalars

Examples of Vectors : Displacement
Velocity
Acceleration
⋮

Examples of Scalars : Temperature
Volume
Area
⋮

Displacement Vector



Note arrow $\vec{r}$

Displacement vector is independent of path.

Some remarks:

1. Magnitude of $\vec{r}$ $|\vec{r}| = r = \sqrt{x^2+y^2}$
2. Direction of $\vec{r}$ $\tan\theta = \frac{y}{x} \Rightarrow \theta = $ —

3. Two vectors are equivalent if they have the same magnitude and direction.

$\Rightarrow$

$\vec{r}' = \vec{r}$

$\Rightarrow$ We can move the vector but always keep the magnitude and direction the same.

4. Two equivalent vectors are parallel

$\vec{r}'$ and $\vec{r}$ are parallel.

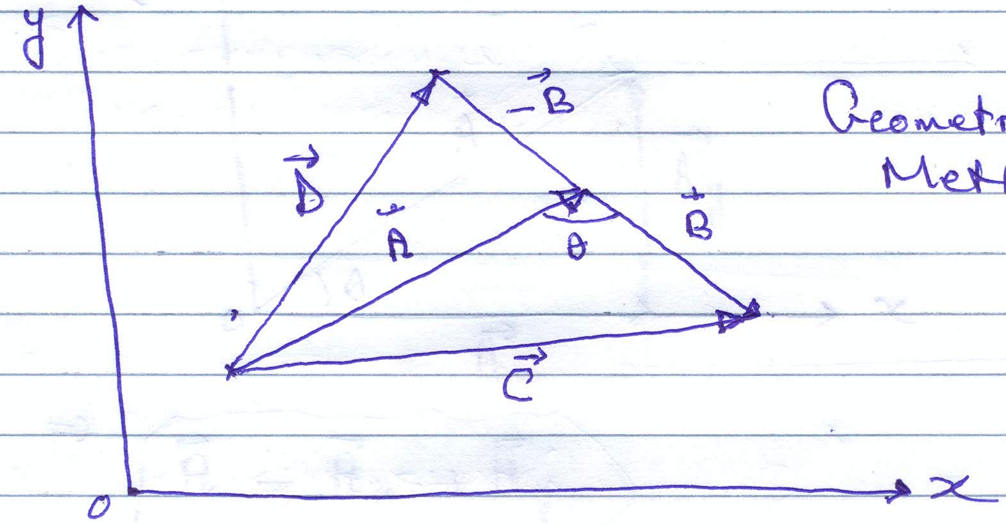
5. If $\vec{A} = -\vec{B} \Rightarrow$ The two vectors have the same magnitude but antiparallel.

Vector Addition and Subtraction

Vector Addition:

Consider two vectors $\vec{A}$, $\vec{B}$ as shown

Need to find $\vec{C} = \vec{A} + \vec{B}$ = resultant



Geometrical Method

From geometry $C^2 = A^2 + B^2 - 2AB\cos\theta$

Vector Subtraction:

$$\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B}).$$

Some remarks:

1. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ Additive Law
2. $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$ Associative law
3. $\vec{A} - \vec{A} = 0$
4. If $\vec{C} = \vec{A} + \vec{B} \Rightarrow C \leq A + B$.
5. $C = A + B$ if $\vec{A}$ and $\vec{B}$ are parallel
6. If $\vec{C} = n\vec{A}$, where n = number

   $\Rightarrow$ Magnitude of $\vec{C}$ = n * magnitude of $\vec{A}$

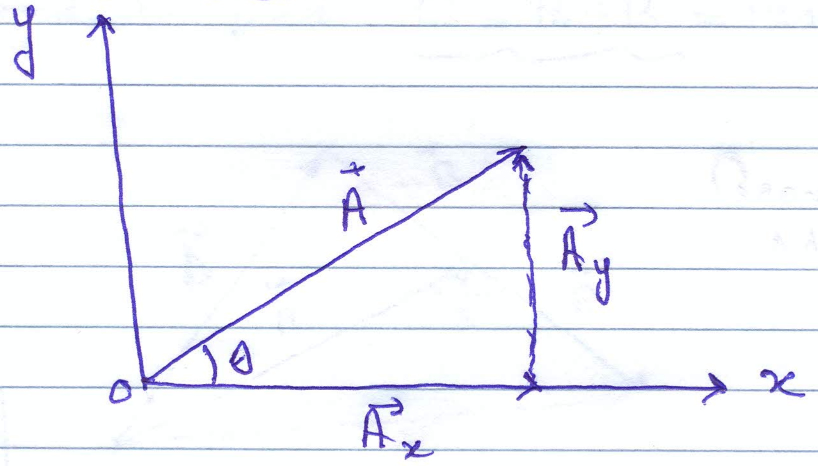
   i.e: $C = |n| A$.

   Example: $\vec{C} = 3\vec{A} \Rightarrow C = 3A$ $\vec{C}$ along $\vec{A}$

   if $\vec{D} = -3\vec{A} \Rightarrow D = 3A$ $\vec{D}$ opposite to $\vec{A}$

Rectangular Components of a Vector:

In two-dimensions



$$\Rightarrow \quad \vec{A} = \vec{A}_x + \vec{A}_y$$

$$A_x = A\cos\theta \quad , \quad A_y = A\sin\theta$$

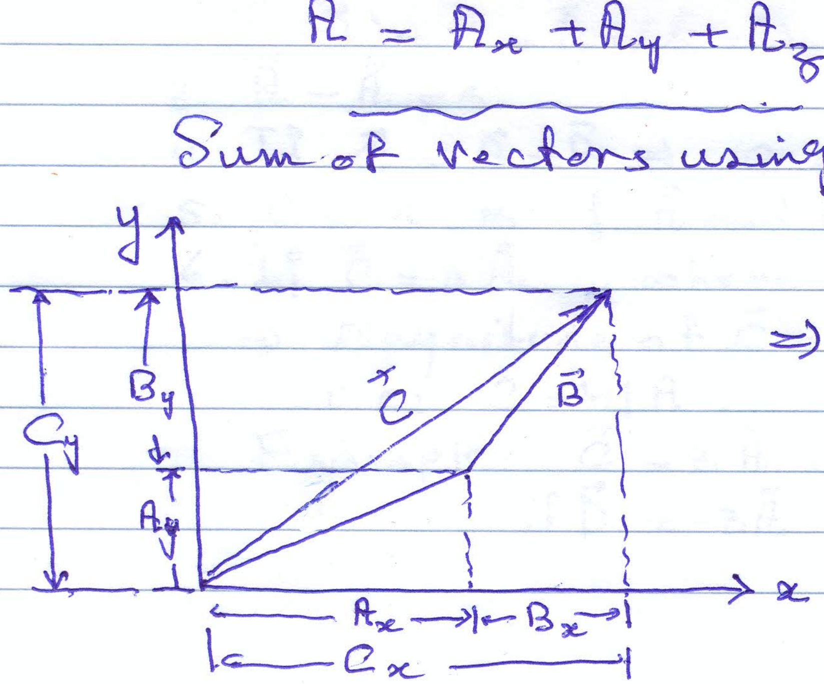
$$A^2 = A_x^2 + A_y^2 \qquad \tan\theta = \frac{A_y}{A_x}$$

$$\Rightarrow \quad A = \sqrt{A_x^2 + A_y^2} \qquad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

In three dimensions

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

Sum of vectors using components



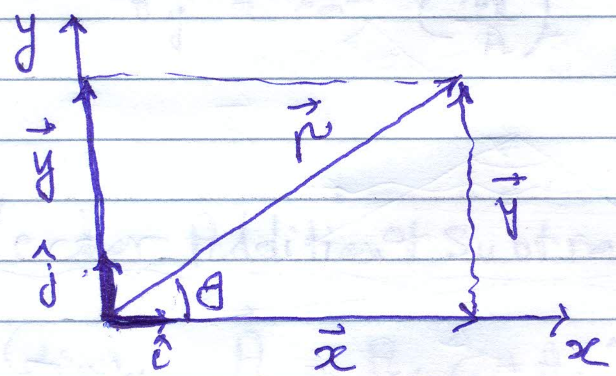
$$\Rightarrow \quad C_x = A_x + B_x$$

$$C_y = A_y + B_y .$$

Now we introduce the concept of

Unit Vectors:

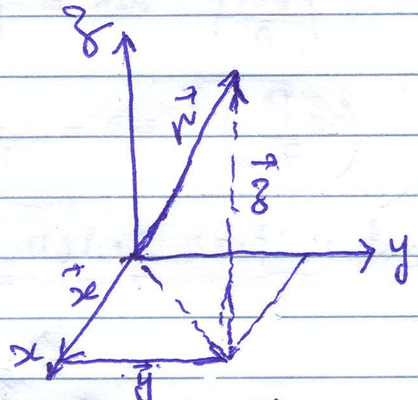
Two dimensions



$\vec{r} = \vec{x} + \vec{y}$

$= \underbrace{x\,\hat{i}}_{x\text{ component}} + \underbrace{y\,\hat{j}}_{y\text{ component}}$

Three-dimensions



$\vec{r} = \vec{x} + \vec{y} + \vec{z}$

$= x\,\hat{i} + y\,\hat{j} + \underbrace{z\,\hat{k}}_{z\text{-component}}$

Unit vectors have magnitude 1.

$\hat{i} \Rightarrow$ x-direction (positive)

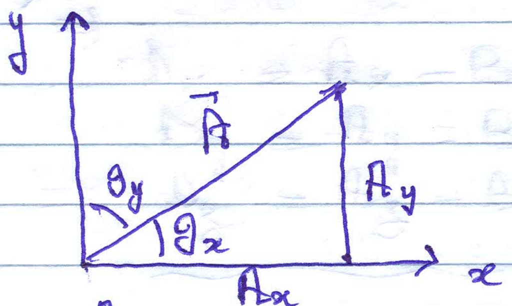
$\hat{j} \Rightarrow$ y-direction (positive)

$\hat{k} \Rightarrow$ z-direction (positive).

In general $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

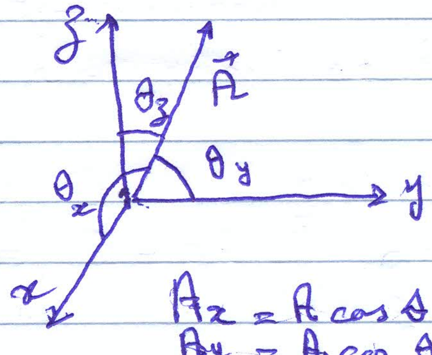
$\Rightarrow \vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$, $\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$

How A_x, A_y, A_z are related to $\vec{A}$?



$A_x = A\cos\theta_x$

$A_y = A\quad\theta_y$



$A_x = A\cos\theta_x$

$A_y = A\cos\theta_y$

$A_z = A\cos\theta_z$

Also

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta_x = \cos^{-1}\left(\frac{A_x}{A}\right)$$

$$\theta_y = \cos^{-1}\left(\frac{A_y}{A}\right)$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\theta_x = \cos^{-1}\left(\frac{A_x}{A}\right)$$

$$\theta_y = \cos^{-1}\left(\frac{A_y}{A}\right)$$

$$\theta_z = \cos^{-1}\left(\frac{A_z}{A}\right).$$

Vector Addition & Subtraction using unit vectors

Given $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\Rightarrow$

1) $\vec{C} = \vec{A} + \vec{B}$

$\Rightarrow \vec{C} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) + (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$\vec{C} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

But

$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$

$\Rightarrow$

$C_x = A_x + B_x$

$C_y = A_y + B_y$.

$C_z = A_z + B_z$

2) $\vec{D} = \vec{A} - \vec{B} \Rightarrow \vec{D} + (-\vec{B})$.

$\Rightarrow$

$D_x = A_x - B_x$

$D_y = A_y - B_y$

$D_z = A_z - B_z$.