Classical Mechanics — Kinematics (No need to know $\vec{F}$)

Classical Mechanics — Dynamics (Must know acting forces $\vec{F}$)

Motion in One Dimension:

Have a record of x versus t

x_i = position at time t_i

x_f = position at time t_f

Define Average velocity $\bar{v}$ during time interval $\Delta t = t_f - t_i$ (change in time).

The displacement $\Delta x = x_f - x_i$

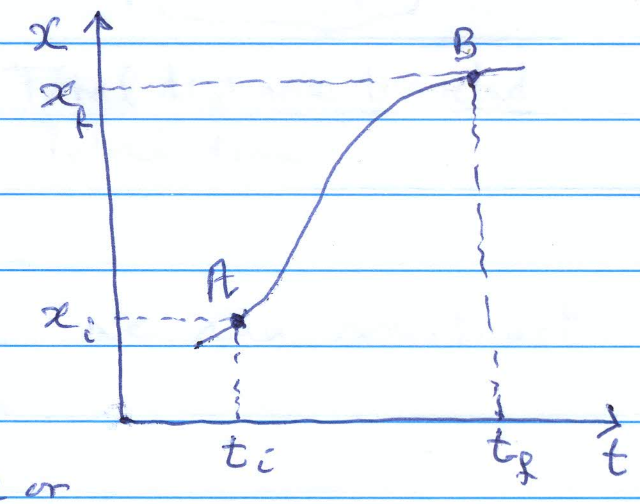
$\Rightarrow$

$$\boxed{\bar{v} = \frac{\Delta x}{\Delta t}}$$

$\Rightarrow$

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i}$$

A possible x versus t relation is:



Some remarks:

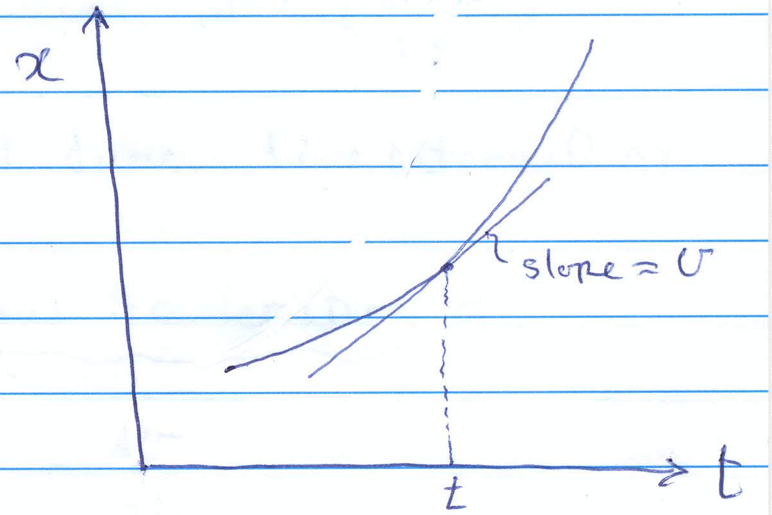
1. We do not care about how x_i changed to x_f.
2. $\bar{v}$ has directional identity (Clear in 2 or 3 dimensional motion).

Define Instantaneous velocity v at one time t.

When $\Delta t \to 0$

$\Rightarrow$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (\text{مشتق } x \text{ نسبت به } t)$$



Define $\boxed{\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}}$

Summary:

Average velocity $\boxed{\bar{v} = \frac{\Delta x}{\Delta t}}$ (during Δt)

Instantaneous velocity $\boxed{v = \frac{dx}{dt}}$ (at time t)

Average speed $= \frac{\text{Total distance traveled}}{\text{Total time}}$

From x versus t, we can construct v versus t also.

instantaneous.

Now we define **Average Acceleration** $\bar{a}$

$$\bar{a} = \frac{\Delta v}{\Delta t} \qquad \Delta v = v_f - v_i$$

instantaneous

$$\Delta t = t_f - t_i$$

Acceleration is measured in m/s^2

$\bar{a}$ is measured during time interval Δt.

Define **Instantaneous Acceleration** a

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Note that $v = \frac{dx}{dt}$

$\Rightarrow$

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

مشتقة x ثانية
بالنسبة t

Motion under constant acceleration:

Constant acceleration $\Rightarrow$ $a_x = \bar{a}_x$

$\Rightarrow$ or $a = \bar{a}$

$$a_x = \frac{v_f - v_i}{t_f - t_i}$$

Let $t_i = 0$, v_i = initial velocity

$t_f = t$, v_f = final velocity

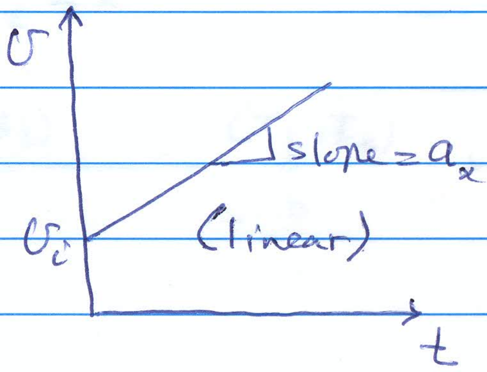
$\Rightarrow$

$$a_x = \frac{v_f - v_i}{t - 0}$$

$\Rightarrow$

$$\boxed{v_f = v_i + a_x t} \quad \text{——— (1)}$$

Plot this equation

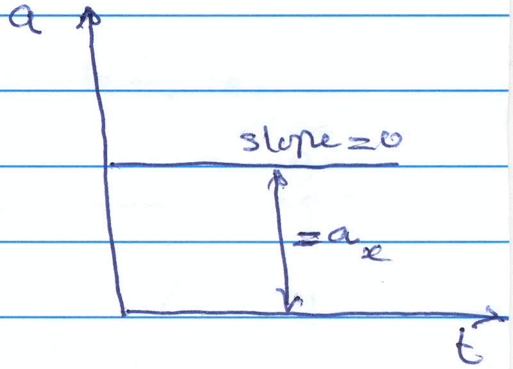


From graph $\Longrightarrow$

$\Rightarrow$ $\bar{v}_x = \frac{1}{2}(v_i + v_f)$

$= \frac{1}{2}(v_i + v_i + a_x t)$

$= v_i + \frac{1}{2} a_x t$



But

$$x_f = x_i + \bar{v}_x t$$

$$\left(\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t - 0} \right)$$

$\Rightarrow$

$$x_f = x_i + \left(v_i + \tfrac{1}{2} a t\right) t$$

$\Rightarrow$

$$\boxed{x_f = x_i + v_i t + \tfrac{1}{2} a t^2} \quad \text{——— (2)}$$

Also $x_f - x_i = \bar{v}(t-0) = \bar{v}\, t$

and

$$\bar{v} = \frac{v_f + v_i}{2} \quad , \quad t = \frac{v_f - v_i}{a_x}$$

$\Rightarrow$

$$x_f - x_i = \left(\frac{v_f + v_i}{2}\right)\left(\frac{v_f - v_i}{a_x}\right)$$

$$= \frac{(v_f + v_i)(v_f - v_i)}{2a_x}$$

$$= \frac{v_f^2 - v_i^2}{2a_x}$$

$\Rightarrow$

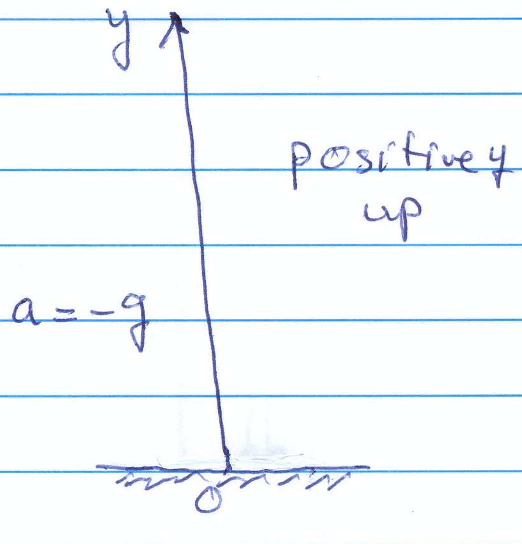
$$\boxed{v_f^2 = v_i^2 + 2a_x(x_f - x_i)} \quad \text{—— (3)}$$

<u>Application</u>: Free falling bodies

$a = -g$ always

$g = 9.81\ m/s^2$

$= 32.2\ ft/s^2$



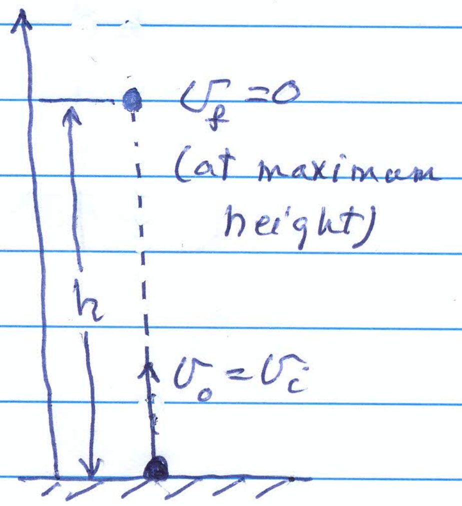
In this case $\Rightarrow$

$$v_f = v_i - gt \quad \text{—— (1)}$$

$$y_f = y_i + v_i t - \tfrac{1}{2} g t^2 \quad \text{—— (2)}$$

$$v_f^2 = v_i^2 - 2g(y_f - y_i) \quad \text{—— (3)}$$

Example: A ball is thrown up with initial velocity $v_i = v_0$. Find the time to reach maximum height h.



We have:

$v_f = 0$

$v_i = +v_0$

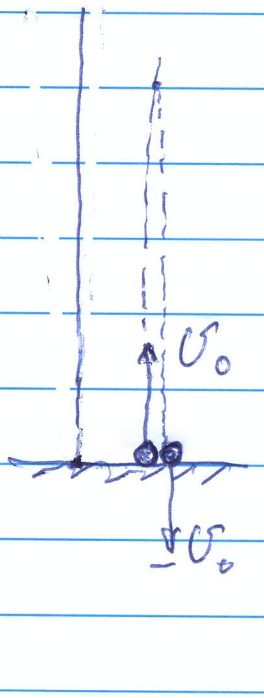
$v_f = v_i - g\,t_{max}$

$0 = v_0 - g\,t_{max}$

$\Rightarrow$

$$t_{max} = \frac{v_0}{g}.$$

Find the time needed to come back to the initial position



In such a case

$v_f = -v_0$

$\Rightarrow$

$v_f = v_i - g\,t_{total}$

$-v_0 = v_0 - g\,t_{total}$

$\Rightarrow$

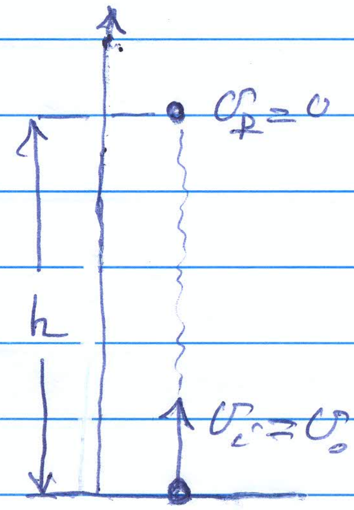
$$t_{total} = \frac{2v_0}{g}$$

This is twice time needed to reach maximum height.

We can also find maximum height.

$y_f = h \quad y_i = 0$

$v_f = 0 \;, \; v_i = +v_0 .$



Use $v_f^2 = v_i^2 - 2g(y_f - y_i)$

$$0 = v_0^2 - 2g(h - 0).$$

$\Rightarrow$

$$\boxed{h = \frac{v_0^2}{2g}}$$